ANALYSIS OF INTERCONNECTED COMMUNICATION NETWORKS

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ABSTRACT

In this paper, a star topology of interconnected network subsystems is analyzed. A general description of the dependent packet process generated by a network subsystem is introduced, based on an underlying Markov chain associated with the operation of the subsystem. The statistical multiplexer, which is formed in the central node of the topology, is analyzed under the described dependent packet arrival processes.

The developed analysis is applied in the case of a specific dependent packet arrival process and exact delay results are obtained. These results are compared with those obtained under the Bernoulli and the Markov (arrival / no arrival) models and some interesting conclusions are drawn.

I. INTRODUCTION

Packet communication networks have been widely adopted as an efficient means of transferring information. Such networks extend from small local area networks to large systems of interconnected networks covering extended geographical areas, [1]. The development of local area networks (or, in general, single networks) has been the focal point of extended research over the last two decades, [1]. The performance evaluation of these networks has been facilitated by the adopted models on the packet generation mechanism. The Bernoulli process has been widely used to model the per user packet generation process of a small (finite) user population system over a fixed time interval (slot). In the case of a large (infinite) user population system, the Poisson model has also been adopted for the cumulative packet generation process. These models are in accordance with the randomness and the unpredictability of the packet generating mechanism, which is captured by the memorylessness nature of these processes.

The single communication network imposes severe limitations on the information exchange capabilities of the supported users. To enchance these capabilities, simple communication networks (or, in general, network subsystems) are interconnected, resulting in larger and more complex systems, [2], [3]. At the same time, new network components are created to support the interconnection of the involved subsystems. These new components form the backbone network. In this paper, a network subsystem is defined as a system that generates packets. Such network subsystems can be local area networks, network switches or repeaters, statistical multiplexers, single user dedicated lines, links which carry the mixed packet traffic of a multi-hop environment, affected by routing decisions, etc.

In this paper we analyze the performance of a star topology of interconnected communication subsystems. The star topology may be the only interconnecting scheme present, or it may be one of the interconnecting points in a Metropolitan Area Network (MAN), supporting a number of different information transfer facilities, [2], [3]. The critical component of a star interconnecting scheme is its central node. The performance of this network component determines the quality of the interconnection. An accurate evaluation of the delay induced by the central node is important to the correct identification of the bottlenecks of the large communication system and may lead to certain effective adjustments resulting in the improvement of the overall network performance.

The characteristics and the operation of the central node of the star topology of interconnected network subsystems, that is considered in this paper, is presented in the next section. The central node is seen as a statistical multiplexer which is fed by the output traffic of N network subsystems. The critical issue in the analysis of such multiplexers is the characterization of the packet arrival process; the latter is the packet traffic generated by the corresponding network subsystem. The arrival processes to the multiplexer, considered in this paper, are described by incorporating a general model which could be an exact description (or a satisfactorily close approximation) of dependent packet departing processes, generated by the network subsystems. The commonly adopted i.i.d. or first order Markov models for the description of the packet departing process can be seen as special cases of our model.

The paper is organized as follows. In the next section, the characteristics of the interconnecting node are described; previous work on related statistical multiplexers is also cited. In section III, the analysis of the equivalent statistical multiplexer is carried out. In section IV, an example is presented and the developed analysis is applied. The example illustrates the dramatic effect of the packet arrival process on the induced delay, by comparing exact results with those obtained under an i.i.d. or first order Markov approximations on the packet arrival processes.

II. THE INTERCONNECTING NODE

In this section, we describe the general model for the central node which receives and forwards traffic from more than one network subsystems. This interconnecting node is the central component of the interconnecting topology investigated in this paper and it can be described in terms of a statistical multiplexer fed by N independent input lines.

