A Limited Sensing Protocol for Multiuser Packet Radio Systems

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Abstract—**In this paper, a protocol for a multiuser packet radio communication channel is proposed. The basic functions of this protocol are determined by a modified stack-type limited sensing collision resolution algorithm. In fact, the developed protocol is a hybrid of pure random access and a reservation scheme. A message consists of a number of packets which are capable of revealing the current activity of the channel. The performance of the system in terms of throughput and average message delay is investigated and analytical results are provided.**

I. Introduction

A LOT of work has been done towards the direction of developing communication protocols which determine how a single common resource can be efficiently shared by a large population of users. By now it is well known that fixed assignment techniques are not appropriate for a system with large population of bursty users. In the latter case, random access protocols are more efficient and many of them have been suggested [1], [2], [3]. In most of the systems, time is divided into slots of length equal to the time needed for a packet transmission (slotted systems).

In local area networks which use cable as a transmission media (e.g., Ethernet), the users can sense the channel and obtain almost immediate feedback information concerning the channel activity. When a collision occurs the users can detect it and abort their transmission. Due to the early channel feedback (limited by the network end to end propagation delay) and the abortion capability, the performance of local area network can be very high, depending on the adopted protocol and the system parameters. In a packet radio environment, fast channel sensing is not possible. It is generally assumed that a significant portion of the packet should be wasted before any reliable information concerning the channel status is obtained.

This paper focuses on random access techniques for multiuser packet radio communication systems. It is assumed that a large number of identical and bursty users share a common communication channel. The users can be static or mobile. The unit of information is the message and the aggregate message generation traffic is assumed to be Poisson distributed. Each message consists of one or more packets. In this paper, a packet is defined as the minimum amount of information whose transmission time is greater than a threshold β; β is the minimum time required in order for the users to obtain the feedback information concerning the channel activity. Proper design of the length of a packet can result in the reduction of the channel capacity wasted in a conflict under the adoption of the proposed protocol.

The first hybrid of a pure random access and a reservation scheme was proposed in [15] with the *R*-ALOHA protocol, to improve the throughput of a satellite channel beyond that of slotted ALOHA. *R*-ALOHA can also be used for any other broadcast media. In [16], an approximate analysis of the *R*-ALOHA is presented. In these systems the user population is finite.

In the next section, we develop a limited sensing stack-type random access protocol which is a hybrid of pure random access and reservation scheme. The protocol developed in this paper is appropriate for a Poisson infinite user population system where limited channel feedback is available. All users compete for the channel until the first successful packet transmission. After that, the channel is reserved for the successful user and the rest of the packets of that user are transmitted conflict free. A stack-type algorithm is used for conflict resolutions, [6]. The performance of the system is expected to increase as the number of packets per message increases. The latter does not imply that a message should be cut into as many packets as possible. The minimum packet detection time, determined by β, sets a lower limit on the packet length. Furthermore, one has to consider the packet overhead which is present in every packet. This overhead results in the reduction of the channel utilization and sets another lower limit on the packet length, if the packet is to carry a significant amount of useful information.

Whenever a reservation scheme is adopted, there is the possibility that the channel be monopolized by certain users, unless restrictions are imposed. For instance, a very large message could be cut into a number of smaller ones. Such issues are beyond the scope of this paper. Due to the statistical similarity of the user population, the users will be equally served in the long run, under stable operation of the system.

In the case of multiuser packet radio system in which frequency hopping transmission is adopted to achieve good performance in fading multipath channels, privacy, [4], [5], and coexistence with other systems, [7]–[10], the proposed protocol can still be applied. In that case, the length of the frequency hopping pattern determines the packet length and the known carrier sensing is equivalent to the frequency hopping pattern sensing. The desired length of the hopping pattern and the packet overhead set a lower limit on the packet length in that case.

II. The Protocol

It is assumed that an infinite population of bursty users share a common communication channel. A user can have at most one message to be transmitted per time. The cumulative message arrival process is modeled as a Poisson process with intensity λ messages per message length. Time axis is slotted and the length of a slot is equal to one time unit. All users are synchronized and a message transmission can start only at the beginning of a slot. A message is assumed to consist of a fixed number M of packets. Each packet has length equal to one time unit.

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A user is considered to be active if he has a message to transmit. An active user keeps sensing the channel starting from the slot that follows its message generation instant until this message is successfully received at the destination (limited channel sensing). At the end of each slot a ternary feedback information is available to all active users revealing the channel status, i.e., whether the channel was idle, involved in a successful transmission, or involved in a collision. These states of the channel correspond to zero, one or more than one transmission attempts in the same slot. Capture phenomena are not considered in this paper; more than one transmissions in the same slot result in the destruction of all messages. If capture can take place then the analysis presented here reflects a worst case performance of the system.

