ANALYSIS OF INTERCONNECTED BUFFERS IN PACKET NETWORKS

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In this paper a system of two interconnected nodes is analyzed. Then, the behavior of the buffers is investigated under some simple routing policies adopted in the first node.

I. Introduction

In this paper, the queueing behavior of the interconnected buffers shown in Fig. 1 is investigated; R^i_{kj} denotes the packet traffic which enters node k and is to be forwarded to node j, through possibly more than one paths; R^o_{kj} denotes the packet traffic departing from node k and forwarded directly to node j. This system of interconnected buffers may be found in the simple topology shown in Fig. 2. At first, the behavior of both buffers of the system shown in Fig. 3 is studied, under a general independent discrete time packet arrival process R^i_{12} and a dependent discrete time packet arrival process R^i_{23}. Notice that no routing is incorporated in node 1 and thus $\lambda^i_{12} = \lambda^o_{12}$, where λ^i_{kj} and λ^o_{kj} denote packet rates associated with the packet processes R^i_{kj} and R^o_{kj}, respectively. Three steps are followed in the analysis of this queueing system. At first, the queueing behavior of buffer 1 is analyzed. Then, the (dependent) packet output process R^o_{12} is described. Finally, buffer 2 is analyzed by incorporating the (dependent) packet processes R^o_{12} and R^i_{23}. In the sequel, the behavior of both buffers of the system in Fig. 3 is studied under some simple routing policies applied to node 1, by following the three steps described before. By maintaining a fixed packet output rate, λ^o_{12} (assuming that λ^i_{23} is constant), it is observed that the routing policies in node 1 result in different queueing behavior of buffer 2. This is due to the fact that, despite the equality of the intensity of the resulting packet rate, different routing policies in node 1 generate statistically different packet output processes R^o_{12}.

II. Analysis of two buffers without routing decisions

In this section, the system shown in Fig. 3 is analyzed. The packet service rates at both nodes are constant and equal to one packet per slot; the slot is defined to be the time distance between two consecutive potential packet arrival instants. The packet input process to the first node, R^i_{12}, is assumed to be a **Generalized Bernoulli Process (GBP)**. That is, the number of packets arriving at node 1 at the potential arrival instants is an independent process and it follows a general distribution. This process is determined by a message (or multi packet) arrival rate, r, and a general distribution, g(j), $1 \le j \le N_B$, of the message (or multi packet) size in packets; N_B is the maximum message length or number of packets which may arrive during a single slot. The packet input process to node 2 is assumed to be a compound process consisted of two independent packet streams. Input R^o_{12} represents the packet output process from the first node. Input R^i_{23} is assumed to be described by a **Markov Modulated Generalized Bernoulli Process (MMGBP)** which is, in general, a dependent process. That is, R^i_{23} is described by a discrete time process $\{a_j\}_{j\ge 0}$ which depends on an underlying Markov chain $\{z_j\}_{j\ge 0}$; $a(z_j)=k$ with probability $\phi(z_j,k)$. Furthermore, it is assumed that there is at most one state, x_0 such that $\phi(x_0,0)>0$ and that the rest of the states of the underlying Markov chain result in at least one (but a finite number of) packet arrivals.

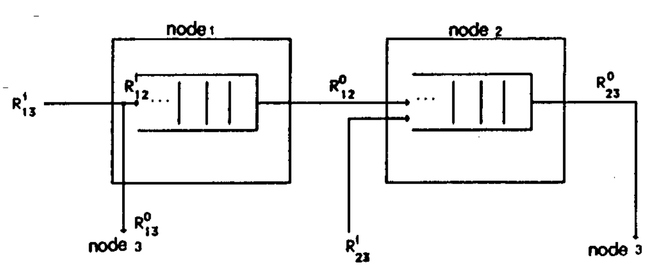

Figure 1
A system of two interconnected buffers with packet routing.

