Analysis of a Class of Star-Interconnected Networks

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ABSTRACT

In this paper, a certain class of multiuser communication networks interconnected according to a star topology is analyzed. A type of a Markov modulated Bernoulli model is developed for the exact characterization of their output processes and the mean packet delay induced by the interconnecting scheme is calculated. The latter is obtained through an approximate analysis of the queueing system which is formulated in the central node of the star topology.

It is shown that ALOHA multi user communication networks belong in the investigated class of networks. Delay analysis of ALOHA networks interconnected according to a star topology is carried out and some numerical results are obtained. These results, and those obtained under Bernoulli approximation on the network output processes, are compared with simulations.

I. Introduction

Although the significance of the interconnection of multi-user communication networks (MUCNs) has been well realized and many (usually ad-hoc) interconnecting schemes have been proposed, [1] - [8], no substantial effort has been directed towards the development of analytical techniques for the performance evaluation of such systems. So far, performance evaluation of network interconnecting schemes has been heavily based on simulation results. The few attempts to analytically evaluate the performance of these systems deal with simple ad-hoc interconnecting schemes and they are usually based on crude model assumptions. These assumptions are usually incorporated in the description of the output process of a multi user communication network. The characterization of this process is of fundamental importance to the analysis of interconnecting schemes. It is the input process to the interconnecting system and affects considerably its operation. In [9], the Bernoulli model for the output process of a CSMA/CD network is implied. In [10] the authors consider the output process of ALOHA and CSMA networks by making the assumptions of the heavy traffic conditions and the memoryless property. Memoryless output processes are also implied in [11] and [12] in the analysis of two-hop ALOHA and CSMA packet radio networks and in [13] in the case of the multi-hop extension. The output process of MUCNs is a highly dependent process and memoryless models are meaningless. The packet interdeparture process of ALOHA and CSMA networks is derived in [14].

In previous work, [15], [16], we derived Markovian approximations on the output process of a certain class of multi user random access communication networks (MURACNs). This class contains all MURACNs whose analysis utilizes the process of the renewal points induced by the operation of the deployed protocol. Most continuous and limited sensing algorithms fall into this category. Then, the performance of such interconnected MUCNs was evaluated by incorporating the Markovian approximations in the output processes of the involved networks.

In this paper we develop a method for the evaluation of the mean delay of a system of networks interconnected according to a star topology. The star topology may be the supporting scheme of an interconnecting facility or it may be an interconnecting node in a Metropolitan Area Network (MAN), supporting not only the interconnection of MUCNs but a number of other information transfer facilities, [17], [18].

The analysis approach presented in this paper introduces a simple new way of describing the output process of a communication network. The communication networks whose output process can be described by incorporating the proposed model, define the class of star interconnected communication networks whose performance analysis can be carried out as outlined in this paper. The finite user population ALOHA network is a MUCN from this class. The performance evaluation of N ALOHA networks interconnected according to a star topology will be carried out as an example of the application of the general procedure.

In the rest of the paper, discussion will be focused on multi user communication networks (MUCN). The output process of such networks is difficult to completely describe, so we have decided to illustrate the developed model by incorporating such networks. It should be clear though that the described class of networks is much wider. As it is emphasized in the conclusions of this work, the analyzed star topology could be an exact or a satisfactory approximate model of other practical systems.

II. The Output Process

Consider a slotted MUCN; it is assumed that the length of a slot is equal to the time required for a packet transmission and that packet transmissions can be attempted only at the beginning of the slots. The Bernoulli packet generation model is adopted for each user, with probability of packet arrival λ packets/slot, in the case of finite user population; the Poisson packet generation model is adopted for the cumulative packet arrival process, with intensity λ packets over a slot, in the case of the infinite user population. In both cases, the packet arrival process is memoryless.

The above commonly adopted model assumptions usually result in a system whose operation can be described by a Markov chain embedded at the beginning or the end of the slots. The MUCNs for which such a Markov chain (with finite state space) can be found, determine the class of MUCNs which are considered in this paper. More precisely consider the following definitions.

Definition 1:

The output process of a slotted MUCN is defined to be the binary discrete time process $\{a_j\}_{j\geq 0}$ of the departing packets; $a_j=1$ if a packet leaves the MUCN at the j^{th} slot and $a_j=0$ otherwise.

Note that in the case of a contention-free MUCN, $\{a_j\}_{j\geq 0}$, is the process of the channel status (or activity). In the case of MURACNs, $\{a_j\}_{j\geq 0}$, is the process of the successfully transmitted packets.