Users are geographically separated and it is assumed that the status of a user (i.e., the existence or nonexistence of a message in that user) cannot be communicated to the rest of the population. The common channel is the only part of the communication system which can be monitored by all users and which can communicate its status to each one of them. In mobile packet radio systems there is usually a central station which broadcasts the channel status to all the users. The channel status is assumed to be the only information available to the whole user population. Thus, a channel access protocol should be based on this information and should operate in a distributed fashion.

The nature of the user population suggests that a random channel access protocol be used. As a result, collisions arise when more than one packets are transmitted in overlapping slots. A collision resolution algorithm that belongs to the class of the stack algorithms is developed, [6], [11]. The concept of the stack is used to illustrate the operation of the algorithm. The content of a counter (assigned to each user) determines the cell of the stack which the particular user belongs to; it also determines the class of users to which he belongs. Users whose counter content equals k, $k \geq 0$, belong to the class B_k. B denotes the class of the inactive users, i.e., those users who do not have a message for transmission. B_0 denotes the class of new active users, i.e., those users who have a message to transmit but no message transmission has been attempted so far. B_1 is the class of active users who attempt packet transmission at the beginning of the slot that follows. Each user's counter content is updated at the end of the slots depending on the outcome of the channel, the steps of the algorithm and the counter content itself; the new value of the counter determines the class which the user enters. Clearly, $\cup_{n=0}^{\infty} B_n$ is the set of all active users. Users who belong to $\cup_{n=1}^{\infty} B_n$ and have entered class B_2 at least once since the instant when they became active for the last time, are the blocked users, i.e., the users who have attempted at least one unsuccessful transmission of their current message.

Each user is assigned a downcounter ω in addition to the counter which determines the class which the user belongs to. The initial value of the downcounter is M and it decreases by one unit per slot, starting from the slot in which the first packet of the message was successfully transmitted. The existence of this counter serves two purposes. The first one is to determine the time when the user becomes inactive. The second one is to provide some protection against the loss of a packet, due to erroneous feedback information. In the latter case, another user might attempt transmission while a successful transmission was in progress, resulting in a collision and destroying the original message. The user who was interrupted still belongs to the class B_1 and thus he attempts transmission at the beginning of the next slot. Thus, message loss is avoided and some priority is given to the unlucky user as well. Since the analysis of the protocol will be based on the assumption of error free channel, such events will not be considered in the analysis.

Let F denote the feedback information; it takes values from the set $\{I, S, C\}$ depending on whether the channel was sensed to be idle, involved in a successful transmission or involved in a collision. The random access algorithm can be described by the following changes of the user classes.

1) If $F = S$, then

$$\text{a) } B_0 \rightarrow B_0, \quad \text{b) } B_1 \begin{array}{l} \nearrow B_1 \text{ if } \omega > 0 \\ \searrow B \ \text{ if } \omega = 0 \end{array}, \quad \text{c) } B_k \rightarrow B_k, \; k \geq 2.$$

2) If $F = C$, then

a) $B_0 \rightarrow B_1$, b) $B_k \rightarrow B_{k+1}$, $k \geq 2$

c) For each $b \in B_1$, $b \begin{array}{l} \nearrow B_2 \text{ with probability } 1-p \\ \searrow B_1 \text{ with probability } p \end{array}$

3) If $F = I$, then

a) $B_0 \rightarrow B_1$

b) If last nonidle slot was involved in a collision, then

For each $b \in B_2$, $b \begin{array}{l} \nearrow B_1 \text{ with probability } 1-p \\ \searrow B_2 \text{ with probability } p \end{array}$,

$B_k \rightarrow B_k$, $k \geq 3$

c) If last nonidle slot was involved in a successful transmission, then

c_i) if the current slot is the first idle slot after the successful one, then $B_k \rightarrow B_k$, $k \geq 2$

c_{ii}) if the current slot is not as in c_i), then $B_k \rightarrow B_{k-1}$, $k \geq 2$.

It should be made clear that a packet collision can be detected by all active users. When this event occurs, the sender aborts his transmission before the end of the current slot. As a result, only one slot is wasted in a collision. Successful transmissions are never interrupted by new or blocked users. We assume that a message transmission is successful once its first packet has been successfully transmitted.

Users in class B_0 (i.e., new active users) are allowed to attempt packet transmission after a slot involved in a collision. This makes sense since the detected collision will be ended before the beginning of the next slot. Step c_i) gives priority to new active users over the blocked ones, after a successful transmission. At that time new active users are given the chance to either transmit successfully or join the blocked users. The probability of having at least one message during a sequence of successful slots (which has length equal to M slots) is significant. This algorithmic step emphasizes the continuous entry characteristic of the proposed protocol.

III. Throughput Analysis

An important quantity in the analysis of the random access algorithm under consideration is the session length. A technical definition of the session can be given via the use of an imaginary marker. The marker is originally placed in cell 0. Upon collision, the marker is placed in cell 2 of a conceptual stack. The position of the marker changes in the same way in which the counter content of the users of class B_3 changes, depending on the channel feedback. The slot in which the marker returns to cell 0 is the first one of the session that follows and it is always idle. When the marker is in cell 0 and a successful transmission occurs the market is placed in cell 1 and moves up or down as described before. Idle slots do not move the marker from cell 0 and they are considered as sessions of length one.