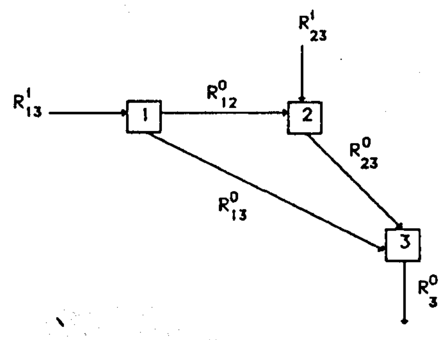

Figure 2
A 3-node element of a topology of a packet communication network.

II.1. Queueing behavior of buffer 1.

The queueing behavior of the buffer in node 1 is studied for the cases of finite $(K<\infty)$ and infinite capacity. The outcome of this study is the derivation of the expressions for the calculation of the first two moments, Q_1 and Q_2, respectively, and the variance, V, of the buffer occupancy and the mean packet delay, D, induced by buffer 1. Let A denote any one of these quantities, $A \in \{Q_1, Q_2, V, D\}$.

II.1-(a) Infinite buffer capacity.

Closed form expressions for the queueing quantities of interest, A, are computed by using the results of the analysis of the multiplexer in [3]; a single state underlying Markov chain describes the GBP. The following closed form expressions are obtained in this case.

$$Q^I_1 = \lambda + \frac{\sigma-\lambda}{2(1-\lambda)}$$

where σ is the second moment of the packet arrival process. By applying Little's theorem to the previous the following expression for the mean packet delay D^I, is obtained.

$$D^I = 1 + \frac{\sigma-\lambda}{2\lambda(1-\lambda)}$$

The variance of the buffer occupancy is given by

$$V^I = Q_2^I - (Q_1^I)^2$$

where Q_2^I is the second moment of the buffer occupancy given by

$$Q_2^I = \frac{1}{3(1-\lambda)}\left[\mu^{3f}+3\mu^{2f}(Q_1^I-1+\lambda)\right] + Q_1^I$$

where

$$\mu^{2f} = \sum_{\nu=0}^{\infty}(\nu-1)(\nu-2)g(\nu) \ , \ \mu^{3f} = \sum_{\nu=0}^{\infty}(\nu-1)(\nu-2)(\nu-3)g(\nu)$$

II.1-(b) Finite (moderate size) buffer capacity

When the capacity K of the buffer in node 1 is finite and of small or moderate size, then the queueing quantities A induced by node 1 can be calculated from the following equations

$$Q_1^I = \sum_{i=0}^{K}\pi(i)i \ , \ V^I = Q_2^I - (Q_1^I)^2 \quad (1a)$$

$$Q_2^I = \sum_{i=0}^{K}\pi(i)i^2 \ , \ D^I = Q_1^I/\lambda \quad (1b)$$

where $\pi(i)$, $0\le i\le K$, are the steady state probabilities of the Markov chain $\{d_j\}_{j\ge0}$, where d_j denotes the number of packets in the buffer of node 1 at the j^{th} slot. Since,

$$d_{j+1} = (d_j-1)^+ + a_j$$

where $(x)^+ = x$ if $x\ge0$ and it is zero otherwise and a_j denotes the packet arrivals during the j^{th} slot, it is clear that $\{d_j\}_{j\ge0}$ is a Markov chain; its transition probabilities can be easily obtained. The steady state probabilities $\pi(i)$, $0\le i\le k$ can be obtained from the equations

$$\sum_{i=1}^{k}\pi(i) = 1 \ , \ \Pi P = \Pi \quad (2)$$

where Π is the vector of the steady states probabilities and P is the matrix of the transition probabilities.

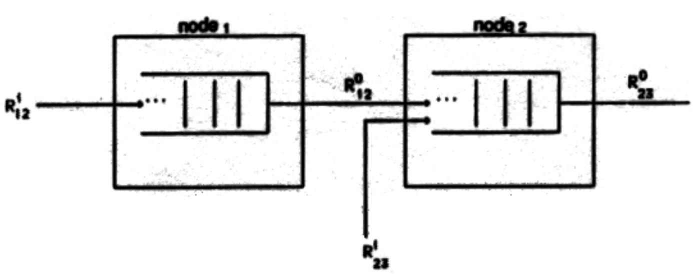

Figure 3
A system of two interconnected buffers without packet routing.