Definition 2

Define C to be the class of slotted MUCNs which satisfy the following:

(a) There exists a finite state Markov chain $\{z_j\}_{j\geq 0}$, embedded at the beginning or the end of the slots, which describes the evolution of the system. Let $S=\{x_0,x_1, \cdots ,x_M\}$ be the state space of $\{z_j\}_{j\geq 0}$.

(b) For any state transition (say from x_i to x_k), there exists a stationary probabilistic mapping $a(x_i,x_k) : SxS\rightarrow\{0,1\}$, which describes the channel activity in the slot over which such a state transition took place. Let $a(x_i,x_k)=1$ with probability $\phi(x_i,x_k)$ and $a(x_i,x_k)=0$ with probability $1-\phi(x_i,x_k)$.

Definition 3

Following definitions 1 and 2 we define the output process of a MUCN from class C to be the process

$$\{a_j\}_{j\geq 0} = \{a_j(x_i,x_k)\}_{j\geq 0}.$$

That is, the output process is described in terms of a type of a Markov modulated Bernoulli process; the output process is a Bernoulli process whose intensity depends on the state transition of an underlying Markov chain. To the best of our knowledge, this is the first time that such a process is incorporated in the description of the output process of a multi user communication network.

III. The Output Process of ALOHA MUCNs.

In this section we describe the output process of a slotted single buffer finite user population ALOHA MUCN, [11], [14], [19], [20], [21], [22]. As it will become clear shortly, this MUCN belongs to class C. The performance of this MUCN has been analyzed through the formulation of an appropriate Markov chain.

Let M be the number of users of the MUCN. A user can be either active (if its buffer is non-empty) or inactive (if its buffer is empty). An active user can be either a backlogged one (if its buffer was non-empty at the beginning of the current slot) or a new one (if its buffer was empty at the beginning of the current slot). The per user packet generation process is assumed to be Bernoulli with per slot probability of packet arrival λ. The single buffer assumption implies that new packets which find the corresponding buffer full are discarded.

Two policies may be considered, the Delayed First Transmission (DFT) and the Immediate First Transmission (IFT). Under the DFT policy, new and backlogged users (at the end of the last slot) transmit at the beginning of the current slot with probability p. Under the IFT policy, the backlogged users transmit at the beginning of a slot with probability p and the new users transmit with probability 1.

Let us assume that the length of a slot equals one. We define the j^{th} slot to be the time interval (j, j+1). Let z_j be the number of active users (under the DFT policy) or the number of backlogged users (under the IFT policy) at the end of the j^{th} slot. Under the IFT policy we assume that the new arrivals over a slot appear at the beginning of this slot, [14], [22]. It is easy to see that $\{z_j\}_{j\geq 0}$, is a Markov chain under both policies with state space $S=\{0,1,2, \cdots ,M\}$. The transition probabilities of this Markov chain have been derived for the analysis at these ALOHA protocols, [13], [21], [22], and are given by

(a) Under DFT policy:

$$p(k,j) = \begin{cases} 0 & j<k-1 \\ \tau_{kj} + \sigma_{kj} & k-1\leq j\leq M \end{cases} \qquad (1a)$$

where

$$\tau_{kj} = [1-b_k(1)] \binom{M-k}{j-k} \lambda^{j-k}(1-\lambda)^{M-j}$$

$$\sigma_{kj} = b_k(1) \binom{M-k+1}{j-k+1} \lambda^{j-k+1}(1-\lambda)^{M-j}$$

(b) Under IFT policy:

$$p(k,j) = \begin{cases} 0 & \text{if } j<k-1 \\ b_k(1)(1-\lambda)^{M-k} & j=k-1 \\ \tau'_{kj} + \sigma'_{kj} & j=k \\ (M-i)\lambda(1-\lambda)^{M-k-1}(1-b_k(0)) & j=k+1 \\ \binom{M-k}{j-k}\lambda^{j-k}(1-\lambda)^{M-j} & j \geq k+2 \end{cases} \quad (1b)$$

where

$$\tau'_{kj} = [1-b_k(1))(1-\lambda)^{M-k}$$

$$\sigma' = (M-k)\lambda(1-\lambda)^{M-k-1}b_k(0)$$

$$b_k(0)=(1-p)^k \text{ , } b_k(1)=kp(1-p)^{k-1} \text{ , } \binom{m}{k}=0 \text{ , } k<0.$$

Given the no buffering assumption and the finiteness of the user population, the system has a well defined steady state behavior for all arrival rates [22]. Let **P** denote the state transition probability matrix. The stationary distribution $\Pi = (\pi(0), \pi(1), \cdots ,\pi(M))$ where $\pi(k) = \lim_{j \to \infty} Pr(z_j=k)$ is simply obtained by solving the system

$$\Pi = \Pi \mathbf{P} \quad (1c)$$

Since $p(k,j)=0$ for $j<k-1$ the system can be solved recursively, [21].