From the definition of the session it is implied that the first slot of a session is a renewal point, i.e., the system regenerates itself statistically after that point. The number of users who attempt packet transmission in the slot which follows the first idle one of the session, determines the multiplicity of that session. All users who attempt packet transmission in that slot plus the one who enter the system before the end of a session, transmit successfully during that session. The last two slots of a session are idle and thus multiplicities of the sessions are independent Poisson distributed random variables with intensity λ,

Let l_k denote the length of a session of multiplicity k (in time slots), $k \geq 0$. The following recursive equations can be written with respect to l_k, $k \geq 0$:

$$l_0 = 1, \quad l_1 = 1 + M + l_{F_M} \tag{1a}$$

$$l_k = 1 + \{1 + l_{k,0}\} I_{\{I_1+F_1=0\}} + \{l_{I_1+F_1} + l_{k-I_1+F_3}\} I_{\{I_1+F_1 \neq 0\}}, \quad k \geq 2 \tag{1b}$$

$$l_{k,0} = \{1 + l_{k,0}\} I_{\{I_2+F_2=0\}} + \{l_{I_1+F_1} + l_{k-I_1+F_4}\} I_{\{I_2+F_2 \neq 0\}}, \quad k \geq 2 \tag{1c}$$

where $I_{\{\cdot\}}$ is the indicator function and F_M, F_1, F_2, F_3, F_4, I_1, I_2 are independent random variables; F_1, F_2, F_3, and F_4 are Poisson distributed random variables over one slot (with probability density P_1), F_M is Poisson over $M + 1$ slots (with probability density P_M) and I_1, I_2 are binomial with probability density

$$b_k(I_1) = \frac{k!}{I_1!(k-I_1)!} p^{I_1}(l-p)^{k-I_1}, \tag{2a}$$

$$b_k(I_2) = \frac{k!}{I_2!(k-I_2)!} p^{I_2}(l-p)^{k-I_2}.\text{(2b)} \tag{2b}$$

The equations in (1) can be explained as follows. a) The session of multiplicity 0 consists only of the idle slot which marks the beginning of this session. b) The length of a session of multiplicity 1 consists of the following parts. i) The idle slot which is always the first of the session. ii) The M slots involved in the successful transmission of the single packet of the system. iii) The length l_{F_M} which is the same as the length of a session of multiplicity F_M. F_M is a random variable associated with the number of message arrivals in $M + 1$ time units; it is Poisson distributed over $M + 1$. c) For $k \geq 2$, the length of the session consists of the following. i) The idle slot which is always the first of the session. ii) Since collision occurs we have to distinguish between two cases. Let I_1 be the number of users which remain in class B_1 after the splitting and let F_1 be the number of new messages which arrive in the slot before the collided one. ii_a) If $I_1 + F_1 > 0$ we add $l_{I_1+F_1} + l_{k-I_1+F_4}$ to the length of the session since the original session is split into two with the corresponding multiplicities. ii_b) If $I_1 + F_1 = 0$, we add another slot to the session since no transmission takes place, plus $l_{k,0}$. The latter quantity is equal to the length of a session of multiplicity k without including the slot of the original collision, i.e., $l_{k,0} = l_k - 1$.

Let L_k be the expected value of a session of multiplicity k. By considering the expectation of both sides of the equations in (1), we obtain the following infinite dimensionality linear system of equations with respect to L_k:

$$L_k = h_k + \sum_{j=0}^{\infty} a_{kj} L_j, \quad k \geq 0. \tag{3}$$

where

$$h_0 = 1, \quad h_1 = M + 1, \; h_k = \frac{1}{1 - b_k(0)P_1(0)}, \; 2 \leq k$$

$$a_{0j} = 0, \; 0 \leq j, \quad a_{1j} = P_{M+1}(j), \; j \geq 0$$

$$a_{k0} = h_k b_k(k) P_1(0), \; 2 \leq k$$

$$a_{kj} = \tau(h_k)\left[b_k(0)P_1(j) + \sum_{i=1}^{j} b_k(i)P_1(j-i) + \sum_{i=0}^{j} b_k(k-i)P_1(j-i)\right], \; 1 \leq j \leq k-1, \; 2 \leq k$$

$$a_{kj} = \tau(h_k)\left[b_k(0)P_1(j) + b_k(0)[1 - P_1(0)]P_1(j-k) + \sum_{i=1}^{k} b_k(i)P_1(j-k+i) + \sum_{i=0}^{k} b_k(i)P_1(j-i)\right],$$

$$k \leq j, \; 2 \leq k.$$

Since it is impossible for the above system to be solved, approximate solutions for L_k, $k \geq 1$, will be found, [6], [11], [12]. These solutions will provide upper and lower bounds on the mean session length and the maximum stable throughput, $S_{\max}$. Before we proceed in the analysis, we state the following definition of the stability.