II.1-(c) Finite (arbitrarily large) buffer capacity.

When the buffer capacity K is finite but large, the accurate values of the quantities of interest can be obtained, in principle, from equations (1). The steady state probabilities $\pi(i)$, $0\le i\le K$, are obtained from the solution of a large number of equations given by (2). When the system operates outside its instability region (i.e. $\lambda<1-\epsilon$, $\epsilon>0$), the expected solutions $\pi(i)$, $0\le i\le K$, given by (2) become vanishingly small as i increases. These solutions may also be inaccurate particularly beyond some k_0, $k_0<K$. To overcome this computational difficulty, bounds on the queueing quantities of interest are derived, by introducing the concept of the dominant systems.

Consider the two nodes C^∞ and C^L which are identical to node 1, except from the capacity of the corresponding buffers. Node C^∞ has an infinite capacity buffer; node C^L has a buffer capacity of size $L<K$, where K is the capacity of node 1. Let A^j, denote the queueing quantity A associated with node C^j, $j=L,\infty$. It is easy to justify that

$$Q_1^L \le Q_1 \le Q_1^\infty \ , \ Q_2^L \le Q_2 \le Q_2^\infty \quad (3a)$$

$$D^L \le D \le D^\infty \quad (3b)$$

$$Q_2^L - (Q_1^\infty)^2 \le V = Q_2 - (Q_1)^2 \le Q_2^\infty - (Q_1^L)^2 \quad (3c)$$

where Q_1, Q_2, V, and D are the queueing quantities, A, associated with node 1. The previous equations establish upper and lower bounds on Q_1, Q_2, V and D. In most practical applications, where buffer overflow is not desired, K should be sufficiently large so that $\pi(K)$ be extremely small. Under such condition it is expected that $A\approx A^\infty$, $A\epsilon\{Q_1,Q_2,D,V\}$.

To analyze the queueing behavior of buffer 2 an accurate description of the packet (output) process generated by node 1 is required.

II.2. The packet output process of node 1, R_{12}^o

Although the packet input process to node 1, described by a GBP, is an independent one, the process of the packets departing from this node is not, due to the dependencies introduced by the operation of the node. This process is shown to be accurately described by a MMGBP.

Let $\{d_j\}_{j\ge0}$ be the Markov chain defined in section II.1-(b) which describes the number of packets in node 1 at the end of the j^{th} slot. Let $S = \{0,1,2, \ ,K\}$ be its state space, where K, $K\le\infty$, is the buffer capacity of node 1. If K is finite and of small or moderate value, then $\{d_j\}_{j\ge0}$ can serve as the underlying Markov chain of the MMGBP to be used for the description of R_{12}^o. If K is very large or infinite, then a new Markov chain, $\{d_j^L\}_{j\ge0}$, with a state space of reduced cardinality L, $L<K$, will be incorporated in the description of R_{12}^o, to lead to tractable computations of the queueing quantities in node 2. Although the description of R_{12}^o based on $\{d_j^L\}_{j\ge0}$ is an approximate one, it turns out that it results in a very accurate calculation of the queueing quantities of interest at node 2. The MMGBP which describes the packet output process R_{12}^o is determined by the probability distribution $\phi(\bar d_j,k)$, where

$$\phi(0,0)=1 \ , \ \phi(\bar d_j,1)= 1 \text{ for } \bar d_j>0 \ , \ \bar d_j\epsilon\{d_j,d_j^L\} \quad (4)$$

since node 1 outputs one packet when $\bar d_j>0$ and zero packets otherwise.

