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ABSTRACT

In this paper, a class of interconnected multi-user packet communication networks is considered. The common characteristic for all members of that class is the existence of a central node which accepts and processes packets originating from more than one multi-access communication network. The queueing problem that appears in the central node is studied under various approximations of the input traffic to this node. Simulation results are also provided.

I. INTRODUCTION

A lot of work has been done towards the direction of developing communication protocols that determine how a single common resource can be efficiently shared by a large population of users. By now, it is well known that fixed assignment techniques are not appropriate for a system with large population of bursty users. In the latter case, random access protocols are more efficient and many of them have been suggested [1], [2]. Usually, the amount of information transmitted per time is of fixed length, called a packet. In most of the systems, time is divided into slots of length equal to the time needed for a packet transmission (slotted systems).

The deployment of a large number of multi-user random access communication networks, brought up the question of how packets whose destination is another network, should be handled. Thus, the issue of network interconnection or multi-hop packet transmission, arises, [3], [4], [5].

A problem that arises when networks are interconnected is how a random access protocol operates in the presence of a node that forwards exogenous traffic coming from other networks. This problem can be avoided by assigning a separate channel to the exogenous traffic. In the latter case, the operation of the network is not affected by the exogenous traffic and the problem of optimum allocation of the available resources (channels) arises. The latter issue has been discussed in [3], where the objective is

to maximize the throughput of the interconnected networks. In [3], delay analysis was not performed and only simulation results were obtained.

In the next section, some systems of interconnected networks are described. In section III, the general queueing problem that appears in a central node under various approximations of the input traffic to this node, is studied. In the last section, results for the mean time that a packet spends in the central node under the approximations are provided and they are compared with those obtained from the simulation of the actual systems.

II. SYSTEMS OF INTERCONNECTED NETWORKS

We consider a system of two or three packet radio slotted communication networks that operate on the same or neighboring areas (Fig. 1). The users of each network can be either static or mobile.

The obvious structure for the above systems would be the one which assigns a central node to each network. This node receives and retransmits packets coming from the specific network. All networks are assumed to operate in different frequency bandwidths to avoid interference. Thus, the number of different channels that are needed is twice the number of the networks (uplinks, downlinks). The central nodes are interconnected via a backbone network which may operate under a fixed or non-fixed assignment protocol. Packets that are destined outside their network are forwarded by these nodes according to the backbone protocol. The backbone network may use wired or radio channels. In the latter case, a frequency bandwidth should be assigned for that purpose.

A probably more efficient structure for the systems described above is the one in which all networks use a common central node. Each network has its own uplink but the downlink is the same for all of them. This structure saves some downlink channels and offers an efficient solution to the problem of the network interconnection. In fact, the latter is provided almost for free. The packet delay in the common central node turns out to be insignificant due to throughput limitation of the random access protocols. The latter implies that the individually per network assigned downlink channels could be idle most of the time. Another approach to increase the utilization of the indivi-

III. THE QUEUEING PROBLEM

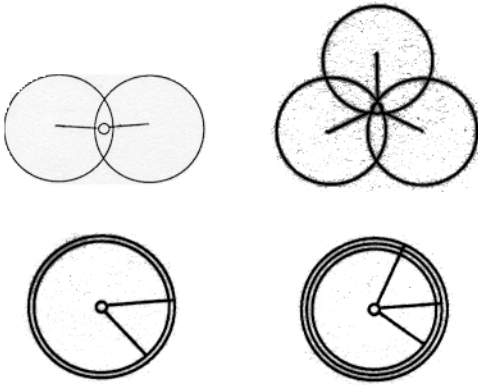


Figure 1.

dual downlinks could be to decrease the transmission rate through these channels. However, this would also increase the packet delay compared to the one induced by a common central node, due to the non-statistical multiplexing of the packets that come from different networks.

Another system of interest is the one in which all networks are separated but the packet destination is another network. Then, a satellite or the input to a transmission line could serve as a common central node (Fig. 2). This configuration is actually a two-hop communication system and can be part of a larger multi-hop system. The work that appears in this paper can be extended to such large multi-hop systems.

A common characteristic of all systems described before is the presence of a central node which is fed by the outputs of more than one random access slotted communication systems. The queueing problem which appears in the central node is hard to analyze. The quantity of interest is the mean time that a packet spends in the central node. The latter quantity is calculated in the next section under various approximations of the input streams to the node, which are the processes of the successfully transmitted packets within the networks.

