ON THE APPROXIMATION OF THE OUTPUT PROCESS OF MULTI-USER RANDOM ACCESS COMMUNICATION NETWORKS

I. Stavrakakis, D. Kazakos
Department of Electrical Engineering
University of Virginia
Charlottesville, VA 22901

Abstract

In this paper, Bernoulli and first-order Markov processes are used to approximate the output process of a class of slotted multi-user random-access communication networks. The parameters of the approximating processes are analytically calculated for a network operating under a specific random access algorithm. The mean time that a packet spends in the central node of the star topology is calculated under the proposed approximations of the output processes of the interconnected networks. The results are compared with simulation results of the actual system. It turns out that the memoryless approximation gives satisfactory results up to a certain per network traffic load. Beyond that point, the first order Markov process performs better.

I. Introduction

A lot of work has been done towards the direction of developing communication protocols which determine how a single common resource can be efficiently shared by a large population of users. By now, it is well known that fixed assignment techniques are not appropriate for a system with large population of bursty users. In the latter case, random access protocols are more efficient and many of them have been suggested [1], [2]. Usually, the amount of information (in bits) transmitted per time is of fixed length, called a packet. In most of the systems, time is divided into slots of length equal to the time needed for a packet transmission (slotted systems).

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The deployment of an ever increasing number of multi-user random access communication networks brought up the question of how packets, whose destination is another network, should be handled. Thus, the issue of network interconnection or multi-hop packet transmission, arises, [3], [6], [7].

The basic problem in analyzing interconnected systems is that of characterizing the output process of a multi-user random access communication system; i.e., the departure process of the successfully transmitted packets. Another problem is how a random access protocol operates in the presence of a node that forwards exogenous traffic coming from other networks. The latter problem can be avoided by assigning a separate channel to the exogenous traffic. In this case, the operation of the system is not affected by the exogenous traffic but the problem of optimum allocation of the available resources (channels), arises. The latter issue has been discussed in [3], where the objective is to maximize the throughput of the interconnected networks. In [3], delay analysis was not performed and only simulation results were obtained.

The output process of a multi-user random-access communication system depends on the protocol that has been deployed. Description of that process is a difficult task and only approximations based on special assumptions have been attempted, [4]-[7].

In this paper we model the output process as a Bernoulli and as a discrete-time first-order Markov process. The detailed description and the motivation behind these approximations are given in the next section.

In section III we introduce the class of random access algorithms whose output processes are approximated. An algorithm from that class is discussed in detail and the parameters of the approximating processes are analytically calculated for the specific algorithm.

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In section IV, a measure of the performance of the proposed approximations is introduced. In section V, the results for the performance of the proposed approximations are given under the employment of the specific algorithm described in section III.

II. The approximation of the output process

A slotted multi-user random access communication network is considered. It is assumed that the packet input rate to the system is λ packets per slot and that λ is in the stability region of the system. For such values of λ, all the processes associated with the description of the system are stationary.

In any system as the above, the communication channel can be in one of the following states: I (idle) if no user is using the channel at that time; S (success) if only one user is transmitting; C (collision) if more than one users are transmitting at that time. In the above channel description we have made the assumption that if only one user transmits, then his packet is considered as a successful one. Successfully transmitted packets appear in the output process of the network, while I and C states of the channel do not result in an output from the network. It is assumed that capture events are not present, [18], and that channel errors cannot occur.

We define the output process, $\{a^j\}_{j\geq 0}$, to be a discrete-time binary process associated with the end of the slots of a slotted multi-user random-access communication system. The random variable a^j takes the value 1 if a successful packet transmission took place in the j^{th} slot, and takes the value 0 otherwise. It is clear that the output process can be interpreted as a two-state channel-status process $\{x^j\}_{j\geq 0}$, where $x^j \varepsilon \{S,NS\}$; by NS we denote the union of the states I and C. The purpose of this interpretation of the output process is to relate it to the channel-status, which is in interaction with the random-access protocol. The latter is true since the evolution of the channel-status process (which determines completely the output proces) depends on the current (and possibly the past) channel state and the employed protocol. This happens because the state of the channel is fed back to the users, who determine their action based on this feedback and the protocol. A type of feedback information is needed by any random-access algorithm.