Let $\{a_j\}_{j\ge0}$ and $\{a_j^L\}_{j\ge0}$ be the packet output processes determined by the probability distribution given by (4) and the underlying Markov chains $\{d_j\}_{j\ge0}$ and $\{d_j^L\}_{j\ge0}$, respectively. As it is mentioned before, the queueing quantities A associated with node 2 are fairly accurately calculated under the approximation of the true packet output process $\{d_j\}_{j\ge0}$ by the process $\{d_j^L\}_{j\ge0}$, as long as node 1 operates outside its instability region. Under the latter conditions, states i, $i>k_0$, for some $k_0<L$, are almost never visited by the true Markov chain $\{d_j\}_{j\ge0}$. Thus, the Markov chain $\{d_j^L\}_{j\ge0}$, for some $L>k_0$, is expected to be a good approximation of $\{d_j\}_{j\ge0}$. In addition, the number of packets, a_j, generated by node 1 in the j^{th} slot, is the same under both Markov chains and independent of the their state, as long as it is a nonzero state (see (4)). Due to the latter observation, an additional refinement in the approximation of $\{d_j\}_{j\ge0}$ by $\{d_j^L\}_{j\ge0}$, as seen from the output process $\{a_j^L\}_{j\ge0}$, is introduced, as long as $L>0$. Finally, the queueing process in node 2 is a complex one and it may also introduce some smoothing on the differences between the processes $\{a_j\}_{j\geq0}$ and $\{a_j^L\}_{j\geq0}$, and consequently deemphasize the difference between $\{d_j\}_{j\geq0}$ and $\{d_j^L\}_{j\geq0}$, as inferred from the values of the queueing quantities A evaluated at node 2. The previous arguments offer some non-rigorous explanations of the expected (and observed) accuracy of the approximation of $\{d_j\}_{j\geq0}$ by $\{d_j^L\}_{j\geq0}$ (and, consequently, of the approximation of $\{a_j\}_{j\geq0}$ by $\{a_j^L\}_{j\geq0}$), as measured by the accuracy of the calculation of the queueing quantities A at node 2.

II.3. Queueing behavior of buffer 2

At this point, the queueing behavior of node 2 is studied under the packet arrival processes R_{12}^o and R_{23}^i, each of which is modeled as a MMGBP; R_{12}^o is the packet output process from node 1 and it is described in section II.2; R_{23}^i is an arbitrary MMGBP as defined at the beginning of section II. The queueing quantities A associated with node 2 are calculated when the cardinalities of the Markov chains associated with R_{12}^o and R_{23}^i are of small or moderate value.

When the capacity of buffer 2 is infinite, then the analysis presented in [3] can be applied and the queueing quantities of interest A, $A\epsilon\{Q_1,Q_2,V,D\}$ be computed. The obtained results are exact if the underlying Markov chain associated with R_{12}^o is the true one (i.e. if the capacity of buffer 1 is of small or of moderate size) and they are approximate otherwise.

When the capacity of buffer 2 is (i) of small or moderate size or (ii) large but finite and the operation of node 2 is away from its instability region, then the queueing quantities of interest can be computed as described in sections II.1-(b) and II.1-(c), where the Markov chain $\{d_j\}_{j\geq0}$ is replaced by the 3-dimensional Markov chain $\{x_j^o,x_j^i,d_j\}_{j\geq0}$; x_j denotes the underlying Markov chain of the packet arrival process R_{12}^o; x_j^i denotes the underlying Markov chain of the packet arrival process R_{23}^i; d_j denotes the buffer occupancy of node 2.

III. Analysis of two buffers under some routing policies

In this section, the queueing system shown in Fig. 1 is studied. The only difference between this system and the one shown in Fig. 3 (studied in section II) is that routing decisions diverse some of the packet input traffic to node 1, R_{13}^i. The system shown in Fig. 2 appears in network topologies such as the one shown in Fig. 2, where the packet traffic which enters node 1 and is to be forwarded to node 3 has two alternate routes; a direct one from node 1 to node 3 and an indirect one through node 2. The routing policies at node 1 may be adopted for the regulation of the rate of the traffic which is forwarded to node 2. As a result, overloading of the links between nodes 2 and 3 and between nodes 1 and 3, may be avoided. The following routing policies will be considered; all packets to be forwarded to node 2 are stored in buffer 12.