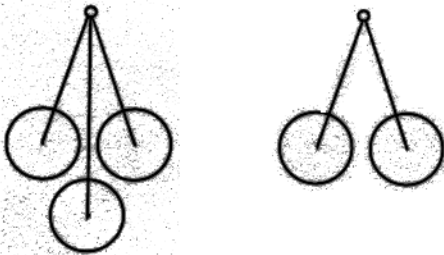


Figure 2.

We consider a discrete time single server queueing system that is fed by N independent input streams (Fig. 3). The service time, T , is constant and equal to one, which is the distance between successive arrival points. The service time policy implies that arriving and departing packets have the same length. The first in - first out policy is adopted and the buffer size is infinite.

When successive arrivals in each input stream are independent, the above queueing system has been studied, [6], and the mean time, D_1 , that a customer spends in the system was then found to be given by the following expression.

$$D_1 = \left[\frac{\sum_{n=1}^N \sum_{m>n}^N \alpha_n \alpha_m}{(1 - \sum_{n=1}^N \alpha_n) \cdot \sum_{n=1}^N \alpha_n} \right] \quad (*)$$

where α_n is the probability of an arrival at the input of the queue, and corresponds to the mean arrival rate of the Bernoulli process that describes the arrival streams.

If the arrival process is a first order ergodic Markov chain with state space $S = \{0, 1\}$, where 1 corresponds to an arrival and 0 to the absence of such an event, then the average time, D_M , that a customer spends in the system is given by the expression below, [7], [8].

$$D_M = \left[1 + \frac{\sum_{n=1}^N \sum_{m>n}^N \alpha_n \alpha_m \left[1 + \frac{\gamma_n}{1-\gamma_n} + \frac{\gamma_m}{1-\gamma_m} \right]}{(1 - \sum_{n=1}^N \alpha_n) \cdot \sum_{n=1}^N \alpha_n} \right] \quad (**)$$

where $\gamma_n = P(1/1) - P(1/0)$ and where α_n is the arrival rate given by the expression

$$\alpha_n = \frac{P(1/0)}{1 - \gamma_n}$$

$P(/)$ denotes conditional probability.

In this section, it is assumed that each input stream is described by a finite-state Markov chain. The cardinality of the state space of the Markov chain associated with the i^{th} stream is denoted by M_i . The arrival process of each input stream corresponds then to a mapping from a finite-state Markov chain onto the set $\{0,1\}$, where 1 represents a single arrival and where 0 represents no arrival. In this case, as it will be shown, the average time that a customer spends in the system can be obtained from the solution of $M_1 \times M_2 \times \dots \times M_N$ linear equations.

Clearly, the Markov arrival system described in [7] is a special case of the general system considered in this paper. In [7], the underlying Markov chain has two states only, and actually coincides with the arrival process. The closed form solution obtained in [7], for the average time

that a customer spends in the system, does not extend to the case where multi-state Markov chains are present. The two state Markov model gives rise to a second order equation, whose roots are used in the derivation of the closed form solution. This procedure does not extend to a larger state space Markov model, since then expressions for the roots of high order equations would be needed.

The queuing system with Bernoulli arrivals, is also a special case of the system considered in this paper. In that case, the arrival process coincides with the underlying Markov process and the state transition probabilities are properly selected. Then, D_1 can be derived from the solution of 4 linear equations.

The arrival processes $\{a_i^j\}_{j \geq 0}$, $i = 1, 2, \dots, N$, are assumed to be synchronized discrete time processes, and at most one arrival can occur in each input line per unit time. The time separation between successive possible arrival points is constant and equal to one. The arrival processes $\{a_i^j\}_{j \geq 0}$, $i = 1, 2, \dots, N$, represent the output processes of

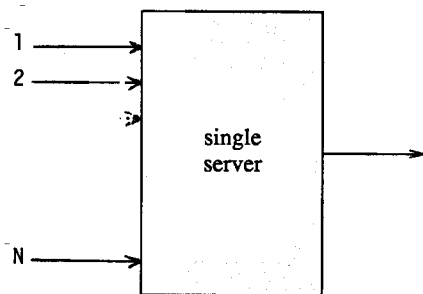


Figure 3.

multi-user random access slotted communication networks, where the arrival points coincide then with the ends of slots. It is obvious that we can have at most one packet arrival per input stream and per unit time. More than one arrivals (from different input streams) that occur at the same arrival point are served in a randomly chosen order.