Suppose that the Markov chain $\{z_j\}_{j\geq 0}$ moves from state k at time j-1 to state i at time j. Let a(k,i) be a binary random variable that describes the channel activity over the j^{th} slot; a(k,i) equals 1 if a successful packet transmission took place in the j^{th} slot and it is 0 otherwise; a(k,i) is a Bernoulli random variable which is completely described by states k and i and the policy under consideration. More specifically, the expressions for the transition probabilities lead to the following:

$$a(k,i) = \begin{cases} 1 & \text{with probability } \phi(k,i) \\ 0 & \text{with probability } 1-\phi(k,i) \end{cases} \quad (2)$$

where

(a) Under DFT policy

$$\phi(k,i) = \begin{cases} 0 & \text{if } i<k-1 \\ \dfrac{\sigma_{ki}}{\sigma_{ki} + \tau_{ki}} & \text{if } k-1 \leq i \leq M \end{cases} \quad (3a)$$

(b) Under IFT policy

$$\phi(k,i) = \begin{cases} 1 & \text{if } i=k-1 \\ \dfrac{\sigma'_{ki}}{\sigma'_{ki} + \tau'_{ki}} & \text{if } i=k \\ 0 & \text{otherwise} \end{cases} \quad (3b)$$

Using definitions 1 - 3 and the above system description, we can easily conclude that the ALOHA MUCNs considered above belong to class *C* and their output process is completely described by the process $\{a(z_{j-1},z_j)\}_{j\geq 0}$.

IV. Analysis of a single server queueing system.

In this section we study a general queueing system to be used in the analysis of MUCNs interconnected according to a star topology. The asymmetry of the system is due to the fact that although all arrival processes are described by the same model, at least one of their parameters is considered to be different for at least two such processes.

Consider a queueing system with N independent input streams which feed a single server. The server has an infinite capacity buffer. The arrival processes $\{a^i_j\}_{j\geq 0}$, $i = 1, 2, \cdots, N$, are assumed to be synchronized discrete time processes, and at most one arrival can occur in each input line per unit time. The time separation between successive possible arrival points is constant and equal to one. The first in - first out (FIFO) policy is adopted and the service time is assumed to be constant and equal to the distance separation between successive time instants. More than one arrivals (from different input streams) that occur at the same time instant are served in a randomly chosen order.

Let $\{z^i_j\}_{j\geq 0}$ denote a discrete time ergodic Markov process associated with the i^{th} input stream, with finite state space $S^i = \{x^i_1, \cdots x^i_{M_i}\}$. Let also a^i be a stationary probabilistic mapping from the set $S^i x S^i$ into the set {0, 1}, where 1 corresponds to an arrival and 0 to the absence of such an event. Then we define the arrival process of the i^{th} input stream to be

$$\{a^i_j\}_{j\geq 0} = \{a^i(z^i_{j-1},z^i_j)\}_{j\geq 0}$$

From the description of the arrival process it is implied that successive arrivals from the same input stream are not independent, but they are governed by an underlying finite state Markov chain, $\{z^i_j\}_{j\geq 0}$. The arrival process can also be seen as the random reward associated with a state transition of a Markov chain, [23]. Given a transition from state k to state j, the arrival process is described by the probabilistic mapping $a^i(k,j)$, where $a^i(k,j)=1$ with probability $\phi^i(k,j)$ and $a^i(k,j)=0$ with probability $1-\phi^i(k,j)$. It is assumed that the underlying processes $\{z_j^i\}_{j\geq 0}$, i = 1, 2, $\cdots$, N, are mutually independent and thus the arrival processes $\{a_j^i\}_{j\geq 0}$, i = 1, 2, $\cdots$, N, are also independent.

