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Incorporating Gabriel graph model for FTTx dimensioning

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Abstract For the realistic generation of synthetic street configurations, used in fiber-to-the-x (FTTx) dimensioning, the Gabriel graph model is proposed. Commencing the analysis with the Primal approach for 100 samples of urban street networks, a great heterogeneity is empirically discovered in their structural properties. Due to the observed morphological complexity, the necessity of a fast abstraction model capturing the complex street patterns is justified. The case study supports the sufficiency of Gabriel graphs for the reproduction of the street networks' basic structural properties such as the average shortest path, the diameter or the average street segment length. The results also demonstrate the sheer superiority of Gabriel graphs for the early estimation of the trenching length of FTTx networks with more than 48 % better accuracy in comparison with the conventional geometric models. Particularly in dense urban areas, the geometric models suffer more serious accuracy shortcomings, whereas the suggested model performs even better.

Keywords Access networks · Complex spatial networks · Fiber-to-the-x (FTTx) · Network planning and design · Optical fiber subscriber loops · Urban morphology

1 Introduction

As the Internet market continues its rapid growth, more and more applications are competing for network bandwidth, and accordingly there are now problems arising from congestion at the network edge. This motivates telecom operators to depart from the existing copper-based technologies and deploy various forms of optical access networks, generally referred to as fiber-to-the-x (FTTx), with the fiber-to-the-building (FTTB) and the fiber-to-the-home (FTTH) being the ones reaching closest to the end-user premises [1].

Regarding the planning of such deployments, it is often spent substantial time and resources to preliminary evaluate telecommunications investments in particular regions or cities. Long techno-economic analyses specific to each area are typical to early determine the feasibility of an FTTx rollout. They usually combine information on telecommunications services' demand and costs with geographic and demographic characteristics and can demonstrate the way that cash flows are affected while upgrading or expanding the broadband access network [2–4].

A major part of such capital investment in a telecommunications network is made in the lower part of the network which connects a subscriber by a physical link to its corresponding Central Office (CO) via intermediate network components. In fixed access networks, it is commonplace that the cables run under streets or pavements in trenches using the road system as a natural guide to reach potential customer locations [1]. Among the deployment costs, trenching is the most crucial in the economics of FTTx, on the grounds that additionally to labor expenditures, any cost for traffic/pedestrian interruption and trenching permits should also be accounted for. According to [2,4] and [5], the cost of digging trenches, installing ducts and cables constitutes by far the largest cost (65–70 %) in an FTTx network deployment.

Since the required trenches depend on the underlying street network, an abstraction of the road system of the installation area is usually taken into consideration in the overall techno-economic methodology to estimate the OutSide Plant

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(OSP) cost [6–8]. The considered abstractions, known as geometric models, are used to estimate important cost components, such as the amount of trenches, ducts, cables and generally the civil works needed in the network deployment. Typically, they assume a regular grid-like structure and resemble lattices. The various analytical models consider highly symmetric graphical models of a uniform customer distribution over a squared or polygon-based area with a recursive structure. The use of geometric models may be optional for the techno-economic evaluation but in the case of the OSP cost estimation of a network installation, it is of the outmost concern, given that an analysis based on a detailed Geographic Information System (GIS), e.g., the FiberPlanIT tool [9] or the AccessPlan tool [10], is rarely a choice as it requires even more time, money and resources. On top of that, GIS approaches cannot be used for the preliminary evaluation of generic scenarios since their street traces are strictly area-specific.

Nonetheless, in practice it can be quite risky to rely on models which regard the spatial structure of the road system to be so simplified and regular. Indeed, recent research demonstrates that the structure of street networks is much more complex [11–14]. Levinson in [15] systematically compares a set of street network structure variables (connectivity, hierarchy, circuity, treeness, entropy, accessibility) across the 50 largest metropolitan areas in the USA and discovers that both network size and structure do vary with city size. Owing to the growth of many world cities over time by accretion rather than being planned from the outset, a regular pattern of even, square or rectangular city blocks is not so common among them. In particular, the urban morphology of an area, in terms of street network topology and geography, may depend on the history, the social processes, the economic activities, the climate, the population density, the introduction of the motor vehicle and many other factors that strongly diversify the overall shape and properties from area to area [12,14,16]. Furthermore, there are broad agreements that the urban patterns affect overlay infrastructure deployment as they define a basic template which strongly constrains the further development of other webs, such as the power grid or communication networks [13].

Due to the existent urban morphological complexity, quantities such as the trenching and fiber lengths, which are fundamental parameters for telecommunications techno-economic models, have been overestimated or underestimated using the conventional geometric models [17–19]. Bearing also in mind the fact that node locations, node capacities, and connection lengths are sensitive to the geometry of the implementation and to the regional specificities, an additional effect probably exists on the quality of service and the overall technical feasibility of the planned installation. This strong dependence of infrastructure network elements and connections on the actual geography of the underlying street network makes the ability to explicitly taking into account the street network in the process of analysis or planning, of the greatest importance for the civil engineering part of the access network. However, a reliable, fast and low-cost model that would capture the irregular street connectivity and in turn contribute to the early FTTx dimensioning is notably missing from literature. The necessity of such an abstraction model in between the conventional geometric models and the GIS solutions is illustrated in Fig. 1.

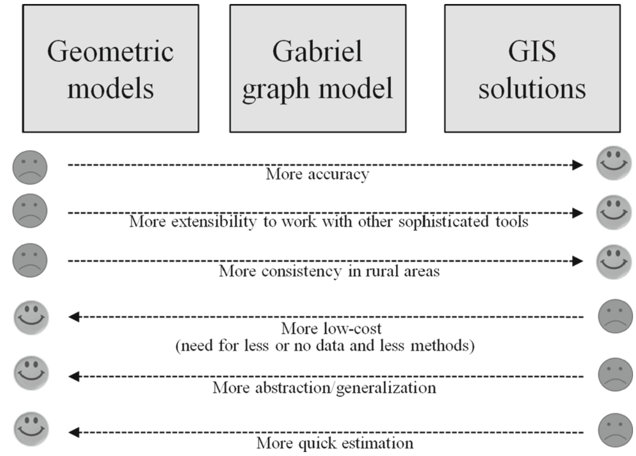

Fig. 1 The necessity of an abstraction model in between the conventional geometric models and the GIS solutions