Let $\{x_i^j\}_{j \geq 0}$ denote a discrete time ergodic Markov process associated with the i^{th} input stream, with finite state space $S^i = \{\sigma_i^1, \dots, \sigma_i^{M_i}\}$. Let also a_i be a stationary mapping rule from the set S_i onto the set $\{0, 1\}$, where 1 corresponds to an arrival and 0 to the absence of such an event. Then the arrival process of the i^{th} input stream is

$$\{a_i^j\}_{j \geq 0} = \{a_i(x_i^j)\}_{j \geq 0} = \{a_i(x_i^0), a_i(x_i^1), \dots\}.$$

From the description of the arrival process it is implied that successive arrivals from the same input stream are not independent, but they are governed by an underlying finite state Markov chain, $\{x_i^j\}_{j \geq 0}$, and a stationary mapping rule a_i .

In this system, it is assumed that the processes

$\{x_i^j\}_{j \geq 0}$, $i = 1, 2, \dots, N$, are mutually independent and thus the arrival processes $\{a_i^j\}_{j \geq 0}$, $i = 1, 2, \dots, N$, are also independent. If $\{b^j\}_{j \geq 0} = \{b^0, b^1, \dots\}$ is the process that describes the total arrivals occurring at a single arrival point, then

$$b^j = \sum_{i=1}^N a_i(x_i^j), \quad j = 0, 1, 2, \dots \quad (2)$$

and $b^j \in \{0, 1, 2, \dots, N\}$.

Referring to the interconnected multi-user random access communication networks, we may define $\{\bar{x}_i^j\}_{j \geq 0}$ to be the process that describes the state of the i^{th} channel at the end of a slot. Let us consider a ternary channel state space

$$S^i \equiv \{0, 1, 2\},$$

where 0, 1 and 2, respectively, denote that 0, 1, or more than 1 packets attempted packet transmission in a single slot. Since a packet appears in the output process only if it is the only one transmitted within the corresponding slot, the arrival process $\{a_i^j\}_{j \geq 0}$ can be clearly described via the mapping

$$a_i(\bar{x}_i^j) = \begin{cases} 1 & \text{if } \bar{x}_i^j = 1 \\ 0 & \text{if } \bar{x}_i^j = 0, 2 \end{cases}$$

The process $\{\bar{x}_i^j\}_{j \geq 0}$ is controlled by the deployed random-access algorithm, and is generally non-Markov. However, this process can be approximated by a Markov process $\{x_i^j\}_{j \geq 0}$ which has the same state space as $\{\bar{x}_i^j\}_{j \geq 0}$ and is ergodic within the stability region of the random-access algorithm.

Let $\pi_i(k)$ and $p_i(k, j)$, $k, j \in S^i$, denote the steady state and the transition probabilities of the ergodic Markov chain, $\{x_i^j\}_{j \geq 0}$, $i = 1, 2, \dots, N$. Let also $p^n(j; \bar{y})$ denote the joint probability that there are j packets in the system at the n^{th} arrival point (arrivals at that point are included) and the states of the Markov chains are y_1, y_2, \dots, y_N , where $\bar{y} = (y_1, y_2, \dots, y_N)$. The vector \bar{y} describes the state of a new ergodic Markov chain that is generated by the N independent Markov chains described before, with steady state and transition probabilities $\pi(\bar{y})$ and $p(\bar{x}, \bar{y})$ respectively, and with state space $\bar{S} = S^1 \times S^2 \times \dots \times S^N$.

The operation of the system can be described by an $N + 1$ dimensional (infinite state space) Markov chain imbedded at the arrival points, with state space $T = (0, 1, 2, \dots) \times \bar{S}$ and state probabilities given by the following equations

$$p^n(j; \bar{y}) = \sum_{\bar{x} \in \bar{S}} p^{n-1}(j+1 - \sum_{i=1}^N a_i(x_i); \bar{x}) p(\bar{x}, \bar{y}), \quad j \geq N+1$$

or

$$p^n(j; \bar{y}) = \sum_{k=1}^{j+1} \sum_{\bar{x} \in F_{j+1-k}} p^{n-1}(k; \bar{x}) p(\bar{x}, \bar{y}) + \sum_{\bar{x} \in F_j} p^{n-1}(0; \bar{x}) p(\bar{x}, \bar{y}), \quad 0 \leq j \leq N,$$

where

$$F_v = \{\bar{x} = (x_1, x_2, \dots, x_N) \in \bar{S} \mid \sum_{i=1}^N a_i(x_i) = v\}. \quad (4)$$

There are totally $M_1 \times M_2 \times \dots \times M_N$ equations given by (3) for a fixed j and all $\bar{y} \in \bar{S}$, where M_i is the cardinality of S^i , $i = 1, 2, \dots, N$.