From the above discussion turns out that the output process, $\{a^j\}_{j\geq 0}$, and the two-state channel-status process, $\{x^j\}_{j\geq 0}$, are identical. The problem of characterizing the output process of a multi-user random-access communication network is identical to that of characterizing the channel-status process, $\{x^j\}_{j\geq 0}$. From now on, we will be referring to the process $\{x^j\}_{j\geq 0}$ rather than to the process $\{a^j\}_{j\geq 0}$, to emphasize the dependence of the output process on the employed random access algorithm and understand qualitatively the implications of the proposed approximations.

Let $\{y^j\}_{j\geq 0}$ be a Bernoulli-type process with probability of success λ; S is the successful event and NS is its complement. By approximating $\{x^j\}_{j\geq 0}$ by the process $\{y^j\}_{j\geq 0}$, we actually assume that the state of the channel at the current slot is independent from the channel state in the previous slot. In a random-access algorithm operating under light traffic, the collision resolution algorithm is "idle" most of the time, since packet collisions are extremely rare. Since it is the collision resolution algorithm that introduces the dependencies among channel states in seccessive slots, it is implied that the Bernoulli process is a reasonable approximation of the channel-status (output) process. Under moderate or heavy traffic load, the collision-resolution algorithm is in effect. In this case, at least intuitively, the Bernoulli-type approximation is not pleasing.

Under moderate and especially under heavy traffic load, the dependencies introduced by the collision resolution algorithm are strong and extend beyond successive slots. We will try to capture some of these dependencies by proposing a discrete-time first-order Markov process, $\{z^j\}_{j\geq 0}$ to approximate the channel-status (output) process $\{x^j\}_{j\geq 0}$; the state space of the proposed Markov process is $\{S,NS\}$. We expect that the Markov approximation will perform better than the Bernoulli-type one, under heavy traffic load.

So far we have not made any assumptions on the type of random-access algorithm which is used in the network. Thus, the previous discussion concerning the characterization of the output process makes sense for any multi-user random-access communication network. The single parameter of the Bernoulli-type process, i.e. the probability of having a successful packet transmission, is trivially calculated for any random access algorithm; it equals the packet input traffic rate, λ, under stable operation of the network. The steady-state probabilities of the discrete-time Markov process, $\{z^j\}_{j\geq 0}$, are also trivially calculated. If $\pi(S)$ is the steady-state probability of the channel process being in state S, and $\pi(NS)$ is the corresponding steady-state probability for the state NS, then it is obvious that $\pi(S)=\lambda$ and $\pi(NS)=1-\lambda$.

The method to analytically calculate the transition probabilities of the discrete-time Markov process, $\{z^j\}_{j\geq 0}$ depends on the random-access algorithm employed. In the next section, a specific random-access algorithm is considered and the transition probabilities are calculated. The method developed can be applied to most of the limited-sensing random access algorithms, [12], [13]. We speculate that the method can be applied to any algorithm whose analysis is based on the concept of the session (explained in the next section) and which operates in statistically identical cycles of finite length. The class of such algorithms is large and includes many well known random-acces algorithms, [14], [19], [20].

III. Transition probabilities for a limited-sensing random-access algorithm

We consider multi-user random-access slotted communication networks in which a binary-feedback, (collision/non-collision, C/NC), limited-sensing collision-resolution algorithm is deployed. The input traffic to the network is assumed to be Poisson with intensity λ packets per slot. This algorithm has been developed and analyzed in [11] and [10]. There, analysis was limited to the derivation of the maximum stable throughput and the average packet delay. In [21] analysis was extended to the calculation of other quantities of interest as well. The characterization of the process of the successfully transmitted packets, i.e. the output process of the network, is still an open problem.

A brief description of the collision-resolution algorithm is provided at this point. Each user is assigned a counter whose initial value is zero (no packet to be transmitted). This counter is updated according to the steps of the algorithm and the feedback from the channel. Upon packet arrival, the counter content increases to one. Users whose counter content is equal to one at the beginning of a slot, transmit in that slot. If the channel feedback is collision (C), the counters whose content is greater than one increase it by one; the counters whose content is one maintain this value with probability p (splitting probability) or increase it to two with probability 1-p. If the channel feedback is non-collision (NC), all non-zero counters decrease their content by one. A detailed description of the algorithm can be found in [10], [11].