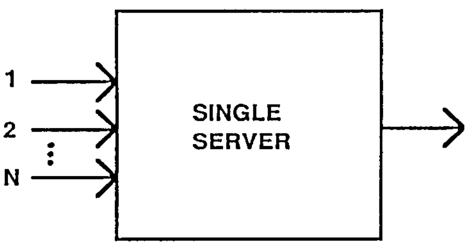

Figure 1.
The central node of a star interconnecting topology.

Consider the queueing system shown in Fig.1. The N input lines are assumed to be the mutually independent links which carry the traffic generated in the corresponding subsystems. These lines are assumed to be slotted, and packet arrivals and service completions are synchronized with the end of the slots. A slot is defined as the fixed service time required by a packet. At most one packet can be served in one slot. The first-in first-out (FIFO) service discipline is adopted. Packets arriving at the same slot are served in a randomly chosen order. The buffer capacity is assumed to be infinite. The packet arrival process associated with line i is defined to be the discrete time process $\{a_j^i\}_{j\geq 0}$, i=1,2,...,N, of the number of packets departing from subsystem i at the end of the j^{th} slot; $a_j^i=k$, $0\leq k\leq\infty$, if k packets leave the subsystem i at the end of the j^{th} slot.

Let $\{z_j^i\}_{j\geq 0}$, be a finite state Markov chain imbedded at the end of the slots, which describes the state of the subsystem i. Let $S^i=\{x_0^i, x_1^i, \cdots, x_{M^i}^i\}$, $M^i<\infty$, be the state space of $\{z_j^i\}_{j\geq 0}$. It is assumed that the state of the underlying Markov chain determines (probabilistically) the packet departure process from the corresponding subsystem. That is, if $a^i(x^i) : S^i \to Z_0$, is a probabilistic mapping from S^i into the nonnegative finite integers, Z_0, then the probability that k packets arrive at the central node at the end of the j^{th} slot is given by $\phi(z_j^i,k)=Pr\{a^i(z_j^i)=k\}$. Furthermore, it is assumed that there is at most one state, x_0^i such that $\phi(x_0^i,0)>0$ and that the rest of the states of the underlying Markov chain result in at least one packet departure, i.e. $\phi(x_k^i,0)=0$, for $1\leq k\leq M^i$. All packet departures are assumed to occur at the end of the slots.

Previous work on similar statistical multiplexers can be found in [4]-[8] (and the references cited there). All previous models differ significantly from the one presented here. In [4], the authors assume a single arrival line and a two state Markov Modulate Poisson arrival process. In [6], the author considers a single input line and arrivals that depend on an underlying two state Markov chain. In [5], [7] and [8], the system of Fig.1 is analyzed. In [5], it is assumed that the packet arrival process of each of the identical input lines depends on an underlying two state Markov chain (active/inactive). In [7] it is assumed that the per line packet arrival process is a first order Markov chain and at most one packet arrival is possible. A closed form solution for the mean packet delay has been derived for the latter case. In [8], a closed form expression for the mean packet delay in the case of Bernoulli per line arrival can be found. The systems presented in [5], [7], [8] (and some special cases of the system in [6]), are special cases of the general system investigated here.

III. ANALYSIS OF THE STATISTICAL MULTIPLEXER

IIIa. General case : Asymmetric system.

In this section we study the statistical multiplexer described before. The asymmetry of the system is due to the fact that although all arrival processes are described by the same general model, no two of them are identical.

Let $\pi^i(k)$ and $p^i(k, j)$, $k,j \in S^i$, denote the steady state and the transition probabilities of the ergodic underlying Markov chain, $\{z_j^i\}_{j\geq 0}$, associated with the i^{th} input line, $i = 1, 2, \cdots, N$. Let also $p^n(j ; \bar{y})$ denote the joint probability that there are j packets in the system at the n^{th} time instant, or beginning of slot, (arrivals at that point are included) and the states of the Markov chains are $y^1, y^2, \cdots, y^N$, where $\bar{y} = (y^1, y^2, \cdots, y^N)$; the arrivals which result from the state $\bar{y}$ are not included at this time instant. The vector $\bar{y}$ describes the state of a new ergodic Markov chain generated by the N independent Markov chains described before. Let $\pi(\bar{y})$ and $p(\bar{x}, \bar{y})$ be the steady state and the transition probabilities, respectively, and $\bar{S} = S^1 x S^2 x \cdots x S^N$ be its state space. The evolution of the buffer occupancy can be described by an N + 1 dimensional Markov chain imbedded at the beginning of the slots, with state space T = (0, 1, 2, $\cdots$)x S and state probabilities given by the following recursive equations