1) Stability Definition: If for an input traffic rate λ $L_k < \infty$ for $k < \infty$, then the operation of the system is stable and λ belongs to the stable region of the throughput. The maximum over all rates λ, which result in a stable system, is defined to be the maximum stable throughput of the system and is denoted by $S_{\max}$.

By following procedures similar to those in [6], the following Lemma can be proved.

Lemma 1: If $\{x_k^u\}_{k=0}^{\infty}$ is an infinite sequence of real numbers which satisfy the following conditions:

1) $0 \leq x_k^u < \infty$, $0 \leq k < \infty$,

2) $h_k + \sum_{j=0}^{\infty} a_{kj} x_j^u \leq x_k^u$, $0 \leq k < \infty$,

3) $h_k \geq 0$, $a_{kj} \geq 0$, for $k \geq 0$, $j \geq 0$,

then the infinite dimensionality linear system of equations

$$h_k + \sum_{j=0}^{\infty} a_{kj} x_j = x_k, \; 0 \leq k < \infty \tag{4}$$

has a unique nonnegative solution $\{x_k\}_{k=0}^{\infty}$ that satisfies $0 \leq x_k \leq x_k^u$, $0 \leq k < \infty$. □

If x_k represents the expected value of a session of multiplicity k, then the range of input traffic rates λ for which a sequence $\{x_k^u\}_{k=0}^{\infty}$ which satisfies the above conditions exists, belongs to the stable region of the throughput. For $\lambda < \lambda^1$ and for some value of the splitting probability p, $0 \leq p \leq 1$, we were able to find a quantity $L_k^u = \beta(\lambda, p)k - \gamma(\lambda, p)$ which satisfies the conditions of Lemma 1 (actually numerical search determined λ^1 for $p \approx 0.48$). Thus, $L_k \leq L_k^u$ for $\beta(\lambda, p)$, $\gamma(\lambda, p)$ and k finite and λ^1 is a lower bound on the maximum stable throughput. The values of λ^1 (in number of messages per message length) are computed for $M = 1, 2, 5, 10, 100$ packets per message. The results are shown in Table I.

An upper bound on the maximum stable throughput can be obtained by considering a truncated version of the infinite

TABLE I
UPPER AND LOWER BOUNDS ON THE MAXIMUM STABLE THROUGHPUT FOR M PACKETS PER MESSAGE. [λ^{lt} IS THE TIGHT LOWER BOUND CALCULATED FROM (14)]

M	λ^{l}	$\lambda^{u} \sim \lambda^{lt}$
1	.273	.283
2	.443	.453
5	.678	.686
10	.812	.817
100	.977	.978

dimensionality system in (3)

$$L_0^1 = 1, \quad L_k^1 = h_k + \sum_{j=0}^{N} a_{kj} L_j^1, \quad 1 \le k \le N. \tag{5}$$

Let λ_N^u be the maximum over all Poisson rates for which the truncated system in (5) has a unique nonnegative solution. Let also $S_{\max}$ be the maximum over all Poisson rates for which the infinite dimensionality system in (4) has a unique nonnegative solution that satisfies the condition

$$\lim_{M\to\infty} \max_{N>M} \left\{ \sum_{j=N}^{\infty} a_{kj} L_j \right\} = 0.$$

Then, λ_N^u decreases monotonically as N increases and $\lim_{N\to\infty} \lambda_N^u = S_{\max}$, [11]. Clearly, λ_N^u is an upper bound on the maximum stable throughput $S_{\max}$. By solving the system in (5) for $N = 24$ and for the cases of $M = 1, 2, 5, 10, 100$ packets per message, an upper bound on $S_{\max}$ is obtained. The results appear in Table I.

IV. DELAY ANALYSIS

In this section, we derive bounds on the mean message delay for input traffic rates $\lambda < \lambda^1$. The important result of this section is the following expression for the average message delay D and for $\lambda < \lambda^1$.

$$D_A + \frac{C^l}{\lambda L^u} \le D \le D_A + \frac{C^u}{\lambda L^l} \text{ (in message lengths)} \tag{6a}$$

where

$$D_A = \frac{6(1-\lambda(M+1)) + \lambda(M+1)(2M+3)}{4M}. \tag{6b}$$

L^l, L^u denote lower and upper bounds on the mean session length L. C^l, C^u denote lower and upper bounds on the mean cumulative in system delay of all messages arriving in a single session. The in system delay is defined to be the time that elapses between the instance when a message enters class B_1 for the fist time and the instant when the whole message has been successfully received by the receiver. The mean cumulative in system delay is defined as the expected value of the sum of the in system delays of all messages arriving during a single session. Let $i = 0$ be the time when the system starts operating and let us define the following quantities.

- l_i: The ith session after the beginning of the operation of the system.
- a_i: The number of message arrivals in the ith session.
- c_i: The cumulative in system delay of the messages arriving in the ith session.