This paper supports that the use of a graph-based model could offer a better alternative street layout, compared to the imprecise geometric models and the costly area-specific GIS solutions. Both operators and researchers would benefit from employing such a model that conjointly meets accuracy, simplicity and generality. For instance, operators willing to early decide about the feasibility of an infrastructure deployment project, that aims to cover a large set of urban areas, would rather use a trustworthy abstract approach than the labor-intensive GIS-based planning. On top of that, avoiding the demanding GIS data gathering and data preparation at this preliminary phase would be quite saving. Furthermore, researchers active in the area of network performance and techno-economics would rather exploit a credible abstract approach, than base their investigations on unrealistic regular street layouts or limit the validity of their results by adopting case study-oriented analyses of particular areas. Principally, the impact assessment of strategic decisions on FTTx planning (e.g., FTTB vs. FTTH, P2P vs. PON, centralized PON vs. cascaded PON) is expected to gain enhanced accuracy from the use of generic yet realistic street layouts.

The current paper commences with the analysis of 100 samples of urban street networks by employing GIS and Graph Theory. The Primal approach [20] is used to turn GIS data into spatial graphs, by associating nodes to street intersections and edges to street segments. Then, various topological and geographic patterns are observed. At the present paper, willing to extend the previous work done in [14] and [19], the use of the Gabriel graph model [21] is proposed for the realistic generation of synthetic street configurations and for further utilization in a telecommunications techno-economic model. Afterward, unlike previous related studies [17,18] which investigate the behavior of only two existing geometric models, here all the five known geometric models are included and compared, namely the Simplified Street Length [8], the Street Length [8], the Double Street Length [8], the SYNTHESYS [6,7] and finally the TITAN/OPTIMUM [6,7] models. However, a full deployment cost model or an optimized design algorithm for FTTx exceeds the scope of this paper. Studies such as [22] comprehensively elaborate on these thematic areas.

The aim of the present study is to propose a graph model as a novel spatial abstraction for FTTx early dimensioning, so as to replace the use of the defective conventional geometric models. More specifically, the analysis will endeavor to contribute to the following: (1) *demonstrate the complex heterogeneity of real urban street networks* (2) *suggest the use of the Gabriel graph model as a synthetic street network generator and indicate its sufficiency for the representation of realistic street structures and* (3) *justify its better fitting compared to the existing geometric models on the FTTx dimensioning and particularly on the early estimation of the main FTTx cost component – the trenching length.*

The rest of this paper is organized as follows. Section 2 introduces the existing geometric models, whereas Sect. 3 describes the Gabriel graph model. Section 4 presents the empirical analysis of the investigated urban real-street samples and illustrates the Gabriel graph model fitting performance to the observed structural properties. Section 5, which follows, reveals the case study comparison results on the FTTx trenching length estimation. Finally, conclusions are drawn in Sect. 6.

2 Conventional geometric models

In this section, the five most well-established geometric models for abstracting the fixed access network deployment area are presented.

The geometric model for a telecommunications access network makes an abstraction of the installation area under consideration and is used as a starting point to design the telecommunications infrastructure. It is commonly based on a set of parameters such as the customer density, the building density, the area size and the average distance between end users and CO and includes an algorithmic or mathematical approach for calculating key geometry-dependent quantities for the cost analysis, e.g., trenching and fiber lengths in the case of fixed street-based[1] (buried) access networks.

In the following, it is assumed that the optical access network will reach each building termination point (BTP) representing a potential customer location. Thus, the term building is used rather than the terms customer, subscriber, house or home while no distinction among them is necessary, as the focus is limited only in the trenching calculations and not in the fiber length itself. This assumption fits the FTTB configuration although it is the common approach in the conventional geometric models. It is further assumed that all buildings are to be connected and no existing passive infrastructure is taken into account (greenfield deployment). The potential customer base is considered to be uniformly distributed over the regarded area. Where applicable, the area is a square of $1\,\text{km}^2$, with the one side containing n buildings (where $n = \lfloor\sqrt{\text{building density}}\rfloor$) and the entire area including n^2 buildings. The distance between two buildings – also known as inter-building spacing (IBS)—is indicated by l (where $l = \sqrt{\text{area size}}/n$). Although the models are described in detail in [6–8], for the sake of completeness, the basic formulations are presented in the subsections below, whereas the notation from the original papers is preserved where possible.

2.1 The Simplified Street Length model

The Simplified Street Length (SSL) model [8] is a very simplified model, assuming that all buildings can be connected in one line through the middle of the building, as seen in Fig. 2a. This simplified Manhattan model could closely resemble a façade installation of the FTTB/FTTH network. In particular, all streets are connected using one divider street. The structure is fully symmetric horizontally as well as vertically.

Regarding the installation length, each row requires an installation length of $(n-1)\cdot l$, whereas there are n rows. Respectively, the divider street requires installation length of $(n-1)\cdot l$. The above combined give a total length (TL) for installation as follows [8]:

$$\text{TL} = \left(n^2 - 1\right)\cdot l \qquad (1)$$

2.2 The Street Length model

The Street Length (SL) model [8] connects all buildings from one street-wise cable along the street. In the calculation of the analytical model, the cable is located at the middle of the street, as shown in Fig. 2b, but could be easily envisaged at one side of the street as well.

[1] There also exist aerial-based geometric models for FTTx deployment in the literature [8], i.e., the diagonal tree and the simplified Steiner tree.