The original assumption concerning the ergodicity of the Markov chains associated with the input streams implies the ergodicity of the arrival processes $\{a_i\}_{i \geq 0}$, $i = 1, 2, \dots, N$. The latter together with the well known condition, [9],

$$\sum_{i=1}^N \sum_{x_i \in S^i} \pi_i(x_i : a_i(x_i) = 1) < 1, \quad (5)$$

imply that the Markov chain described in (3) is ergodic and there exist steady state (equilibrium) probabilities. Thus, we can consider the limit of the equations in (3) as n approaches infinity and obtain similar equations for the steady state probabilities.

By considering the generating functions of the steady state probabilities given by (3) and manipulating the resulting equations, we obtain the following system of linear equations

$$P(z; \bar{y}) = \sum_{v=0}^{\infty} \sum_{\bar{x} \in F_v} z^{v-1} [P(z; \bar{x}) + (z-1)p(0; \bar{x})] p(\bar{x}, \bar{y}), \quad \bar{y} \in \bar{S}, \quad (6)$$

where $P(z; \bar{y})$ is the generating function of the steady state distribution of the $N+1$ dimensional imbedded Markov chain.

From the independence of the Markov chains associated with the input streams and the state description of the imbedded $N+1$ dimensional Markov chain, it is obvious that

$$\pi(\bar{x}) = \prod_{i=1}^N \pi_i(x_i), \quad p(\bar{x}, \bar{y}) = \prod_{i=1}^N p_i(x_i, y_i), \quad (7a)$$

$$p(0; \bar{x}) = p_0 \prod_{i=1}^N \pi_i(x_i). \quad (7b)$$

If $P(z)$ is the generating function of the distribution of the number of packets in the system, then

$$P(z) = \sum_{j=0}^{\infty} p(j)z^j = \sum_{j=0}^{\infty} \sum_{\bar{y} \in \bar{S}} p(j; \bar{y})z^j = \sum_{\bar{y} \in \bar{S}} P(z; \bar{y}) \quad (8a)$$

$$P'(z) = \sum_{\bar{y} \in \bar{S}} P'(z; \bar{y}), \quad (8b)$$

where $P'(z; \bar{y})$, $\bar{y} \in \bar{S}$, can be derived by differentiating (6) and are given by

$$P'(z; \bar{y}) = \sum_{v=0}^{\infty} \sum_{\bar{x} \in F_v} (v-1)z^{v-2} [P(z; \bar{x}) + (z-1)p(0; \bar{x}) +$$

$$+ z^{v-1} [P'(z; \bar{x}) + p(0; \bar{x})] p(\bar{x}, \bar{y}), \quad (9)$$

for $\bar{y} \in \bar{S}$.

Since $P'(1)$ is the average number of packets in the system, Q , from (8b) we have that

$$Q = \sum_{\bar{y} \in \bar{S}} P'(1; \bar{y}). \quad (10)$$

$P'(1; \bar{y})$, $\bar{y} \in \bar{S}$, are in fact the solutions of the following $M_1 \times \dots \times M_N$ dimensional linear system of equations, which are obtained from (9) by setting $z=1$:

$$P'(1; \bar{y}) = \sum_{v=0}^{\infty} \sum_{\bar{x} \in F_v} \left[(v-1)P(1; \bar{x}) + P'(1; \bar{x}) + p(0; \bar{x}) \right] p(\bar{x}, \bar{y}), \quad \bar{y} \in \bar{S}, \quad (11)$$

where

$$P(1; \bar{x}) = \pi(\bar{x}) = \prod_{i=1}^N \pi_i(x_i) \quad (12)$$

The $M_1 \times \dots \times M_N$ linear equations with respect to $\bar{y} \in \bar{S}$ that appear in (11) are linearly dependent. This is usually the case when the equations have been derived from the state transition description of a Markov chain. By summing up the equations in (11), solving with respect to $P'(1)$, and using L' Hospital's rule we obtain an additional linear equation with respect to $P'(1; \bar{y})$, $\bar{y} \in \bar{S}$, which is linearly independent from those in (11) and is given by

$$\sum_{v=0}^{\infty} \sum_{\bar{x} \in F_v} \left[2(v-1)P'(1; \bar{x}) + (v-1)(v-2)P(1; \bar{x}) + 2(v-1)p(0; \bar{x}) \right] = 0. \quad (13)$$