Most of this section is devoted to the calculation of the transition probability p(S/NS). The rest of the transition probabilities are, then, trivially calculated at the end of this section. To calculate the transition probability p(S/NS) it seemed easier to compute the joint probability p(NS,S) at first; i.e. the probability of having an NS slot followed by an S slot in the approximating process $\{z^j\}_{j\geq 0}$. The transition probability p(S/NS) is then calculated from the joint probability p(NS,S) and the steady-state probability π(NS). The joint probability is calculated as the probability that a pair (NS,S) of consecative slots occurs in the channel-status process, $\{x^j\}_{j\geq 0}$, under stable operation of the network.

An important quantity for the analysis of most of the limited and continuous-sensing random-acces algorithms is the session. A session is defined as the time interval between two renewal points of the operation of the system. The length of such sessions is easy to describe via recursive equations. The multiplicity of a session is defined as the number of packet transmission attempts in the first slot of the session. The following quantities are useful in the analysis that is presented in this section.

(NS,S) pair : A pair of consecative slots with the first slot being in state NS and the second in state S.

internal (NS,S) pair : An (NS,S) pair is internal if both slots belong to the same session.

l_k : Length of a session of multiplicity k (in slots).

L_k : Expected value of l_k.

L : Expected value of L_k with respect to k.

$\tau_k^{NS,S}$: Number of internal (NS,S) pairs in a session of multiplicity k.

$T_k^{NS,S}$: Expected value of $\tau_k^{NS,S}$.

$T^{NS,S}$: Expected value of $T_k^{NS,S}$ with respect to k.

i_k : A random variable associated with the last slot of a session of multiplicity k; i_k=1 if that slot is idle; i_k=0 if that slot is involved in a successful transmission.

I_k : Expected value of i_k.

I : Expected value of I_k with respect to k.

As it will become clear later, an important quantity for the calculation of the joint probability p(NS,S) is the mean session length, L. The latter can be calculated by following procedures similar to those that appear in [11], [12], [13], [14]. In fact, for the specific algorithm under consideration, L has been calculated in [10] and [11]. We believe that the recursive equations with respect to l_k which describe the operation of the system will be very helpful for the better understanding of the procedure for the calculation of p(NS,S). For this reason we start by calculating L.

From the description of the algorithm the following equations can be written, with respect to l_k, k=1,2,... .

$$l_0 = 1 \quad , \quad l_1 = 1 \qquad (1a)$$

$$l_k = 1 + l_{\phi_1 + f_1} + l_{k-\phi_1 + f_2} \quad , \quad k \geq 2. \qquad (1b)$$

f_1 and f_2 come from two independent Poisson random variables over T=1 (length of a slot) with probability function $P_f(\,)$ and intensity λ ; ϕ_1 comes from a Binomial with parameters k and p (p = .5) and probability function $b_k(\,)$. Equation (1b) can be explained as follows: The length of a session of multiplicity $k \geq 2$ consists of the slot wasted in the collision, plus the length of the sub-session of multiplicity ϕ_1+f_1 (which will be initiated in the next slot), plus the length of the sub-session of multiplicity $k-\phi_1+f_2$, (which will be initiated after the end of the sub-session of multiplicity ϕ_1+f_1). Sub-sessions are statistically identical to the sessions of the same multiplicity. ϕ_1 is the number of users whose counter content remained one after the splitting; f_1, f_2 is the number of new users which will be activated (have a packet for transmission) and enter the system in the first slot of the corresponding sub-session.

By considering the expected values in (1) with respect to all random variables involved, we obtain an infinite dimensional linear system of equations of the form

$$L_k = h_k + \sum_{j=0}^{\infty} a_{kj} L_j \quad , \quad k \geq 0. \qquad (2)$$

The most widely used definition of stability is the one which relates it with the finiteness of L_k, for $k<\infty$. In [10], [11] it has been found that the system is stable for Poisson input rates $\lambda < S_{max} = .36$ (packets/packet length). The authors in [10], [11] were actually able to find a (linear) upper bound on L_k which is finite for $k<\infty$. S_{max} is then defined as the supremum over all rates λ for which such a bound, L_k^u, was possible to obtain.