(P_1) Buffer 12 stores all packets up to a maximum number $\Theta<\infty$.

(P_2) Buffer 12 stores half (or a portion) of the packets arriving over a slot (or half of them plus one in case of an odd number of packet arrivals), according to a deterministic splitting, up to a maximum Θ_2; Θ_2 can be infinite.

III.1. Queueing behavior of node 1.

The queueing quantities of interest A, $A\epsilon\{Q_1,Q_2,V,D\}$, are computed by applying the analysis presented in section II. 1-(b), where the buffer capacity K is set to be equal to Θ under policy P_1. The analysis presented in sections II.1-(b), II.1-(c) or II.1-(a) is applied for the calculation of A, $A\epsilon\{Q_1,Q_2,V,D\}$, under policy P_2, depending on whether the buffer capacity Θ_2 is small, large or infinite, respectively. Notice that the splitting of the packets which arrive over the same slot modifies the message length distribution g(j), as seen from buffer 12, resulting in a better randomized packet output process under this routing policy.

III.2. The packet output process from node 1, R_{12}^o

The packet output process R_{12}^o is modeled as a MMGBP, as described in section II.2. The (exact) underlying Markov chain $\{d_j\}_{j\geq0}$ is incorporated in the description of the packet output process $\{a_j\}_{j\geq0}$ under policy P_1, (if Θ_2 is small). The (approximate) underlying Markov chain $\{d_j^L\}_{j\geq0}$ is incorporated in the description of the (approximate) packet output process $\{a_j^L\}_j$, under policy P_2, when Θ_2 is very large or infinite.

III.3 Queueing behavior of node 2.

The queueing quantities of interest A, $A\epsilon\{Q_1,Q_2,V,D\}$ are computed by applying the analysis approach presented in section II.3.

IV. Numerical results

The following parameters for the input process to node 1 are considered: N_B=5, g(1)=.1, g(2)=.3, g(3)=.3, g(4)=.1, g(5)=.1 and various values of the message arrival rate r. Let r_{in} and r_{out} be the packet input and the packet output rates associated with node

At first, the system shown in Fig. 3 is considered. For various values of r and K (the capacity of buffer 1), $1\leq K\leq\infty$, the queueing quantities A, associated with node 1, are exactly computed. The results are shown in Table 1. From these results, the monotonicity of the quantities Q_1, Q_2 and D with respect to the buffer capacity K, as indicated by equations (3), is clearly observed. Notice also that the values of A, computed for the case of the largest finite value of K shown in Table 1, practically coincide with those obtained under infinite buffer capacity. The latter suggests that the closed form expressions mentioned in II.1-(a) may be used for the computation of A, for any buffer capacity which is larger than a certain value.

When the buffer capacity of buffer 1 is finite and equal to 20 and the capacity of buffer 2 is infinitely large, the queueing quantities of interest, A, associated with node 2 are are shown in Table 2; γ is the clusterness coefficient associated with the packet output process from node 1, R_{12}^o, defined as $\gamma = p(\bar{0},\bar{0})-p(0,\bar{0})$, where 0 is the zero packet generating state of the underlying Markov chain of the MMGBP which models the process R_{12}^o (as described in section II.2), and $\bar{0}$ is the union of all the packet generating states of this Markov chain; $p(0,\bar{0})$ denotes the transition probability from 0 to o. As it is observed below, the parameter γ affects the values of the queueing quantities of interest associated with node 2. The (independent from R_{12}^o) packet arrival process R_{23}^i is assumed to be a 2-state MMGBP with clusterness coefficient γ'=.2 and packet arrival rate equal to .9$-r_{out}$, where r_{out} is the packet rate of R_{12}^o and .9 is the total packet traffic load offered to node 2. A 2-state MMGBP is completely determined by r and γ. Notice that for r≤.20 the behavior of buffer 1 is identical to that of a buffer with infinite capacity ($r_{out}^{20}=r_{out}^{\infty}$, where r_{out}^{K} is the packet output rate of a buffer of capacity K) and that the clusterness coefficient γ is identical to that corresponding to the packet output process of an infinite capacity buffer. The latter is true since practically no packet rejection, which would modify the clusterness of the packet output process, takes place and, thus, γ is completely determined by the message size distribution g(j) and not by the message input rate to node 1, r. For r=.3, some packet rejection takes place ($r_{out}^{20}=.777 < .78 = r_{out}^{\infty}$) which leads to a slightly reduced clusterness coefficient γ.