Since the operation of the algorithm represents a renewal process and the multiplicities of the sessions are independent and identically distributed random variables, we conclude that the random variables l_i, $i = 1, 2, 3, \cdots$ are independent and identically distributed. Clearly, the same holds for the random variables, a_i and c_i, 1, 2, 3, $\cdots$. The following theorem is a direct application of the strong law of large numbers (p. 126, [14]).

Theorem 1: For $\lambda < \lambda^1$, the mean in system delay D_s is given by

$$D_s = \frac{C}{\lambda L}, \text{ with probability 1}$$

where $C = E\{c_i\}$ and $L = E\{l_i\}$.

Proof: Since each of the l_i, c_i, a_i, $i = 1, 2, \cdots$, are independent and identically distributed random variables and since we known that $L = E\{l_i\} < \infty$, for $\lambda < \lambda^1$, we conclude that $C = E\{c_i\} < \infty$ and that $E\{a_i\} = \lambda L < \infty$. The mean in system delay is given by

$$D_s = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} a_i} = \lim_{n\to\infty} \frac{\frac{1}{n}\sum_{i=1}^{N} c_i}{\frac{1}{n}\sum_{i=1}^{n} a_i} \overset{(*)}{=} \frac{E\{c_i\}}{E\{a_i\}} = \frac{C}{\lambda L}$$

with probability 1; equality (*) is justified by the strong law of large numbers. □

In the sequel, we define the access delay A to be the time period that elapses between a message arrival instant and the instant when the message enters class B_1 for the first time. The next theorem provides an expression for the mean access delay.

Theorem 2: The mean access delay D_A is given by

$$D_A = \frac{6(1-\lambda(M+1)) + \lambda(M+1)(2M+3)}{4} \text{ for every } \lambda < \lambda^1.$$

Proof: Let θ be a random variable associated with each message arrival that takes the values IC or S depending on whether the first time slot following the message arrival is idle/involved in a collision or involved in a successful transmission, respectively. If A denotes the access delay of a random message, then

$$E\{A/\theta = \mathrm{IC}\} = \frac{3}{2M} \text{ and} \tag{7a}$$

$$E\{A/\theta = S\} = \frac{2M+3}{4M} \text{ (message lengths).} \tag{7b}$$

Clearly, $E\{A\} = D_A = E\{A/\theta = \mathrm{IC}\}P\{\theta = \mathrm{IC}\} + E\{A/\theta = S\}P\{\theta = S\}$. To calculate the probabilities involved in the previous expression we define the following random variables. Let τ_i^{IC} be the number of slots of the ith session which were idle/involved in a collision, and τ_i^S be the number of slots of the ith session which were involved in a successful transmission. The random variables τ_i^{IC}, $i \ge 0$ are independent and identically distributed; the same holds for τ_i^S, $i \ge 0$. Since $E\{\tau_i^{\mathrm{IC}}\} < \infty$ and $E\{\tau_i^S\} < \infty$ for every $\lambda < \lambda^1$, by applying the strong law of large numbers we have

$$P\{\theta = \mathrm{IC}\} = \lim_{n\to\infty} \frac{\frac{1}{n}\sum_{i=1}^{n} \tau_i^{\mathrm{IC}}}{\frac{1}{n}\sum_{i=1}^{n} l_i} = \frac{E\{\tau_i^{\mathrm{IC}}\}}{E\{l_i\}} = \frac{E\{\tau_i^{\mathrm{IC}}\}}{L}$$

with probability 1

and

$$P\{\theta=S\}=\lim_{n\to\infty}\frac{\frac{1}{n}\sum_{i=1}^{n}\tau_i^S}{\frac{1}{n}\sum_{i=1}^{n}l_i}=\frac{E\{\tau_i^S\}}{E\{l_i\}}=\frac{E\{\tau_i^S\}}{L}$$

with probability 1.

Now since

$$E\{\tau_i^S\}=\lambda E\{l_i\}=\lambda L \text{ and } E[\tau_i^S\}+E\{\tau_i^{IC}\}=E\{l_i\}=L$$

we obtain that $E\{\tau_i^{IC}\} = (1 - \lambda(M + 1))L$. The last expressions together with (7) complete the proof of the theorem. □

The expressions in (6) become obvious in view of the Theorems 1 and 2. In the following sections we derive upper and lower bounds on the mean session length L and the mean cumulative in system delay C.

A. Bounds on the Mean Session Length

Definition: If

$$x_k=A_k+\sum_{j=0}^{\infty}B_{kj}x_j,\quad 0\le k\le\infty \text{ and} \tag{8a}$$

$$y_k=a_k+\sum_{j=0}^{\infty}b_{kj}y_j,\quad 0\le k\le\infty \tag{8b}$$

are infinite dimensionality linear systems of equations with $A_k \geq |a_k|$ and $B_{kj} \geq |b_{kj}|$, $0 \leq k \leq \infty$, $0 \leq j \leq \infty$, then we say that the system in (8a) is a majorant for the system in (8b); similarly, the system in (8b) is a minorant for the system in (8a).