The existence of $L_k^u < \infty$, for $k<\infty$, implies that (2) has a non-negative solution, L_k; the solution $\tilde{L}_k$ of the finite dimentional system of equations

$$\tilde{L}_k = h_k + \sum_{j=0}^{J} a_{kj} \tilde{L}_j \quad , \quad 0 \leq k \leq J, \qquad (3)$$

is a lower bound on L_k and $\tilde{L}_k \rightarrow L_k$ as $J \rightarrow \infty$, [11], [14], [15]. It turns out that for sufficiently large J (e.g. 15), $\tilde{L}_k$ is extremely close to L_k and thus, for practical purposes, L_k is considered to be equal to L_k, especially for λ outside the neighborhood of S_{max} . By solving (3), we calculate the mean session length of multiplicity k. Since the multiplicities of successive sessions are independent and identically distributed random variables, the mean session length, L, is calculated by averaging L_k over all k ; k is the number of arrivals in a slot from a Poisson process with intensity λ. In fact, the average for $k \leq J$ is sufficient.

From the description of the algorithm it can be concluded that the last slot of a session can be either idle or involved in a successful transmission. We proceed by calculating the probability that the last slot of a session be idle, since this probability is used in the calculation of p(NS,S).

By thinking in a way similar to that in the derivation of the recursive equations for l_k, the following equations are obtained for i_k; i_k is the indicator function of the event "the last slot of a session of multiplicity k is idle".

$$i_0 = 1 \quad , \quad i_1 = 0 \quad , \quad i_k = i_{k-\phi_1+f_2} \quad , \quad k \geq 2. \qquad (4)$$

By considering the expected values in (4) we obtain the following infinite dimensional system of linear equations

$$I_0 = 1 \quad , \quad I_1 = 0 \qquad (5a)$$

$$I_k = \sum_{f_2=0}^{\infty} \sum_{\phi_1=0}^{k} P_f\,(F_2{=}f_2)\; b_k\,(\Phi_1{=}\phi_1)\; I_{k-\phi_1+f_2} \cdot \qquad (5b)$$

Notice that I_k is the probability that the last slot of a session of multiplicity k is idle; $I_k \leq 1 < \infty$, for $k<\infty$. The system in (5) is of the form of that in (2). By using the same arguments as those used in the calculation of L^k we can solve a truncated, up to J=15, version of (5) and obtain a very good approximation of I_k. By averaging the latter over all $k \leq J$, we can approximate I; I is actually the probability that the last slot of a session is idle.

Up to this point, the average session length, L, and the probability that the last slot of a session is idle, I, have been calculated. The objective is to calculate the joint probability p(NS,S). As a last step before the calculation of this probability, we calculate the average number of internal (NS,S) pairs in a session. The following recursive equations are obtained with respect to $\tau_k^{NS,S}$; $\tau_k^{NS,S}$ denotes the number of internal (NS,S) pairs in a session of multiplicity k.

$$\tau_0^{NS,S} = 0 \quad , \quad \tau_1^{NS,S} = 0 \qquad (6a)$$

$$\tau_k^{NS,S} = \tau_{\phi_1+f_1}^{NS,S} + \tau_{k-\phi_1+f_2}^{NS,S} + 1_{\{i_{\phi_1+f_1}=1\ ,\ k-\phi_1+f_2=1\}}$$

$$+ 1_{\{\phi_1+f_1=1\}}\ ,\ k\geq 2. \qquad (6b)$$

Notice that the idle slots which are the last of a session and are followed by a session of multiplicity 1 (that would give an (NS,S) pair), are not considered by the expressions in (6).

By considering the expected values in (6), we obtain an infinite dimensional system of linear equations with respect to $T^{NS,S}$. Since $T_k^{NS,S} \leq L_k < \infty$ for $k<\infty$, the comments that were made in the calculation of L_k apply to this case again. Thus, $T_k^{NS,S}$ can be calculated by solving a truncated version of the system in (6). The resulting finite dimensional system is of the form of that in (3). The average number of internal (NS,S) pairs in a session, $T^{NS,S}$, is then approximated by averaging $T_k^{NS,S}$ over all $k \leq J$.