$$p^n(j\,;\,\bar{y}) = \sum_{\bar{x}\in S} \sum_{\nu=0}^{R} p^{n-1}(j+1-\nu\,;\,\bar{x})$$

$$p(\bar{x},\bar{y})\, g_{\bar{x}}(\nu) \quad , \quad j \geq R+1 \qquad (1a)$$

$$p^n(j;\bar{y})=\sum_{\bar{x}\epsilon S} \sum_{k=1}^{j+1} p^{n-1}(k;\bar{x})p(\bar{x},\bar{y})g_{\bar{x}}(j+1-k)$$

$$+\sum_{\bar{x}\epsilon S} p^{n-1}(0;\bar{x})\, p(\bar{x},\bar{y})g_{\bar{x}}(j) \;,\; 0\leq j\leq R \qquad (1b)$$

where R is the maximum number of arrivals from all input lines over a slot, $\bar{x}$ is the state of the N-dimensional Markov chain at time instant n-1 and

$$g_{\bar{x}}(\nu)=\Pr\{\sum_{i=1}^{N} a^i(x^i,)=\nu\} \qquad (2a)$$

$$\text{with} \quad \mu_{\bar{x}}=\sum_{\nu=1}^{R} \nu g_{\bar{x}}(\nu) \;,\; \sigma_{\bar{x}}=\sum_{\nu=1}^{R} \nu^2 g_{\bar{x}}(\nu) \qquad (2b)$$

$g_{\bar{x}}(\nu)$ is the probability that the N dimensional underlying state $\bar{x}$ results in ν packet arrivals. There are totally $M^1xM^2x \cdots xM^N$ equations given by (1) for a fixed j and all $\bar{y} \epsilon$ S, where M^i is the cardinality of S^i, i = 1, 2, $\cdots$, N.

Ergodicity of the Markov chains associated with the input streams implies the ergodicity of the arrival processes $\{a_j^i\}_{j\geq 0}$, i = 1, 2, $\cdots$, N. The latter together with the ergodicity condition for the total average input traffic λ

$$\lambda = \sum_{\bar{x}\epsilon S} \mu_{\bar{x}}\pi(\bar{x}) < 1 \qquad (3)$$

imply that the Markov chain described in (1) is ergodic and there exist steady state (equilibrium) probabilities. Thus, we can consider the limit of the equations in (1) as n approaches infinity and obtain similar equations for the steady state probabilities. By considering the generating function of these probabilities, manipulating the resulting equations, differentiating with respect to z and setting z=1, we obtain the following system of linear equations.

$$P'(1;\bar{y})=\sum_{\bar{x}\epsilon S} P'(1;\bar{x})\; p(\bar{x},\bar{y})$$

$$+\sum_{\bar{x}\epsilon S}(\mu_{\bar{x}}-1)p(\bar{x},\bar{y})\pi(\bar{x})+\sum_{\bar{x}\epsilon S} p(0;\bar{x})p(\bar{x},\bar{y}) \;,\; \bar{y}\epsilon S \qquad (4)$$

where

$$\pi(\bar{x})=\prod_{i=1}^{N} \pi^i(x^i) \;,\; p(\bar{x},\bar{y})=\prod_{i=1}^{N} p^i(x^i,y^i)$$

$$p(0;\bar{x})=p_0 p(\bar{x}_0,\bar{x})$$

where p_0=1−λ is the probability that the buffer of the multiplexer is empty and $\bar{x}_0=(x_0^1,x_0^2, \cdots ,x_0^N)$ is the only state that results in no packet arrival.