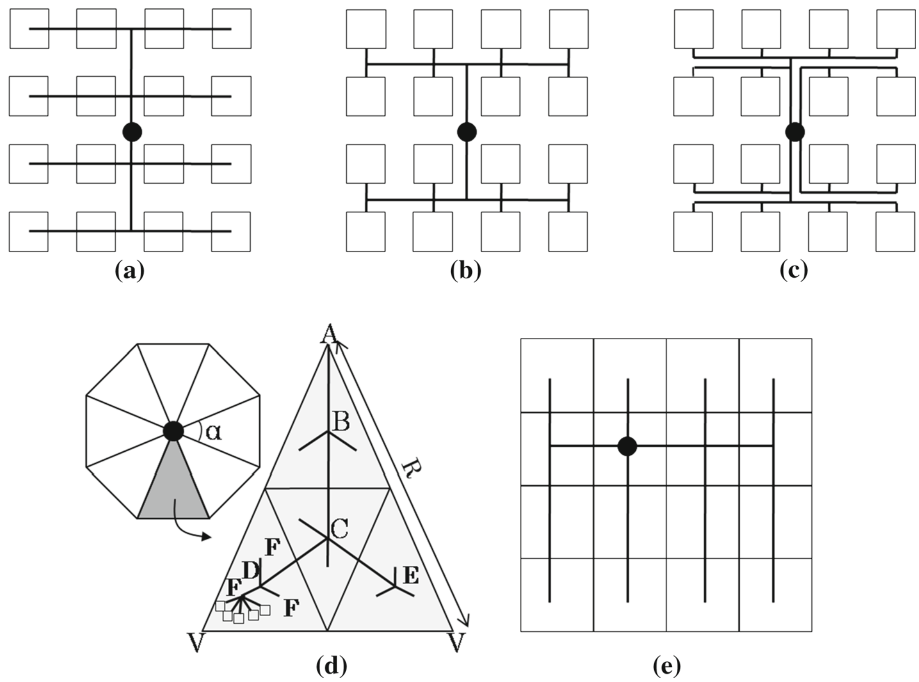

Fig. 2 Indicative illustrations of the five considered geometric models: **a** Simplified Street Length, **b** Street Length, **c** Double Street Length, **d** SYNTHESYS and **e** TITAN/OPTIMUM

With regard to the installation length, this structure can group all buildings per 2. In order to connect all couples of buildings in two adjacent rows (one street), the required installation length is $n \cdot l$, whereas there are $n/2$ such adjacent rows. Likewise, in order to link these previously connected couples of buildings into one fully connected street, the installation length needed is $(n-1) \cdot l$, and again in $n/2$ adjacent rows. In addition, the divider street has a length of $(n-2) \cdot l$. Finally, the above give a total installation length as shown [8]:

$$\mathrm{TL} = \left(n^2 + \frac{n}{2} - 2\right) \cdot l \quad (2)$$

2.3 The Double Street Length model

The Double Street Length (DSL) model [8] considers a street to consist of two sides and therefore reduces the number of the street crossings to a minimum, as in Fig. 2c.

Regarding the installation length, the structure mentioned in SL model for the grouping of two buildings is used once again. Nevertheless, in this case, the adjacent buildings are not directly connected to each other as there is no crossing of the street with distance w. The installation length in this part is $(l-w)$ and there are again $n^2/2$ such adjacent buildings. As for the connection in the rows, an installation length is needed at both sides of the street. In all cases except the upper street side of the upper row and the lower street side of the lower row, an installation length of $(n-1) \cdot l$ is required minus the street width w of the divider street which is not crossed. There are in total $n/2$ streets and n street sides. With respect to the divider street, the installation length at both sides of the street will be the same. Again the horizontal streets are spaced at a distance of $2 \cdot l$ and the length at one side to connect two streets is $2 \cdot l - w$. The number of streets to connect is $n/2$, and the number of connectors (at both sides) is $2 \cdot (n/2-1) \cdot (2 \cdot l - w)$. In this calculation, one should still add the length of installation crossing the streets at both sides for every two streets, except for the top and bottom street, where the street is only crossed at one side. After all, there is also one street crossing for connecting both sides of the street at the CO. In the present paper, it is subsequently assumed that $w = \frac{3}{4} \cdot l$. The trenching length can be calculated by [8]:

$$\mathrm{TL} = \left[\frac{3 \cdot n^2}{2} + n - 4\right] \cdot l - \left[\frac{n^2}{2} + 2 \cdot n - 4\right] \cdot w + (n-1) \cdot w \quad (3)$$

2.4 The SYNTHESYS model

The SYNTHESYS model developed by RACE R1044 project [23] is a polygon-based geometric model for the access network [6,7]. The first-level links consist of A–B, B–C, C–D and C–E links. Second-level links are denoted D–F links. Third-level links are the links between F and the subscriber premises entrance. The abstraction is shown in Fig. 2d.

A set of equations has been derived in order to calculate trenching length estimates at different network levels. In these equations, the following variables are used: d_b denotes the number of buildings per km^2; N_h denotes the number of buildings per hub; n_r is the rank of the polygon; M is the number of potential users per branching box. The radius of the polygon is:

$$\left|\overline{AV}\right|^2 = R^2 = \frac{2 \cdot N_h}{d_b \cdot n_r \cdot \sin(\alpha)} \tag{4}$$

where α is the peak angle of the triangular sector ($\alpha = 360°/n$). Using these variables, length estimates can be expressed as follows:

$$\left|\overline{AB}\right| = \left|\overline{BC}\right| = \frac{R}{3} \cdot \cos\left(\frac{\alpha}{2}\right) \tag{5}$$

$$\left|\overline{CD}\right| = \left|\overline{CE}\right| = \frac{R}{6} \cdot \sqrt{1 + 8 \cdot \sin^2(\alpha/2)} \tag{6}$$

$$\left|\overline{DF}\right| = R \cdot \left(0.132 + \frac{0.336}{n_r}\right) \tag{7}$$

Equations (4)–(6) are directly derived from the geometry, whereas Eq. (7) is obtained by simulation. Next, in order to calculate the average distance l_b between the branching box and building entrance, the relevant surface corresponding to a branching box ($S = M/d_b$) is approximated to be equivalent to a circle, i.e., $\pi \cdot r^2$. Then, this additional distance is:

$$l_b = \frac{2}{3} \cdot \sqrt{\frac{M}{\pi \cdot d_b}} \tag{8}$$

The total trenching length can be obtained by summing up the above distances for all triangles. The formula for the total trenching length is:

$$\begin{aligned}\mathrm{TL} = n_r \cdot \left(\left|\overline{AB}\right| + \left|\overline{BC}\right| + \left|\overline{CD}\right| + \left|\overline{CE}\right| + n_s \cdot \left|\overline{DF}\right|\right) \\ + n_b \cdot l_b\end{aligned} \tag{9}$$

where n_s is the number of splitters and n_b is the number of buildings in the polygon.