The following theorem can be found in [13].

Theorem 3: If a majorant for a given system has nonnegative solutions x_k, $k \geq 0$, then the given system has the solution y_k which satisfy $|y_k| \leq x_k$, $0 \leq k \leq \infty$. □

Note that the infinite dimensionality linear system of equations in (4) is a majorant for its truncated version in (5) and the system in (4) has a nonnegative solution for every $\lambda < \lambda^1$ by Theorem 1. Thus, Theorem 3 implies that, for every $\lambda < \lambda^1$, the solutions L_k^1 of the finite dimensionality system in (5) are lower bounds on the expected value of the length of a session of multiplicity k, L_k. A lower bound L^1 on the mean session length for input traffic rates $\lambda < \lambda^1$ can now be obtained by using the expression

$$L^1=E\{L_k^1\}=\sum_{k=0}^{\infty}P(k)L_k^1 \tag{9}$$

where $P(k)$ is the Poisson (over one slot) distribution of the multiplicities of the sessions. L_k^1, $0 \leq k \leq N$, are the solutions of the system in (5) and L_k^1 equals zero for $k > N$; the latter choice makes sense since despite the fact that zero may not be a tight lower bound on L_k for $k > N$, the probability that a session of multiplicity $k > N$ occurs is extremely small for large N and the input rates of interest.

The values of L_k^1, $0 \leq k \leq N$, for some values of $\lambda < \lambda^1$ and for $M = 1, 2, 5, 10, 100$ packets per message, are computed by solving the finite dimensionality system in (5) for $N = 24$. The mean session length is then calculated from the expression in (9). The results appear in Table II.

An upper bound on the expected value of the length a session of multiplicity k has already been calculated and it is given by L_k^u. An upper bound on the mean session length L_k can now be obtained by considering the expectation of L_k with respect to k, i.e.,

$$L^u=E\{L_k^u\}=\beta(\lambda,p)\lambda-\gamma(\lambda,p)[1-P(0)]+P(0).$$

The values of L^u were found to be very close to those of L^1 for $\lambda < \approx 0.8\ \lambda^1$. As λ approaches λ^1, the upper bound increases rapidly. Later in this chapter, a tighter upper bound on the mean session length will be calculated.

TABLE II
UPPER AND LOWER BOUNDS ON L AND C AND VALUES OF D (IN MESSAGE LENGTHS) FOR $\lambda < \lambda^1$ AND $M = 1, 2$ PACKETS PER MESSAGE

M	λ	$L^1 \sim L^u$	$C^1 \sim C^u$	D
	.010	1.020	.010	2.535
	.050	1.117	.069	2.735
	.100	1.290	.222	3.200
	.150	1.584	.643	4.132
1	.200	2.211	2.300	6.602
	.250	4.760	19.239	17.542
	.260	6.580	41.559	25.662
	.270	11.157	135.282	46.273
	.273	14.280	230.128	60.394
	.010	1.015	.010	1.762
	.050	1.082	.057	1.823
	.100	1.184	.139	1.938
	.150	1.317	.265	2.114
	.200	1.498	.484	2.396
2	.250	1.764	.912	2.849
	.300	2.199	1.896	3.662
	.350	3.049	4.868	5.355
	.400	5.474	21.032	10.405
	.440	20.075	359.548	41.510
	.443	25.541	592.653	53.184

(a)

TABLE II
UPPER AND LOWER BOUNDS ON L AND C AND VALUES OF D (IN MESSAGE LENGTHS) FOR $\lambda < \lambda^1$ AND $M = 5, 10, 100$ PACKETS PER MESSAGE

M	λ	$L^1 \sim L^u$	$C^1 \sim C^u$	D
	.010	1.012	.010	1.305
	.100	1.137	.118	1.337
	.200	1.362	.301	1.477
	.300	1.607	.641	1.735
5	.400	2.077	1.428	2.158
	.500	3.045	3.981	3.090
	.600	6.231	21.617	6.292
	.650	14.321	130.382	14.533
	.670	31.075	647.895	31.652
	.678	59.217	2404.230	60.506
	.010	1.011	.010	1.193
	.100	1.124	.114	1.211
	.200	1.285	.274	1.302
	.300	1.506	.524	1.438
10	.400	1.831	.974	1.651
	.500	2.356	1.944	2.012
	.600	3.360	4.688	2.730
	.700	6.062	18.028	4.695
	.770	14.647	118.671	10.999
	.812	116.156	8037.010	85.706
	.010	1.010	.010	1.019
	.100	1.112	.119	1.070
	.200	1.253	.257	1.141
	.300	1.435	.461	1.234
	.400	1.680	.771	1.360
	.500	2.027	1.294	1.538
100	.600	2.557	2.304	1.812
	.700	3.467	4.672	2.284
	.800	5.395	12.425	3.288
	.900	12.214	70.183	6.842
	.950	33.395	552.386	17.893
	.970	109.410	6065.790	57.647
	.977	539.488	150042.000	285.163