When the capacity of the buffer 1 is infinite, then the queueing results in node 2 are obtained as presented in II.1-(a), where the MMGBP describing the packet output process R_{12}^o is approximately described based on a truncated Markov chain with state space of cardinality L, as described in section II.2. The results for the queueing quantities of interest associated with node 2 are shown in Table 3, for various message input rates to the first node, r, and different values of L. For message input rates r less than .20, a truncation of the true underlying Markov chain associated with R^o_{12} at L=20 gives result which remain, in essence, unchanged for any L>20; the latter is observed at L=50 when r=.3. If processes R^o_{12} and R^i_{23} are approximated by a Bernoulli process, then the mean packet delay induced by node 2 is given in Table 3, for the various input rates r and total traffic load offered to node 2 equal to .9. By comparing the mean delay results in node 2, shown in Table 3, under the MMGBP and the Bernoulli process modeling R^o_{12}, it is easily established that the Bernoulli approximation leads to significant underestimation of the queueing problems induced by node 2.

Finally, when the packet routing policies P_1 and P_2 are considered in buffer 1, the queueing quantities of interest associated with node 1 and 2 are shown in Tables 4 and 5, respectively. The message size probability distribution g(j), $1 \le j \le 5$, remains the same as before. The message arrival rate is different in each case and it is such that the output rate r^o_{12} be equal to .45; r^i_{23} = .45 and γ^i=.3. Various values of the buffer limit Θ have been considered under policy P_1. Notice that the value of A, increases in both nodes as Θ increases, although r^o_{12} remains constant, due to the increased clusterness γ (affecting the value of A associated with node 2) and the increased buffer size Θ (affecting the value of A associated with node 1), under the same probability distribution g(j), $1 \le j \le 5$. Finally, the results under policy P_2 and for various values of the buffer constrain Θ_2 (denoted by Θ) are given in the same table. The new probability distribution $g'(j)$, $1 \le j \le 3$, is given by $g'(1)=g(1)+g(2)=.5$, $g'(2)=g(3)+g(4)=.4$ and $g'(3)=g(5)=.1$. Notice that as the buffer constrain Θ increases the values of A, also increase for the reasons stated before. Notice that the values of A under policy P_2 are smaller than these under policy P_1 corresponding to the same value of Θ, the reason being that the probability distribution $g'(j)$, $1 \le j \le 3$, has been changed under policy P_2 and less clustered packets are generated by $g'(j)$. The values of A obtained for Θ=10, under policy P_2 remain unchanged if a larger value of Θ is considered. Thus, these values of A can be considered to be equal to the ones obtained for $\Theta=\infty$.