The analysis of the queueing system described above can be carried out by following an approach similar to that used in the analysis of the statistical multiplexer described in [30]. Let $\pi^i(k)$ and $p^i(k, j)$, $k,j \in S^i$, denote the steady state and the transition probabilities of the ergodic Markov chain, $\{z_j^i\}_{j\geq 0}$, i = 1, 2, $\cdots$, N. Let also $p^n(j ; \bar{y})$ denote the joint probability that there are j packets in the system at the n^{th} time instant (arrivals at that point are included) and the states of the Markov chains are $y^1, y^2, \cdots, y^N$, where $\bar{y} = (y^1, y^2, \cdots, y^N)$. The vector $\bar{y}$ describes the state of a new ergodic Markov chain that is generated by the N independent Markov chains described before, with steady state and transition probabilities $\pi(\bar{y})$ and $p(\bar{x}, \bar{y})$ respectively, and with state space $\bar{S} = S^1 x S^2 x \cdots x S^N$. The operation of the system can be described by an N + 1 dimensional Markov chain imbedded at the time instants, with state space T = (0, 1, 2, $\cdots$)x $\bar{S}$ and state probabilities given by the following recursive equations

$$p^n(j;\bar{y})=\sum_{\bar{x}\in\bar{S}}\sum_{\nu=0}^{N} p^{n-1}(j+1-\nu;\bar{x})p(\bar{x},\bar{y})g_{\bar{x}\bar{y}}(\nu)\ ,\ j\geq N+1 \quad (4a)$$

$$p^n(j;\bar{y})=\sum_{\bar{x}\in\bar{S}}\sum_{k=1}^{j+1} p^{n-1}(k;\bar{x})p(\bar{x},\bar{y})g_{\bar{x}\bar{y}}(j+1-k)+$$

$$+\sum_{\bar{x}\in\bar{S}} p^{n-1}(0;\bar{x})\ p(\bar{x},\bar{y})g_{\bar{x}\bar{y}}(j)\ ,\ 0\leq j\leq N \quad (4b)$$

where $\bar{x}$ is the state of the N-dimensional Markov chain at time instant n-1 and

$$g_{\bar{x}\bar{y}}(\nu)=P(\sum_{i=1}^{N} a^i(x^i,y^i)=\nu)\ ,\ \ p(\bar{x},\bar{y})=\prod_{i=1}^{N} p^i(x^i,y^i) \quad (5)$$

$g_{\bar{x}\bar{y}}(\nu)$ is the probability that ν out of the N state transitions determined by $\bar{x}, \bar{y}$, result in an arrival. There are totally $M^1 x M^2 x \cdots x M^N$ equations given by (4) for a fixed j and all $\bar{y} \in \bar{S}$, where M^i is the cardinality of S^i, i = 1, 2, $\cdots$, N.

Ergodicity of the Markov chains associated with the input streams implies the ergodicity of the arrival processes $\{a_j^i\}_{j\geq 0}$, i = 1, 2, $\cdots$, N. The latter together with the ergodicity condition for the total average input traffic λ

$$\lambda = \sum_{\bar{x}\in\bar{S}}\ \sum_{\bar{y}\in\bar{S}} \mu_{\bar{x}\bar{y}}p(\bar{x},\bar{y})\pi(\bar{x}) < 1 \quad (6)$$

where

$$\mu_{\bar{x}\bar{y}} = E\{\sum_{i=1}^{N} a^i(x^i,y^i)\} = \sum_{i=1}^{N} E\{a^i(x^i,y^i)\} = \sum_{i=1}^{N} \phi^i(x^i,y^i)$$

and

$$\pi(\bar{x})=\prod_{i=1}^{N}\pi^i(x^i) \quad ,$$

imply that the Markov chain described in (4) is ergodic and there exist steady state (equilibrium) probabilities. Thus, we can consider the limit of the equations in (4) as n approaches infinity and obtain similar equations for the steady state probabilities. By considering the generating function of these probabilities, manipulating the resulting equations, differentiating with respect to z and setting z=1, we obtain the following system of linear equations.

$$P'(1;\bar{y})=\sum_{\bar{x}\in\bar{S}} P'(1;\bar{x})\ p(\bar{x},\bar{y})+\sum_{\bar{x}\in\bar{S}}(\mu_{\bar{x}\bar{y}}-1)p(\bar{x},\bar{y})\pi(\bar{x})+$$

$$+\sum_{\bar{x}\in\bar{S}} p(0;\bar{x})p(\bar{x},\bar{y})\ ,\ \bar{y}\in\bar{S} \quad (7)$$