(b)

B. Bounds on the Mean Cumulative in System Delay, C

To derive bounds on the mean cumulative in system delay, we follow a procedure similar to the one employed in the derivation of the bounds on the mean session length. The following recursive equations for the cumulative in system delay c_k can be written by thinking in a similar way as in (1) c_k is the cumulative in system delay of a session of multiplicity k.

$$c_0=0, \quad c_1=M+c_{F_M}$$

$$c_k=k+[c_{I_1+F_1}+(k-I_1)c_{I_1+F_1}+c_{k-I_1+F_3}]I_{\{I_1+F_1\neq 0\}} + [k-c_{k,0}]I_{\{I_1+F_1=0\}} 2<k<\infty$$

$$c_{k,0}=[k+c_{k,0}]I_{\{I_2+F_2\neq 0\}}+[c_{I_2+F_2}+(k-I_2)c_{I_2+F_2} + c_{k-I_2+F_4}]I_{\{I_2+F_2=0\}} 2<k<\infty.$$

By considering the expectation of both sides of the previous equations we obtain an infinite dimensionality linear system of equations of the form of (3)

$$C_k=g_k+\sum_{j=0}^{\infty} a_{kj}C_j, \quad k\geq 1. \tag{10a}$$

where a_{kj} is as in (3) and g_k is given by

$$g_0=0, \quad g_1=M \tag{10b}$$

$$g_k=\tau(h_k)\left[k+\sum_{I_1=0}^{\infty}\sum_{F_1=0, I_1+F_1\neq 0}^{\infty} \cdot b_k(I_1)P(F_1)[k-I_1]L_{I_1+F_1}\right], \quad k\geq 2. \tag{10c}$$

By setting the upper (lower) bound on $L_{I_1+F_1}$ in (10c), we obtain an infinite dimensionality linear system of equations which is a majorant (minorant) for the system in (10a). Upper and lower bounds on C_k (C_k^u and C_k^l, respectively) can be computed by following procedures similar to those used for the derivation of the corresponding bounds on the mean session length of multiplicity k, $k \geq 0$. An upper bound on C on the form

$$C_0^u=0, \quad C_k^u=v_1k^2+v_2k+v_3, \quad k\geq 1$$

was obtained for all input traffic rates $\lambda < \lambda^1$ where v_1, v_2, v_3 are some finite constants, which depend on λ and p. An upper bound C^u on the mean cumulative in system delay can be calculated by considering the expectation of C_k^u with respect to k. Thus,

$$C^u=E\{C_k^u\}=(\lambda+\lambda^2)v_1+\lambda v_2+(1-P(0))v_3.$$

By solving a truncated version ($N = 24$ equations) of the infinite dimensionality system of equations in (10a), we obtain lower bounds on C_k. Then by using $C_k^l = 0$, $k > 24$, and considering the expectation of C_k^l with respect to k, we obtain a lower bound on the mean cumulative in system delay C^l. The values of C^l for some values of $\lambda < \lambda^1$ and for $M = 1, 2, 5, 10, 100$ packets per message, are shown in Table II.

The lower bounds on L and C that were calculated by solving finite systems for $N = 24$, are the same with those found for much smaller value of N. Since

$$\lim_{N\to\infty} L_k^l=L_k \text{ and } \lim_{N\to\infty} C_k^l=C_k$$

our results indicate that the lower bounds are very tight. On the other hand, the upper bounds which were calculated previously are arbitrary and probably very loose. The next theorem is employed for the derivation of tighter upper bounds; its proof is based on Lemma 1 and the theory of majorant systems, [13].

Theorem 4: Let $\{x_k^u\}_{k=0}^{\infty}$ be a sequence of real numbers which satisfies

$$(\alpha)\ 0\leq x_k^u<\infty,\ 0\leq k<\infty \text{ and } (\beta)\ h_k+\sum_{j=0}^{\infty} a_{kj}x_j^u\leq x_k^u$$

with (γ) $h_k \geq 0$, $a_{kj} \geq 0$, for $0 \leq k, j \leq \infty$. Then the following hold.

a) The finite dimensionality system of linear equations

$$x_k^{ut}=h_k+\sum_{j=N+1}^{\infty} a_{kj}x_j^u+\sum_{j=0}^{N} a_{kj}x_j^{ut} \tag{12}$$

has a nonnegative solution x_k^{ut} which satisfies $x_k^{ut} \leq x_t^u$, $0 \geq k \leq N$.

b) If x_k is a nonnegative solution of the system

$$x_k=h_k+\sum_{j=0}^{\infty} a_{kj}x_j,\ k\geq 0, \tag{13}$$

then $x_k \leq x_k^{ut}$, $0 \leq k \leq N$. □

By employing the sequences $\{L_k^u\}_{k=0}^{\infty}$ and $\{C_k^u\}_{k=0}^{\infty}$ in the place of the sequence $\{x_k^u\}_{k=0}^{\infty}$ in the above theorem and solving the resulting finite dimensionality linear systems of equations given by (12), tight upper bounds on L_k and C_k were obtained, for $k \leq N = 24$. By considering the tight upper bounds on $L_k(L_k^{ut})$ and on $C_k(C_k^{ut})$ for $k \leq N$ and the upper bounds L_k^u and C_k^u for $k > N$, tight upper bounds on $L(L^{ut})$ and $C(C^{ut})$ are obtained, respectively. The values of L^{ut} and C^{ut} for some values of $\lambda < \lambda^1$ and for $M = 1, 2, 5, 10, 100$, are shown in Table II. Note that the tight upper bounds coincide with the lower bounds (up to at least the first three decimal digits).