By invoking the strong law of large numbers, [16], and the ergodic theorem for stationary processes, [17], it can be shown that the joint probability p(NS,S) is given by the following expression

$$p(NS,S) = \frac{T^{NS,S}}{L} + \lambda e^{-\lambda} \frac{I}{L}. \qquad (7)$$

The last term in the above equation takes care of the non-internal (NS,S) pairs of slots. The transition probability p(NS,S) is then calculated from the expression.

$$p(S/NS) = \frac{p(NS,S)}{\pi(NS)} = \frac{p(NS,S)}{1-\lambda}$$

The rest of the transition probabilities are computed from the following expressions.

$$p(NS/NS) = 1 - p(S/NS)$$

$$p(S/S) = 1 - p(S/NS)\frac{\pi(NS)}{\pi(S)}$$

$$p(NS/S) = 1 - p(S/S)$$

The approximating Markov process, $\{z^j\}_{j\geq 0}$, is now completely determined since the steady-state and the transition probabilities have been calculated.

IV. Performance of the approximations on the output process

The most interesting, probably, application for which the characterization of the output process of a multi-user random-access communication network is of great importance, is that of analyzing the performance of systems of interconnected multi-user random access communication networks. In such systems one can find star topologies of interconnected networks. In such topologies, the mean time that a packet spends in the central node is an important measure of the performance of the interconnection and it is usually desired that this quantity be calculated. This is the reason for the selection of the previous mean time as a performance measure of the proposed approximations. The value of the mean time is not by itself a measure of the performance of the approximations. It is the comparison of this quantity, calculated under the various approximations on the input traffic to the central node, with the one from the simulation of the actual system, that indicates how good the approximations are.

A star topology of interconnected networks is shown in Fig. 1. Each input stream represents the output process from a multi-user random-access slotted communication system. Let λ_n be the output rate (in packets per slot) of the n^{th} network. A

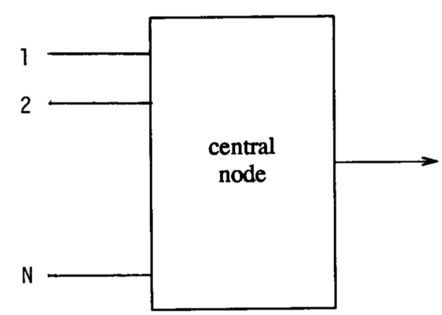

Figure 1.
A star topology of interconnected networks.

packet arrival in the central node is declared at the end of the slot in which the packet was successfully transmitted. Thus, the arrival process of each input line is a discrete process. The arrival points in all streams coincide; that is, the networks are assumed to be synchronized and all slots are of the same length.

The service time in the central node is constant and equal to one, which is assumed to be the length of a slot. This implies that arriving and departing packets have the same length. The first in-first out (FIFO) service policy is adopted. More than one arrivals (from different input streams) that occur at the same arrival point are served in a randomly chosen order. The buffer capacity of the central node is assumed to be infinite.

If the output process of a network is approximated by the Bernoulli-type process $\{y^j\}_{j\geq 0}$, then the resulting queueing system in the central node has been studied and the mean time that a packet spends in the central node, D_I, is given by, [8],

$$D_I = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda_n \lambda_m}{(1 - \sum_{n=1}^{N} \lambda_n) \sum_{n=1}^{N} \lambda_n} \right]$$

If the output process of a network is approximated by the Markov process $\{z^j\}_{j\geq 0}$, then the resulting queueing system in the central node has been studied in [9] and the mean time that a packet spends in the central node, D_M, is given by

$$D_M = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda_n \lambda_m \left[1 + \frac{\gamma_n}{1-\gamma_n} + \frac{\gamma_m}{1-\gamma_m} \right]}{(1 - \sum_{n=1}^{N} \lambda_n) \sum_{n=1}^{N} \lambda_n} \right]$$

where $\gamma_n = P(S/S) - P(S/NS)$.

V. Results and conclusions

We consider systems of N=2 and N=3 multiuser random-access communication networks interconnected according to the star topology described in the previous section. It is assumed that the limited-sensing collision resolution algorithm, described in section II, is deployed in each of the networks. The output process of each of the networks is approximated by a Bernoulli-type and a discrete-time Markov process. The parameters of the approximating processes are calculated according to the procedures developed before.

In Table 1, the values of the transition probabilities p(S/NS), calculated for various per network input rates λ and according to the procedures developed in section III, are compared with the corresponding values obtained from the simulation of the actual system. The coincidence (up to the third decimal point) between the analytical and the simulation results, shows that the estimation of this probability by solving truncated systems of J=15 linear equations, is extremely good.