2.5 The TITAN/OPTIMUM model

The TITAN [24] and OPTIMUM [25] projects have developed a geometric model, namely the TITAN/OPTIMUM model [6,7], which allows modeling of clustered areas where subscribers are not homogeneously distributed. The topology can either be a star, ring or bus, or be a combination of these. The model is based on a layered structure in which each layer uses the same basic geometric model, although with different parameters. A model layer represents a specific type of flexibility point (FP) and is characterized by FP area density and distribution ratio. The distribution ratio at a given layer or network level represents the number of lower-level flexibility points linked to this flexibility point. Link levels (LL) interconnect the flexibility points of different levels. Total trenching length in the model area derives by simply adding lengths from different layers.

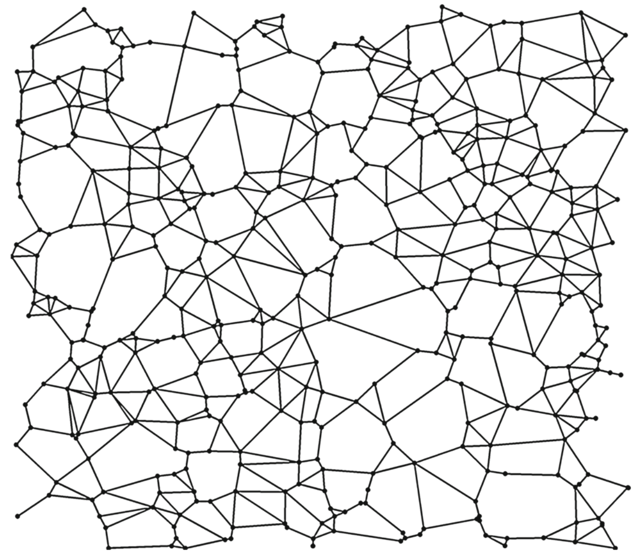

Fig. 3 An instance of artificial Gabriel graph for the representation of urban street network

The basic model area in each layer is rectangular. The area has a length of a units and a width of b units. The total trenching length (star or bus topology) for one layer, as in Fig. 2e, is given by:

$$\mathrm{TL} = a \cdot b - 1 \tag{10}$$

3 The Gabriel graph model

From the family of planar graph models [26] (nearest neighbor graph, minimum spanning tree, relative neighbor graph, Gabriel graph, Delaunay triangulation, planar Erdős-Rényi model, etc.), the Gabriel graphs seem to be visually closer to the layout of real-street networks (Fig. 3).

The Gabriel graphs are named after Gabriel, who introduced them in a paper with Sokal [21]. In this connection scheme, two nodes are connected directly if and only if there are no other nodes that fall inside the circle associated with the diameter that has the two nodes as endpoints. Mathematically, two nodes i and j, from a set of N nodes (N is the only parameter of the model), are connected if the square of the distance between them is less than the sum of the squared distance between each of these points and any other point k. An undirected graph is constructed by adding edges between nodes i and j if for all nodes k, $k \neq i, j$, where d expresses the Euclidean distance:

$$d\,(i,j)^2 \leq d\,(i,k)^2 + d\,(j,k)^2 \tag{11}$$

Put differently, two nodes i and j are connected directly, unless there exists some other node k such that in the triangle ikj, the angle subtended at k is above 90 degrees.

In this model, the installation would follow all streets and connect buildings (subscribers) which are all assumed to be distributed along the streets (graph edges) for simplicity reasons. The total length can be derived by simulations which only require the parameter N. It is thus necessary for the length estimation to be able to calculate the number of intersections N i.e., counting road intersections on the map. It is later seen in Sect. 4 that these simulations result in a simple power-law equation.

The Gabriel graphs are useful in modeling graphs with geographic connectivity that resemble grids [21], but additionally incorporate more complex traits. The Gabriel graph model has already been suggested to capture the structure of telecommunications networks in the physical backbone level [27,28]. Nonetheless, it is quite intriguing to examine whether the Gabriel model can capture the structure of the urban street network—the basis of the access network. From this viewpoint, recent research has already focused on neighboring graphs of Gabriel, or entire graph families. Actually, authors in [29] have investigated how closely the β-skeleton graphs (supersets of the Gabriel graphs) resemble real-street networks and found considerable agreement.

Similar rationale has motivated researchers to use or to invent other models, departing from the conventional geometric ones, though with vague fit to real-street data. For instance, the mathematical framework of stochastic geometry [30] has been used to derive analytical formulas for distributions of connection lengths. The Stochastic Subscriber Line Model (SSLM) [31] is a stochastic–geometric model for fixed access networks. Nevertheless, the estimation success depends on the fitting of optimal tessellations to the considered road system. Furthermore, authors in [32,33] have suggested the use of novel graph models for generating a realistic street network. For the problem of suboptimal Passive Optical Network (PON) design algorithm, authors in [34] have evaluated their method using a Delaunay triangulation graph in order to approximate a road network.

4 Empirical analysis of urban street networks

In this section, the structural properties of urban street networks of varying population/ household/ building density are analyzed. The findings indicate a great heterogeneity on the topological and geometric properties among the street network samples.

As increasing amounts of pervasive geographic data are becoming available, new approaches are suggested [11–13] that make use of Graph Theory and Complex Network Theory to characterize and compare the topology of street networks. Typically, street networks are spatial, that is to say a special class of networks whose nodes are embedded in a two (or three-)-dimensional Euclidean space and whose edges do not define relations in an abstract space, but are real physical connections [12].

4.1 Dataset

The case of Greece is chosen since it is anticipated to exhibit a large differentiation on key determinants of the street morphology, i.e., geographic restrictions (hillslope, soil properties, etc.) and historical development (high urbanisation, etc.), among the individual areas along its long history.

More specifically, in Greece there are more than 6000 municipal departments (MDs) with only a small portion of them able to be considered as urban. Here, it is decided to regard as urban the 100 MDs which have the highest population density. In these MDs, the population density varies between 1200 and 27,000 people/km^2, the household density is between 400 and 10,000 households/km^2, whereas the building density spans between 250 and 3200 buildings/km^2 [35]. Therefore, the street dataset represents a diverse set in terms of population/ household/ building density. There are collected 1-square-kilometer street samples of these 100 distinct urban MDs, as in Fig. 4a. The street data are obtained from the collaborative project OpenStreetMap [36] and the census data from the Hellenic Statistical Authority [35]. The samples are square delimited, in order to introduce an equivalent artificial limit for all samples following a procedure common in relevant studies [12,13].