The $M^1x \cdots M^N$ linear equations with respect to $\bar{y} \epsilon$ S that are described by (4) are linearly dependent. This is the case when the equations have been derived from the state transition description of a Markov chain. By adding up all the equations in (4) and by using L'Hospital's rule, we obtain an additional linear equation with respect to $P'(1\,;\,\bar{y})$, $\bar{y} \epsilon$ S, which is linearly independent from those in (4) and is given by

$$\sum_{\bar{x}\epsilon S}\Big[2(\mu_{\bar{x}}-1)P'(1;\bar{x})+2(\mu_{\bar{x}}-1)p(0;\bar{x})$$

$$+[\,2+\sigma_{\bar{x}}-3\mu_{\bar{x}}]\pi(\bar{x})\Big]=0 \qquad (5)$$

By solving the $M^1x \cdots xM^N$ dimensional linear system of equations that consists of (5) and any $M^1x \cdots xM^N - 1$ equations taken from (4), we compute $P'(1\,;\,\bar{x})$, $\bar{x} \epsilon$ S. Then, the average number of packets in the system, Q, can be computed by summing up all the solutions. The average time, D, that a packet spends in the system can be obtained by using Little's formula as the ratio Q/λ.

Consider the special case in which the per stream arrival process is Bernoulli. The underlying Markov chain has one state and the equations (4) and (5) become

$$p(0)=1-\mu \qquad (4')$$

and

$$2(\mu-1)P'(1)+2(\mu-1)p(0)+[2+\sigma-3\mu]=0 \qquad (5')$$

where

$$\mu=\sum_{i=1}^{N}\lambda^i \;,\; \sigma=\sum_{i=1}^{N}\lambda^i(1-\lambda^i)+\mu^2$$

sp 1 where λ^i is the rate of the i^{th} network. From (5'), by substituting (4') and manipulating the resulting expression, we get the following equation with respect to P(1)

$$P(1)=Q_B=\frac{\sum_{i=1}^{N}\sum_{j\geq 1}^{N}\lambda^i\lambda^j + \mu(1-\mu)}{(1-\mu)}$$

where Q_B is the average number of packets in the system. The mean packet delay, D_B is given by Q_B/μ, which is a known result, [8].

IIIb. Special case : Symmetric system.

Let us now assume that the parameters of the input processes are identical, i.e. the parameters of all such processes are identical. Let M be the cardinality of each of the involved one dimensional Markov chains. As it will be shown shortly, the number of equations which need to be solved for the calculation of the mean delay in the multiplexer, is reduced significantly. This can be easily seen by observing that the unknown quantities in (4) and (5), $P'(1;\bar{x})$, are the same for certain values of $\bar{x}$. For instance, the quantity that corresponds to state $\bar{x}=(x_1,x_2,x_3, \cdots ,x_N)$ is equal to that of state $\bar{x}=(x_2,x_1,x_3, \cdots ,x_N)$.

If $\bar{v}(\bar{x})=(v_1(\bar{x}),v_2(\bar{x}), \cdots v_M(\bar{x}))$ is an M-dimensional vector with $v_i(\bar{x})$, i=1,2,...,M, denoting the number of input processes in state x_i, then each such vector $\bar{v}(\bar{x})$, with the constraint $\sum_{i=1}^{M} v_i(\bar{x})=N$, represents a class of equivalent states $\bar{x}$. The number of equivalent states $\bar{x}$ in a class $\bar{v}(\bar{x})$ is given by (pp. 20, [9])

$$c(\bar{x})=\begin{pmatrix} N \\ v_1(\bar{x}),v_2(\bar{x}), \cdots v_M(\bar{x}) \end{pmatrix} = \frac{N!}{v_1(\bar{x})!v_1(\bar{x})! \cdots v_M(\bar{x})!}$$