r	r_{in}	r_{out}	K	Q_1	Q_2	V	D
.10	.26	.100	1	.100	.100	.090	1.000
.10	.26	.253	5	.579	1.693	1.358	2.288
.10	.26	.258	7	.621	1.980	1.594	2.404
.10	.26	.260	10	.636	2.116	1.710	2.450
.10	.26	.260	∞	.638	2.141	1.733	2.455
.20	.52	.200	1	.200	.200	.160	1.000
.20	.52	.484	5	1.234	3.963	2.440	2.550
.20	.52	.506	7	1.457	5.565	3.444	2.877
.20	.52	.517	10	1.610	7.066	4.473	3.117
.20	.52	.520	15	1.676	7.951	5.143	3.225
.20	.52	.520	20	1.685	8.125	5.286	3.241
.20	.52	.520	∞	1.687	8.159	5.314	3.243
.30	.78	.300	1	.300	.300	.210	1.000
.30	.78	.680	5	1.949	6.843	3.044	2.868
.30	.78	.752	10	3.228	18.804	8.381	4.293
.30	.78	.777	20	4.293	39.892	18.462	5.527
.30	.78	.780	30	4.540	43.737	23.118	5.825
.30	.78	.780	∞	4.598	46.028	24.885	5.895

Table 1
Queueing behavior of buffer 1 without routing policies.

r	r_{out}	Bernoulli	Q_1	Q_2	V	D
.05	.130	2.112	3.390	22.963	13.859	3.767
.10	.260	2.849	5.402	59.330	35.554	6.000
.15	.390	3.210	6.897	99.634	58.960	7.664
.20	.520	3.195	7.796	131.000	78.026	8.662
.30	.777	2.060	6.257	87.895	55.000	6.953

Table 2
Queueing behavior of buffer 2 without routing policies in node 1 and buffer 1 capacity K=20 (γ=.615).

r	$r_{in}=r_{out}$	L	Q_1	Q_2	V	D	Bernoulli
.10	.26	1	5.642	64.928	38.734	6.269	2.849
.10	.26	5	5.292	55.643	32.932	5.880	2.849
.10	.26	10	5.401	59.283	35.516	6.000	2.849
.10	.26	20	5.402	59.344	35.565	6.002	2.849
.20	.52	1	6.532	83.163	47.032	7.257	3.195
.20	.52	5	7.176	104.446	60.124	7.974	3.195
.20	.52	10	7.753	128.475	76.121	8.614	3.195
.20	.52	20	7.803	131.403	78.317	8.670	3.195
.30	.78	1	3.568	21.707	12.547	3.964	2.040
.30	.78	5	4.800	44.093	25.854	5.333	2.040
.30	.78	10	6.014	78.058	47.201	6.682	2.040
.30	.78	20	6.569	102.015	65.422	7.300	2.040
.30	.78	30	6.641	106.611	69.145	7.379	2.040
.30	.78	50	6.651	107.516	69.922	7.391	2.040

Table 3
Queueing behavior of buffer 2 without routing policies in node 1 and infinite capacity of buffer 1 (γ=.615).

P	r	r_{in}	γ	Θ	Q_1	Q_2	V	D
P1	.2794	.7264	.687	2	.687	1.160	.689	1.526
P1	.1845	.4797	.590	5	1.128	3.571	2.297	2.506
P1	.1738	.4519	.614	10	1.306	5.311	3.605	2.901
P1	.1731	.4501	.615	20	1.331	5.718	3.945	2.958
P2	.3210	.5136	.288	2	.642	1.024	.612	1.424
P2	.2822	.4515	.373	5	.796	1.784	1.150	1.769
P2	.2813	.4508	.375	10	.808	1.882	1.229	1.795

Table 4
Queueing behavior of buffer 1 under some routing policies with r_{out}=.45

P	r	r_{in}	γ	Θ	Q_1	Q_2	V	D
P1	.2794	.7264	.379	2	4.247	30.810	17.039	4.716
P1	.1845	.4797	.590	5	6.161	73.569	41.771	6.846
P1	.1738	.4519	.614	10	7.223	108.527	63.581	8.025
P1	.1731	.4501	.615	20	7.395	116.005	68.715	8.216
P2	.3210	.5136	.288	2	4.126	28.924	16.028	4.584
P2	.2822	.4515	.373	5	4.959	45.720	26.081	5.511
P2	.2813	.4508	.375	10	5.041	47.912	27.535	5.602

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