4.2 The Primal approach

Data from GIS vector maps, using geometrical segments such as points and lines (coordinate pairs or series of coordinate pairs) to represent objects, can rapidly be transformed into graphs. Among the new techniques that have emerged, such as the angular-segment maps [37] and the continuity maps [38], there are two principal modeling approaches which can be applied to represent the street network as a graph: the Primal approach [20] and the Dual approach [39].

In the Primal approach, the streets are turned into spatial, undirected, weighted graphs, where street intersections and end points are represented as nodes while street segments between successive intersections are represented as edges, in the way that is shown in Fig. 4b. The Dual approach is the opposite, where named streets are represented as nodes and the intersections between the streets are represented as graph edges. In recent studies, e.g., [12], as well as in the current paper, the analysis of the real-street networks is based on the production of Primal graphs that encapsulate both topological and geometrical properties of urban networks. The beneficial consideration of the Primal approach lies in its ability to preserve the geometry of the urban space— indispensable for the present study, whereas the Dual approach can only

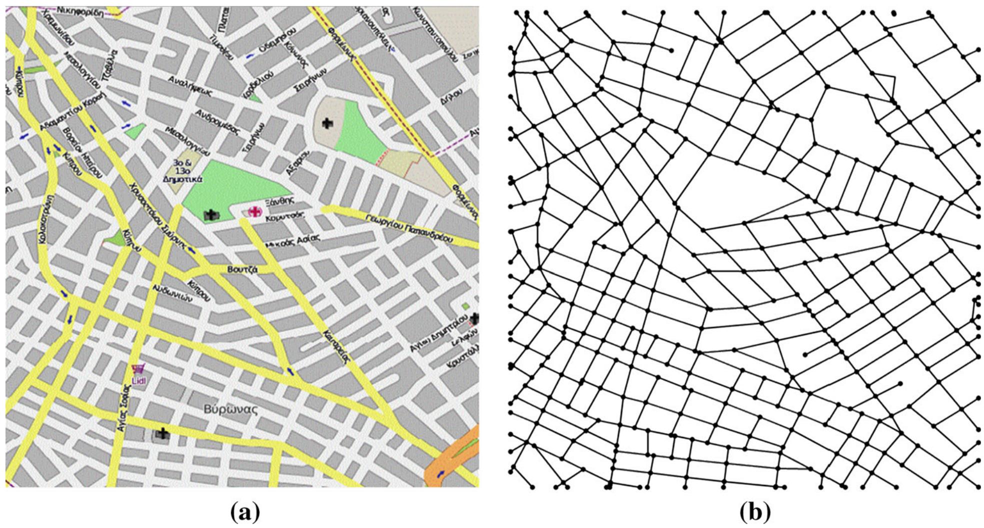

Fig. 4 The sample of Vironas MD: **a** the conventional street map, **b** the corresponding spatial Primal graph

preserve the topological properties with the geometric ones disappearing in the end.

4.3 Utilization of graph theory

The derived spatial network whose nodes are embedded in a two-dimensional Euclidean space can be represented as a graph $G(V, E)$, which consists of a finite set of nodes V and a finite set of edges E. The graph nodes have precise position on the planar map $\{x_i, y_i\}_{i=1,...,|V|}$, while the links follow the footprints of the real streets and are associated with a set of real positive numbers representing the street lengths, $\{l_a\}_{a=1,...,|E|}$. In the following, the graph representing an area is described by the adjacency $|V| \times |V|$ matrix A, whose entry a_{ij} is equal to 1 when there is an edge between i and j and 0 otherwise, and by a $|V| \times |V|$ matrix L, whose entry l_{ij} is the weight (physical length) associated with the edge connecting i and j. In this way, both the topology and the geography metrics are taken into account.

Beyond the number of nodes $|V|$ and edges $|E|$, the basic statistical metrics, which can abstract the properties of a complicated network structure, are calculated [40]; the graph density measures the ratio of the number of edges to the maximum number of possible edges, the average node degree is the average number of edges connected to a node, the average shortest path length is defined as the average number of steps along the shortest paths for all possible pairs of network nodes, and the diameter is the length (in number of edges) of the longest shortest path between any two nodes in the network. Moreover, the average street segment length is the ratio of the total length to the number of edges, where the total length or cost of construction can be quantified [12] by using the measure W defined in formula:

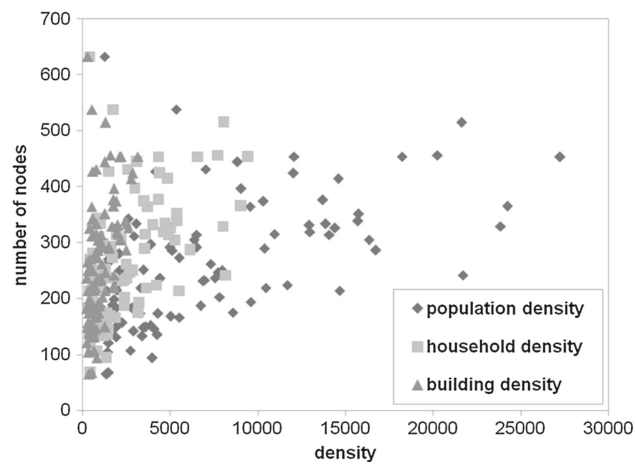

Fig. 5 The number of nodes of the considered samples versus the corresponding population/household/building density

$$W = \sum_{i,j,i \neq j} a_{ij} l_{ij} \qquad (12)$$

4.4 Structural properties and Gabriel graph fit

The basic statistical properties of interest can reveal the differences among the considered urban networks. The peculiar historical, cultural and socioeconomic mechanisms have shaped distinct urban networks in different ways, e.g., in Fig. 5 the diversity in the number of nodes is obvious among the investigated area samples. Their size varies greatly, ranging from 65 to 633 nodes (per km^2), and they all define a single connected component.