Let F be the set of representative states $\bar{x}$ of the symmetric system (i.e. no two states $\bar{x}\epsilon F$ belong to the same class of equivalent states); let $v(\bar{x}_o)$ be the class of the equivalent to $\bar{x}_o$ states. For each $\bar{x}_o,\bar{y}_o \epsilon F$, equations (4) and (5) can be written as follows.

$$P'(1;\bar{y}_o)=\sum_{\bar{x}_o\epsilon F}\{ \sum_{\bar{x}\epsilon V(\bar{x}_o)} p(\bar{x},\bar{y}_o) \}P'(1;\bar{x})$$

$$+\sum_{\bar{x}\epsilon S}(\mu_{\bar{x}}-1)p(\bar{x},\bar{y}_o)\pi(\bar{x})+\sum_{\bar{x}\epsilon S}p(0;\bar{x})p(\bar{x},\bar{y}_o) \quad , \quad \bar{y}_o\epsilon F \qquad (4a)$$

$$\sum_{\bar{x}_o\epsilon F} c(\bar{x}_o)2(\mu_{\bar{x}_o}-1)P'(1;\bar{x}_o)$$

$$+\sum_{\bar{x}\epsilon S}\Big[2(\mu_{\bar{x}}-1)p(0;\bar{x})+[2+\sigma_{\bar{x}}-3\mu_{\bar{x}}]\pi(\bar{x})\Big]=0 \qquad (5a)$$

By solving the above equations with respect to $P'(1;\bar{x}_o)$, $\bar{x}_o\epsilon F$, we obtain the average number of packets in the multiplexer under input state $\bar{x}_o$, for each $\bar{x}_o\epsilon F$. Then the average delay in the queueing system can be obtained from

$$D_s = \frac{\sum_{\bar{x}_o\epsilon F} P'(1;\bar{x}_o)c(\bar{x}_o)}{\lambda}$$

where λ is given by (3). Depending on the number of input streams, the reduced number of equations, K, in (4a) and (5a) is easily computed. For the practical case of N=2 and 3 input streams (or interconnected network subsystems) the number of those equations is given by the next theorem. Similar expressions for N>3 can be easily derived.

<u>Theorem</u>

Let M be the cardinality of the state space of the 1-dimensional underlying Markov chain defined before. The number of classes of equivalent N-dimensional states $\bar{x}=(x_1,x_2, \cdots ,x_N)$, , $x_k \epsilon S^1$, k=1,2, ⋯ ,M, is given by K, where

$$K=M+\frac{M(M-1)}{2} \quad \text{for} \quad N=2$$

$$K=M(M+\frac{(M-1)(M-2)}{6}) \quad \text{for} \quad N=3$$

Proof:

The proof is based on the enumeration of all M dimensional vectors $\bar{v}(\bar{x})=(v_1(\bar{x}),v_2(\bar{x}), \cdots v_M(\bar{x}))$ with $\sum_{i=1}^{M} v_i(\bar{x})=N$, and where $v_i(\bar{x})$ is the number of input processes in state x_i (pp. 20, [9]).

The above theorem indicates that significant reduction in the number of equations can be achieved under symmetric inputs. In the later case the required number of linear equations is K versus M^N for the general asymmetric case. Partially symmetric inputs will also result in a significant reduction of the number of the equations.

IV. RESULTS AND CONCLUSIONS

In this section we use the results of the previous analysis to evaluate the mean packet delay induced by the central node of a star topology of interconnected network subsystems. Each of the input lines is assumed to carry at most one packet over a slot. The following traffic situations are considered.

(a) Bernoulli arrivals per slot (arrival / no arrival).

(b) First order Markov arrivals per slot (arrival / no arrival).

(c) Arrivals appear in blocks of length L (slots), where L follows a general distribution. The arrival of the first packet after an idle slot (occurring with probability r) is assumed to be followed by consecutive packet arrivals over the next L-1 slots.