By summing up the equations in (9), solving with respect to $P(1)$, and using L' Hospital's rule, we calculate the steady state probability that there is no packet in the system, denoted by p_0 . As it was expected it was found that

$$p_0 = 1 - \sum_{i=1}^N \sum_{x_i \in S^i} \pi_i(x_i : a_i(x_i) = 1). \quad (14)$$

By substituting (7b), (12) and (14) into (11) and (13) and solving the $M_1 \times \dots \times M_N$ dimensional linear system of equations that consists of (13) and any $M_1 \times \dots \times M_N - 1$ equations taken from (11), we compute $P(1; \bar{x})$, $\bar{x} \in \bar{S}$. Then, the average number of packets in the system can be computed by (10).

The average time, D , that a packet spends in the system can be obtained by using Little's formula and it is given by

$$D = \frac{Q}{\sum_{i=1}^N \sum_{x_i \in S^i} \pi_i(x_i : a_i(x_i) = 1)} \quad (15)$$

The denominator in the above expression corresponds to

the total input traffic to the queueing system.

IV. RESULTS AND CONCLUSIONS

As an example, we consider systems of networks in which a binary feedback (c/nc) limited sensing collision resolution algorithm is deployed. A description of this algorithm whose maximum stable throughput is .36, can be found in [10], [11]. The steady state and state transition probabilities of the output process of such networks have been calculated in [12].

The values of the mean time that a packet spends in the central node under the various approximations on the input traffic to the central node (i.e. the output traffic from the networks), are shown in Tables 2 and 3. The simulation results of the actual system together with the network induced packet delay, [10], appear there as well. It can be observed that the 3-state Markov model performs very well, especially when the total traffic to the central node is less than .99. For input per stream traffic less than .25, all examined models give good results for practical purposes. When the network traffic increases beyond .25, the number of collisions within a network becomes significant. In the latter case only the 3-state Markov model performs well

λ = Poisson input rate to a network

λ	Indep.	2-Markov	3-Markov	Sim.	Net. Del.
.10	1.06	1.02	1.03	1.00	1.97
.20	1.08	1.17	1.09	1.02	3.33
.25	1.25	1.14	1.03	1.05	5.30
.30	1.37	1.23	1.10	1.12	11.38
.33	1.48	1.31	1.21	1.21	30.00
.35	1.58	1.41	1.31	1.30	87.70

Table 1.(N=2)

λ	Indep.	2-Markov	3-Markov	Sim.	Net. Del.
.10	1.14	1.05	1.06	1.02	1.97
.20	1.50	1.24	1.37	1.21	3.33
.25	2.00	1.55	1.91	1.70	5.30
.30	4.00	2.82	4.24	4.27	11.38
.31	5.43	3.74	5.94	6.25	15.00
.32	9.00	6.03	10.16	11.37	20.00
.33	34.00	22.34	39.87	48.89	30.00

Table 2.(N=3)

since it is the only one of the suggested models which can distinguish between idle and collided slots.

The results show that the packet delay in the central node is of the order of one slot (2 networks), or less than a third of the network induced packet delay (3 networks).

Thus, the usage of a single downlink in the networks of Fig. 1, is justified. The additional delay is small compared to the network induced delay. Furthermore, the frequency bandwidth that is saved can be used to increase the throughput/delay performance of the system.

In a two-hop environment in which the network topologies that are shown in Fig. 1 and Fig. 2 are present, the 3-state Markov model can be used to approximate the process of the successfully transmitted packets within a network. Then, the packet delay in the central node can be calculated by following the procedures described in this paper. The queueing system that has been analyzed, can also model a central node which accepts and processes packets coming from any number of networks, and whose destination is another network. In this case, it is assumed that only a portion of the successfully transmitted packets within a network are forwarded to the central node, whose total input traffic must be less than one packet per packet length.

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