λ	Anal.	Simul.
.01	0.009	0.009
.10	0.095	0.095
.20	0.186	0.186
.30	0.274	0.274
.33	0.300	0.300

Table 1.
Analytical and simulation results on the transition probability p(S/NS) of the channel-status process of a single network with Poisson packet generation process of intensity λ (packets per slot).

The mean time that a packet spends in the central node of the star topology was calculated from the expressions given in the previous section. The results (in slots) are shown in Tables 2 and 3, together with the results obtained from the simulation of the actual system. The network induced mean packet delay is also shown in the last column of these tables; it is the average time between the packet generation instant and the time when this packet is successfully transmitted (and appears in the output process). The results were taken from [10] and are provided to indicate the average total delay (in the network and in the node) that a packet undergoes. The maximum per network output rate under stable operation of the particular algorithm is .36 packets per slot. On the other hand, the queueing system of the star topology is stable for total input rates less than .99 packets per slot ,[22].

In the case of N=2 interconnected networks, the maximum total input rate to the central node is .72 packets per slot, far from the stability limit of the queueing system. The latter implies that the queueing problems will not be severe in the case of two interconnected networks. Indeed, it turns out that the queueing delay is less than .5 slots. Both approximations perform satisfactorily in this case in which the queueing problem is not significant.

In the case of N=3, the total input traffic to the central node can be as high as the stability limit of the queueing system (in fact even above that). Under such conditions the performance of the two approximations begin to differentiate. As the results in Table 3 indicate, when the per network traffic is large ($\lambda > .3$) and the queueing problem in the central node signifficant (total input traffic > .9), then the Markov approximation performs better than the Bernoulli-type one. This is due to the fact that the Markov model captures some of the strong dependencies introduced by the collision-resolution algorithm. These dependencies affect considerably the mean time that a packet spends in the central node only when there is a signifficant queueing problem.

λ	Indep.	Markov	Sim.	Net. Del.
.10	1.06	1.06	1.00	1.97
.22	1.20	1.21	1.02	3.85
.25	1.25	1.29	1.05	5.30
.30	1.37	1.44	1.13	11.38
.33	1.48	1.57	1.21	30.00
.35	1.58	1.70	1.30	87.70

Table 2.

Results for the mean packet delay in the central node of a star topology of 2 interconnected networks; λ is the per network input (output) rate. The results are under the Bernoulli-type approximation, under the Markov approximation and from the simulation of the actual system. Net. Del. is the algorithm induced delay within the network (in slots).

λ	Indep.	Markov	Sim.	Net. Del.
.10	1.14	1.15	1.01	1.97
.20	1.50	1.56	1.21	3.33
.25	2.00	2.16	1.70	5.30
.30	4.00	4.54	4.28	11.38
.31	5.42	6.23	6.25	15.00
.32	9.00	10.47	11.37	20.00
.33	34.00	40.14	48.89	30.00

Table 3.

Results for the mean packet delay in the central node of a star topology of 3 interconnected networks; λ is the per network input (output) rate. The results are under the Bernoulli-type approximation, under the Markov approximation and from the simulation of the actual system. Net. Del. is the algorithm induced delay within the network (in slots).

As a last comment on the performance of the Bernoulli-type approximation, we note that the good performance of this model under moderate per network traffic (e.g. N=3 and $\lambda = .2$) is due to the fact that , although dependencies are introduced by the collision-resolution algorithm, some independency is introduced to the cumulative input traffic to the central node from the mutually independent input streams. We expect that as the number of interconnected networks increases (and the per network output rate decreases, for the stability of the queue) the Bernoulli-type model will perform satisfactorily ,with respect to the performance measure under consideration, for increasingly larger cumulative input rates to the central node.

More than 3 interconnected networks can also be considered. In such a system one should make the assumption that only a portion of each network traffic is destined outside the particular network and thus needs to be forwarded to the central node. This assumption is clearly necessary for the stability of the queue. The output process of a network in such a system is not determined by the two-state channel-status process, $\{x^j\}_{j\geq 0}$. It is a combination of the channel-status process and a binomial process, if there is a probability that the destination of a successfull packet is outside the particular network. In the latter case, an S-state will result in an output with some probability.

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