Accordingly, considerable variation is observed at all other measures, presented in Figs. 6 and 7, which are cor-

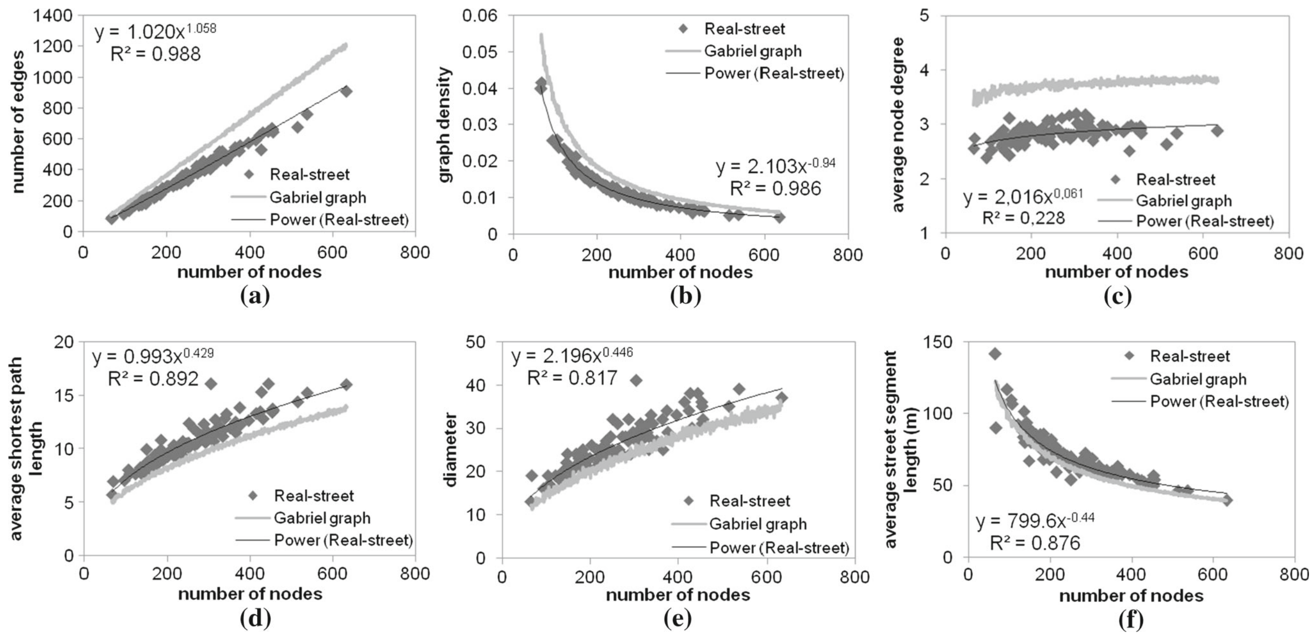

Fig. 6 Empirical findings of topological and geometric patterns in real-street networks (power-law) can confirm the Gabriel graph estimation capability

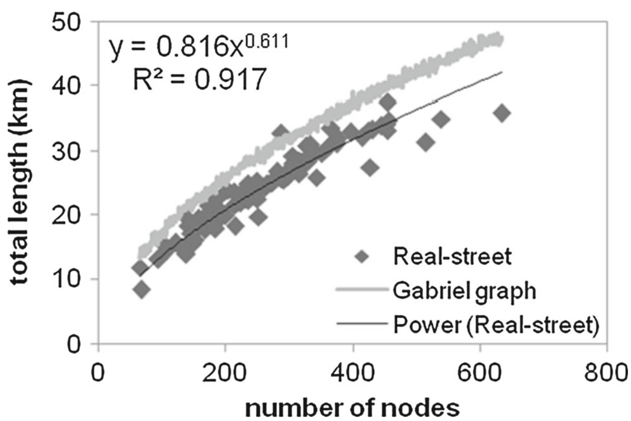

Fig. 7 The total length of real-street networks (power-law) can confirm the Gabriel graph estimation capability

related with the varying number of nodes. The number of edges, the graph density, the average shortest path, the diameter, the average street segment length and the total length all receive values in a great range, and all fit well a power-law with the number of nodes. More specifically, as the number of nodes increases, more edges are constructed and average node degree slightly increases, though on the contrary, the graph density decreases. At the same time, both the average shortest path and the diameter increase their values, while the average street segment length decreases. This strong dependence on the number of nodes, and thus on the overall structure of the street network, is apparent in the total length as well, which increases as the number of nodes increases.

Furthermore, it is deemed worthy to stress that among the examined urban street networks, a large heterogeneity is evident associated with the perceived values in the above set of structural properties. For instance, an area including 100 nodes can have: 130 edges, graph density about 0.02, average shortest path length around 7, diameter near 18, average street segment length of 100 m and total length of 13 km. On the contrary, another area, for example, one comprising of 600 nodes, is associated with: 900 edges, graph density close to 0.005, average shortest path length about 15, diameter close to 38, average street segment length of 40 m and total length of more than 30 km.

Moreover, Table 1 by making use of the statistic Mean Absolute Percentage Error (MAPE), as well as Figs. 6 and 7, all demonstrate that the Gabriel graph model is able to produce synthetic networks[2] quite similar to the real-street networks. The statistical properties of this model are in fine agreement with the observed empirical patterns.

However, the analysis was conducted in a particular country, and it is possible that the discovered patterns are country-specific. Thus, it is intriguing to investigate whether the above good fitting of Gabriel graphs to real-street data is verified in other countries too. Likewise, it is imperative to examine whether the good fitting holds for larger surface samples, given that the 1-square-kilometer surface in the dataset samples might be regarded too small: (1) to eradicate possible

[2] Each point representing a Gabriel graph measurement is the averaged outcome of 100 runs simulating a Gabriel graph of a given size.