V. Results and Conclusions

By substituting the values for the tight upper and lower bounds on the mean session length and the mean cumulative in system delay into expression (6a), upper and lower bounds on the average message delay are obtained. Since $L^{ut} \approx L^1$ and $C^{ut} \approx C^1$ we obtain the approximate expression

$$D\approx D_A+\frac{C^1}{\lambda L^1}$$

for the mean message delay. The accuracy of the previous expression is restricted by the accuracy of the fourth, or beyond that, decimal digit in C^1 and L^1. The values of the mean message delay for some values of $\lambda < \lambda^1$ and for $M = 1, 2, 5, 10, 100$ packets per message are shown in Table II. A plot of the mean message delay D versus the input traffic rate for $M = 1, 2, 5, 10, 100$ appears in Fig. 1.

The lower bound on $S_{\max}$ that appears in Table I is not tight. In fact, we can assert that $S_{\max} \approx \lambda^u$ (where $\lambda^u = \lambda_{24}^u$). The fact that the upper bound λ_{24}^u, which is calculated by solving the finite system in (5) for $N = 24$, is the same (up to the third decimal digit) with λ_5^u, which is obtained by solving (5) for $N = 5$, indicates that λ^u is very close to the value $S_{\max}$ given by the infinite dimensionality system; the latter is the limit of λ_N^u as N approaches infinite. The previous argument can be justified by repeating the procedure of finding λ^1 and using for the upper bound x_k^u of Lemma 1 the expressions, [12],

$$x_0^u=1, \quad x_k^u=(1+\epsilon)L_k^1, \quad 1\leq k\leq 7 \tag{14a}$$

$$x_k^u=\beta(\lambda, p)k-\gamma(\lambda, p),\ 8\leq k<\infty \tag{14b}$$

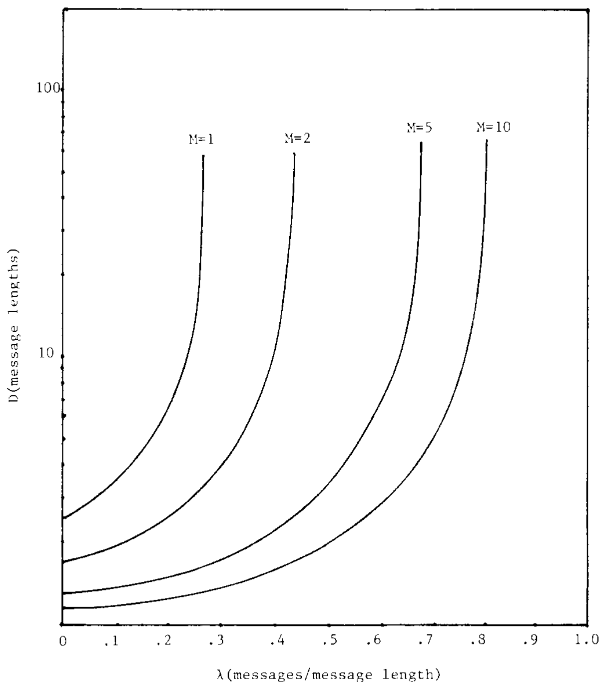

Fig. 1. Analytical results for the average packet delay D, versus the input traffic rate λ with the number of packets per message M as a parameter.

where ϵ is an arbitrarily small positive number. A sequence $\{x_k^u\}_{k=0}^{\infty}$ as in (14) which satisfies the conditions of Lemma 1, was possible to obtain for $\lambda \leq \lambda^{u}$ (where $\lambda^{u} \approx \lambda^u$ up to the third decimal digit), and thus $S_{\max} \approx \lambda^u$. Then, the mean message delay D, for $\lambda^1 \leq \lambda \leq \lambda^u$, was computed and it was found that it increases rapidly to infinity, as it was expected.

From Table I and Fig. 1 it can be concluded that the performance of the system increases substantially as the number of packets per message M increases. This was expected since the portion of time that it involved in the channel sensing (in message lengths) decreases as M increases. As a result, the suggested protocol turns out to be efficient in systems in which a message is formed by a number of packets, the length of the latter being determined by the minimum time required for the channel status identification.

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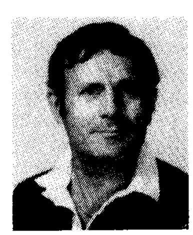

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