Model (c) may be incorporated in the description of the traffic of a message switched line (or node), where a message may consist of more than one packets. A message switched line may be one dedicated to an important user (information is generated by a message generating mechanism, in this case), or a single message buffered switch (messages are received by the switch only if it is empty). Model (c) may also describe the output process of a multi user communication network (one successful transmission is possible over a slot). Particularly, the output traffic of a reservation multi user communication network could be described by a general distributed number of packets, transmitted over a number of consecutive slots, during a reservation period. Notice that in those message arrival models, packets are transmitted one at a slot and the resulting packet arrival process is different from one which would assume simultaneous arrivals of all packets of a single message.

To describe the arrival process in terms of the general model introduced before, we define the state of line i at the end the j^{th} slot to be given by z_j^i, where $z_j^i=0$, if no packet arrived in the j^{th} slot and $z_j^i=k$, $1 \le k \le L$, if there are k packets of a message to be transmitted over the next k slots, starting with the $j+1^{st}$ slot. In this environment, a message describes a block of packets arriving througth the same line over consecutive slots. According to the message arrival model described above, a message is generated during a slot with probability r if the slot is empty, and with probability 0 if the slot is occupied. This scenario of the message arrival process could describe the output of a reservation multi user random access slotted communication network, where an idle slot is necessary for the release of the channel. If no such a slot is necessary, we allow a nonzero message generation process over slots in states 0 and 1; in this case the next message transmission may start right after the end of the previous one (the coming end is declared by the line state 1). This second scenario may also represent the output of a single message buffer which can receive a new message while in the last stage of the transmission of the previous one. We can also generalize, by defining the line state to be the content of the buffer at the other end of the line; L in this case denotes the buffer capacity. Different message acceptance disciplines may also be incorporated. For instance, if the length of the new message arriving at the buffer exceeds the available capacity at that time, the message can either be rejected or be accepted in part. All these cases can easily be translated into the appropriate transition probabilities of the process $\{z_j^i\}$, which can be easily shown to be a Markov chain with state space $S=\{0,1, \ldots ,L\}$.

The process $\{z_j^i\}$ in the case of the initial scenario (a message is generated only when the line state is 0), is a Markov chain with state transition probabilities given by (we omit the superscript i for simplicity)

$$p(k,j) = \begin{cases} 1 & j=k-1 \;,\; 1 \le k \le L \\ rc & k=0 \;,\; 1 \le j \le L \\ 1- & k=j=0 \\ 0 & \text{otherwise} \end{cases}$$

where d(j) is the probability that the length of a message (block) is j, $1 \le j \le L$. The probabilistic mapping in this case is

$$a(k,j) = \begin{cases} 1 & 1 \le k \le L \\ 0 & k=0 \end{cases}$$

The steady state probabilities of $\{z_j^i\}$ can be easily obtained from the system of equations

$$\Pi = \Pi P$$

$$\sum_{k=0}^{L} \pi(k)=1$$

where Π is the vector of steady state probabilities and P is the matrix of the transition probabilities.

The equivalent Bernoulli model for the packet arrival process described before has parameter p (packet arrivals per slot) equal to $p=1-\pi(0)$. The equivalent first order Markov model (arrival=1, no arrival=0) has the following parameters

$$\pi_m(0)=\pi(0) \;,\; \pi_m(1)=1-\pi_m(0) \;,$$

$$p_m(0,0)=1-p_m(0,1) \;,\; p_m(1,0)=1-p_m(1,1)$$

where

$$p_m(0,1)=r \;,\; p_m(1,1)=1-p(0,1)\frac{\pi_m(0)}{\pi_m(1)}$$

For this model, we define the burstiness coefficient γ to be equal to

$$\gamma=p_m(1,1)-p_m(0,1)$$

At this point we consider N=3 network subsystems interconnected according to a star topology. It is assumed that the packet output process generated by each of them is described by the message (block) arrival process described before. Let L=5 and d(1)=.1, d(2)=.2 d(3)=.3 d(4)=.3 d(5)=.1. The exact value of the mean packet delay in the central node of the interconnection, D_E, can be obtained by solving the equations (4) and (5). The approximate delay results under the Bernoulli (D_B) and the Markov (D_M) models are calculated from the closed form expressions that are available for these cases, [7], [8], and are given by