Table 1 Gabriel model fitting to the basic street network properties

	Edges/ density/ av. node degree	Av. shortest path length	Diameter	Average street segment length	Total length
MAPE	0.317	0.149	0.139	0.095	0.201

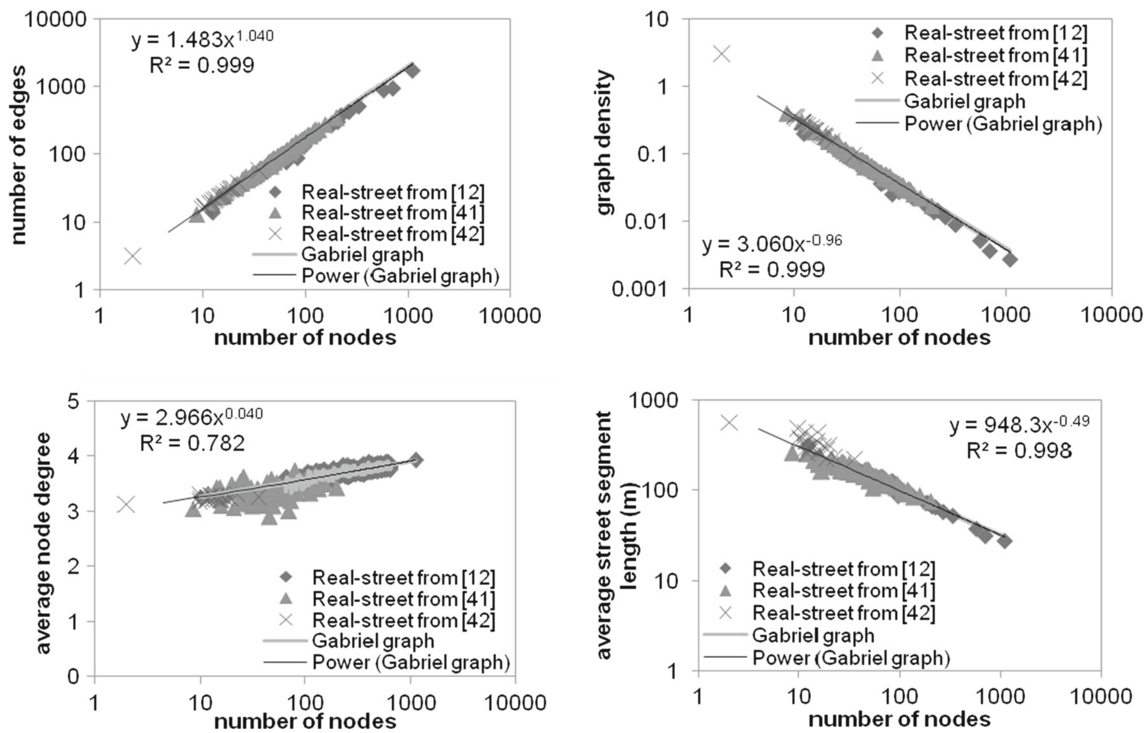

Fig. 8 Empirical findings (log–log plot) of topological and geometric patterns in literature datasets (of larger surface samples) can confirm the Gabriel graph estimation capability (power-law)

side effects (i.e., "edge effects"[3] due to node creation at the samples' boundaries during the clipping process) and (2) to consider it as a typical access network service area. In order to alleviate these concerns, three additional datasets—provided in related empirical studies—are supplementary taken into account [12,41,42]. Their samples are from diverse areas around the world (20 world cities, 118 US urban areas and 21 German cities, respectively) and correspond to large surface areas (2.59, 9.12–38.62 and 141–891 square kilometers, respectively); therefore, they are evidently pertinent to shed light on the above concerns. Though, as only partial information is available for these additional datasets (the area surface size, the number of nodes, the number of edges and the total length), thus Figs. 8 and 9 depict the obtained data along with the Gabriel graph measurements and the associated Gabriel graph regression lines (power-law relation). For comparison reasons, all calculations have been normalized (scaled down)

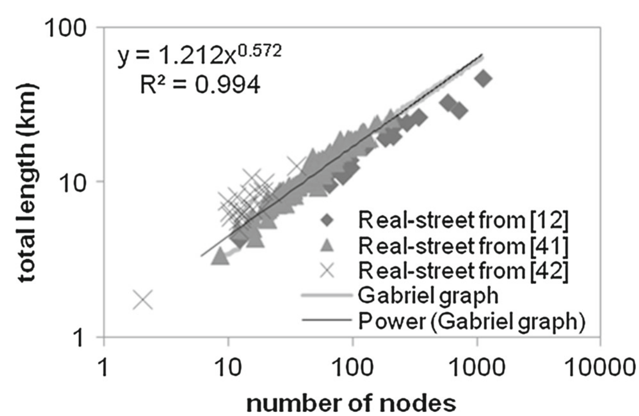

Fig. 9 The total length (log–log plot) of literature empirical data (of larger surface samples) can confirm the Gabriel graph estimation capability (power-law)

to correspond to 1-square-kilometer area. The key findings are repeatedly verified for these real-street datasets (Table 2).

Regarding the crucial property of total length, it appears in Fig. 7 to take values as a function of the number of nodes (intersections) $|V|$. Additionally, the Gabriel graph estima-

[3] These "edge effects" are probably the main factor behind the inversely proportional relation between the samples' surface size and the MAPE referring to the edges/density/average node degree (Tables 1 and 2).

Table 2 Gabriel model fitting to data from literature

	Real-street data from [12]	Real-street data from [41]	Real-street data from [42]
MAPE (total length)	0.249	0.072	0.222
MAPE (edges/ density/ av. node degree)	0.278	0.041	0.021
MAPE (average street segment length)	0.076	0.078	0.186

tion lies quite close to the three real-street calculations, as observed in Fig. 9, and the MAPE statistic, presented in Tables 1 and 2, is up to the level of ~25 %. Despite the fact that the above samples represent very diverse geographic cases, the relation between the total length and the number of nodes can well fit a power-law, i.e., $\sim |V|^p$. As well, the recent report [43], where a street network is sampled at different times of its growth, gives a power-law with a similar exponent value $p = 0.54$.

In the case of Gabriel synthetic graphs, this particular equation, estimating the total length by only using the number of nodes, is:

$$W = 1.212 \cdot |V|^{0.572} \quad (13)$$

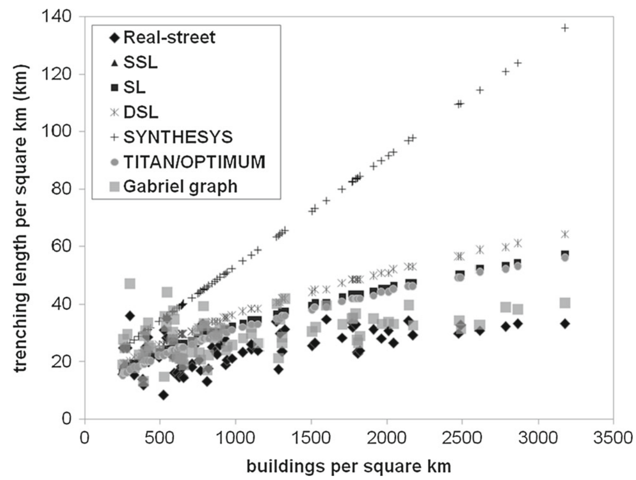

Fig. 10 Trenching length versus building density, with the Gabriel graph fitting best the real-street trenching

5 Estimation of trenching length

In this section, a case study is presented concerning the FTTB/FTTH deployment in the selected 100 urban areas of Greece. All five traditional geometric models along with the Gabriel graph model are applied in order to estimate the required trenching length of the installation.

As mentioned before, the potential customer base in each area is uniformly distributed over a square of dimensions 1 km × 1 km and the CO is situated in the middle of the square. The only exception is the SYNTHESYS model, where—by definition—a polygon of 1 km^2 surface is employed ($n_r = 8$, $n_s = 12$).

As for the real-street trenching length, measured and presented in Figs. 7 and 10, it can take values between 8 and 35 km. However, some remarkable variances among the results of the simple geometric models can be clearly observed, which in turn lead to errors higher than 38.9 %, as seen in Table 3. The numerical results indicate that the solution by the Gabriel model outperforms the existing solutions by the conventional geometric ones. Even more significant is the observation that the proposed Gabriel model leads to an approximately 20 % error that is to say at least 48 % better accuracy (up to 85 %) than any other geometric model. Hence, the models may be ranked from the best to the worst as follows concerning the estimation of trenching length: Gabriel graph, TITAN/OPTIMUM, SSL, SL, DSL and SYNTHESYS.

In addition, it is achievable to quantify the inaccuracy of the considered models, caused by the irregular street connectivity, discriminated in different area types: Semi-Urban (250–500 buildings/km^2), Urban (500–1000 buildings/km^2) and Dense Urban (more than 1000 buildings/km^2). Once again, it is visible that the Gabriel approach offers much higher accuracy than geometric models do, especially in highly populated areas, where the existing models diverge even more from real data (see Fig. 11).

Although in the current study the comparison between the Gabriel graph model and the conventional models refers to one real dataset i.e., Greece, any additional verification for other datasets, corresponding to other countries, is employable but would require supplementary data (mainly the building density per sample) which are not readily available.

As well, it is important to note the restrictive hypothesis of buildings scattered along the entire length of the streets in the Gabriel model calculations. Of course, more realistic spatial distributions may be incorporated to the Gabriel model to define the buildings locations. As the Gabriel graph itself represents the street lines, another layer of points may serve as the building locations. In that case, the point pattern could follow a spatial distribution such as the homogeneous Poisson (also known as complete spatial randomness) or the Ripley's K-function. However, this would require modifying the calculations and transcending the solely mathematical graph

Table 3 Models' fitting to the real-street trenching length

	SSL	SL	DSL	SYNTHESYS	TITAN/ OPTIMUM	Gabriel graph
MAPE	0.415	0.429	0.562	1.326	0.389	0.201

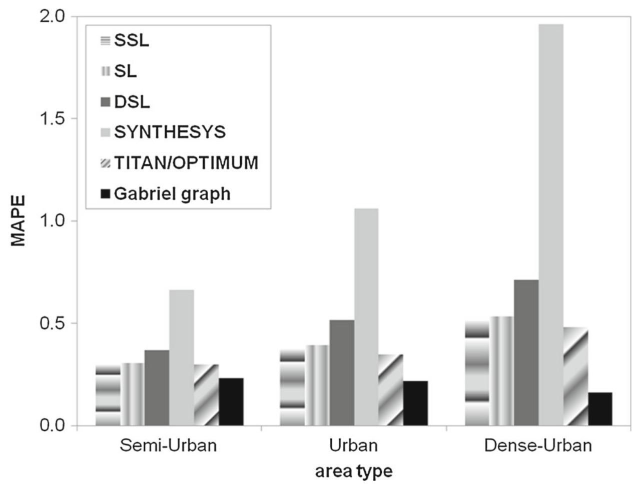

Fig. 11 MAPE behavior under different area types: Semi-Urban (250–500 buildings/km^2), Urban (500–1000 buildings/km^2) and Dense Urban (more than 1000 buildings/km^2)

handling met in this paper. Similar modification would be suggested in a brownfield network deployment case, where the reuse of previous infrastructure becomes an additional challenge to deal with.

6 Conclusion

In the current paper, the Gabriel graph model is proposed and validated by real-street data for the representation of the underlying street network. Besides, the conventional geometric models for FTTx dimensioning are proven inadequate to offer a realistic perspective.

In particular, the real-street topologies are found to possess a remarkable complex heterogeneity in their structure and deviate from simple regular patterns such as square grids. The proposed approach of the Gabriel graph model in contrast to the conventional geometric ones can sufficiently capture the irregular connectivity of the urban street networks in order to obtain reasonably accurate estimates for the key cost quantities of an FTTx installation – exclusively the trenching length quantity. On the other hand, the traditional geometric models are verified to suffer from inaccuracy problems and thus may lead to wrong conclusion of the early techno-economic assessment, especially in dense urban service areas, where striking differences are found between estimations and real-street calculations.

In these terms, the use of the proposed Gabriel model instead of the conventional geometric models not only is of importance to the telecommunications practitioners, but can also provide supplementary contribution to research efforts in the access network field. More specifically, it can be favorably applied in studies where design algorithms are evaluated or implementation scenarios are investigated, such as [2,3,34] (e.g., different splitting ratios, P2P vs. PON). Moreover, the empirical features found here and the proposal of the Gabriel graph model may be useful for the planning of other infrastructure networks whose development occurs in planar constraints and is based on the complex road system as a natural guide to reach customers, such as transport, energy or water networks.

Although the Gabriel graphs appear to satisfactorily fit all examined metrics, there still remains space for exclusively ascribing these metrics to engineering metrics or utilizing them to solve dimensioning problems. As well, the calculation and the accuracy evaluation for the rest of the FTTx cost factors, i.e., fiber length, number of splitters, by employing the Gabriel model is an open issue. Meanwhile, having a legitimate trenching cost, one can form rough estimations of the remaining cost components from the total budget, e.g., by using percentage cost breakdown derived from similar project cases, also known as analogous estimating.

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