$$D_B = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda^n \lambda^m}{(1 - \sum_{n=1}^{N} \lambda^n) \sum_{n=1}^{N} \lambda^n} \right.$$

and

$$D_M = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda^n \lambda^m \left(1 + \frac{\gamma^n}{1-\gamma^n} + \frac{\gamma^m}{1-\gamma^m}\right)}{(1- \sum_{n=1}^{N} \lambda^n) \sum_{n=1}^{N} \lambda^n} \right.$$

where λ^i is the packet arrival rate of the i^{th} line.

The delay results for different values of per network message (block) arrival rate r, which result in a per line packet arrival rate r_{out}, and a total packet arrival rate r_{tot}, together with the corresponding burstiness coefficient γ, are shown in Table I. From these results, a number of interesting conclusions may be drawn. It can be noticed

r	r_{out}	r_{tot}	D_E	D_B	D_M	γ
.05	.134	.403	1.654	1.224	1.982	.63
.10	.236	.710	2.997	1.816	4.045	.58
.12	.271	.813	4.330	2.453	6.113	.56
.14	.303	.908	8.064	4.287	11.97	.54
.15	.317	.952	14.84	7.643	22.47	.53

Table I.
Results for the mean packet delay in the central node of the star topology of N=3 interconnected network subsystems, under dependent packet arrival processes.

r	r_{out}	D_E	D_B - error	D_M - error
.05	.134	4.861	3.771 -22.4%	5.693 +17.1%
.10	.237	5.229	3.967 -24.1%	6.350 +21.4%
.20	.383	4.809	3.943 -18.0%	5.953 +23.7%
.30	.482	4.105	3.724 -09.3%	5.082 +23.8%
.40	.554	3.481	3.464 -00.05%	4.282 +23.0%
.50	.608	2.984	3.210 +07.6%	3.636 +21.8%
.60	.650	2.597	2.977 +14.6%	3.128 +20.4%

Table II.
Results for the mean packet delay in the central node of the star topology of N=3 interconnected network subsystems, under dependent packet arrival processes.

that the Bernoulli approximation results in smaller delay than the one calculated under the Markovian approximation. This is always the case; the latter can be shown directly from the corresponding equations, keeping in mind that γ=0 in the case of the Bernoulli model while γ>0 under the Markovian model. The latter fact can be explained intuitively as well. Under the Markov model, packet arrivals tend to arrive in bursts. Whenever simultaneous bursts of arrivals occur, the content of the buffer of the node will keep increasing until the end of all but one burst and cannot start decreasing before the end of all bursts. Clearly, this situation (not present under the Bernoulli model) results in the increased packet delays. We believe that the latter behavior of the Markov model (or the geometrically distributed message length) is the reason for the larger delay results obtained under this approximation, when the true arrival process has the general length distribution described before. The Markov model creates concrete blocks of packets of average length equal to the average length of the generally distributed message length. On the other hand, generally distributed message lengths result in better randomized empty slots which reduce the intensity of the queueing problems.

In Table II, similar results are presented. In this case it is assumed that two of the packet arrival processes are exactly described by the Bernoulli model and one by the message length distribution used before. The total input traffic is .90 packets per slot; r_{out} is the intensity of the non-Bernoulli line and $(.9-r_{out})/2$ is the intensity of each of the Bernoulli lines. The delay error introduced by the adoption of the Bernoulli model for the packet arrival process of the non-Bernoulli line, is also shown. Notice that the error is significant (~20%) even when the dependent input line carries less than 15% of the total load. This observation implies that even if more than 85% of the total traffic is accurately described, the error in the approximation can still be large.

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