The exact calculation of the boundary joint probability $p(0;\bar{x})$ is not possible; we use the following expression to estimate its value

$$p(0;\bar{x})=p_0\frac{\sum_{\bar{z}\in\bar{S}} p(\bar{z},\bar{x})\pi(\bar{z})q_0(\bar{z},\bar{x})}{\sum_{\bar{z}\in\bar{S}}\pi(\bar{z})\sum_{\bar{y}\in\bar{S}} p(\bar{z},\bar{y})q_0(\bar{z},\bar{y})}$$

where $p_0=1-\lambda$ is the probability that there is no customer in the system and $q_0(\bar{z},\bar{x})$ is the probability that the state transition from $\bar{z}$ to $\bar{x}$ results in no customer arrival; the latter probability is easily obtained from the probabilistic mappings in the independent streams and it is given by

$$q_0(\bar{z},\bar{x}) = \prod_{i=1}^{N}(1-\phi^i(z^i,x^i))$$

Notice that the calculation of the above boundary probability is the only point of approximation in this work and it is checked against simulations.

The $M^1 x \cdots M^N$ linear equations with respect to $\bar{y} \in \bar{S}$ that appear in (7) are linearly dependent. This is the case when the equations have been derived from the state transition description of a Markov chain. By manipulating the original equations and using L'Hospital's rule we obtain an additional linear equation with respect to $P'(1 ; \bar{y})$, $\bar{y} \in \bar{S}$, which is linearly independent from those in (7) and is given by

$$\sum_{\bar{x}\in\bar{S}}\Big[2(\mu_{\bar{x}}-1)P'(1;\bar{x})+2(\mu_{\bar{x}}-1)p(0;\bar{x})+$$

$$+[2+\sigma_{\bar{x}}^2+(\mu_{\bar{x}})^2-3\mu_{\bar{x}}]\pi(\bar{x})\Big]=0 \quad (8)$$

where

$$\mu_{\bar{x}} = E_{\bar{y}}[\mu_{\bar{x}\bar{y}}] = \sum_{j=0}^{M}\sum_{i=1}^{N}\phi^i(x^i,y^j)p^i(x^i,y^j)=\sum_{i=1}^{N}\phi^i(x^i) \quad (9)$$

$$\phi^i(x^i)=\sum_{j=0}^{M}\phi^i(x^i,y^j)p^i(x^i,y^j)$$

$$\sigma_x^2 = \sum_{i=1}^{N} \phi^i(x^i)(1-\phi^i(x^i))$$

By solving the $M^1 x \cdots x M^N$ dimensional linear system of equations that consists of (8) and any $M^1 x \cdots x M^N - 1$ equations taken from (7), we compute $P'(1\,;\,\bar{x})$, $\bar{x} \in \bar{S}$. Then, the average number of packets in the system, Q, can be computed by summing up all the solutions. The average time, D, that a packet spends in the system can be obtained by using Little's formula as the ratio Q/λ.

When the arrival processes, significant reduction of the number of equations can be achieved (see [30]). The special case in which the arrival processes are modeled as Bernoulli can be easily derived from the above equations, [30], and it is given by

$$Q_B = \frac{\sum_{i=1}^{N} \sum_{j\geq 1}^{N} \lambda_i \lambda_j + \mu(1-\mu)}{(1-\mu)}$$

where Q_B is the average number of packets in the system. The mean packet delay, D_B is given by Q_B/μ, which is a known result, [29].

V. Delay analysis of interconnected ALOHA MUCNs.

In this section we use the results of the previous analysis to evaluate the performance of a system of N networks interconnected according to a star topology. More specifically, the $\mathcal{C}$ - ALOHA MUCNs whose output process was described in section III will be considered to illustrate the general procedure. At the same time exact analysis of interconnected $\mathcal{C}$ - ALOHA MUCNs will be performed. By $\mathcal{C}$ - ALOHA MUCNs we will refer to the finite user, single buffer ALOHA MUCNs under either the immediate or the delayed first transmission policy, as described in Section III. To simplify the discussion and without loss of generality, we assume that each of the interconnected $\mathcal{C}$ - ALOHA MUCNs supports M users. From the discussion in section III follows that the output process of a $\mathcal{C}$ - ALOHA MUCN depends on an underlying finite state Markov chain $\{z_j\}_{j\geq 0}$ (z_j is the number of either the active or the backlogged users at the end of slot j, depending on the policy under consideration) with state transition probabilities given by (1); the output process is given by the probabilistic mappings $a_j(z_{j-1},z_j)$ described in (2) and (3).

Consider N $\mathcal{C}$ - ALOHA MUCNs interconnected according to a star topology. The central node of this topology is assumed to have the characteristics of the single server described in section IV. All networks are synchronized and have identical slot lengths. A packet departure from a network occurs at the end of a slot involved in a successful packet transmission and is declared as an arrival to the central node at the beginning of the next slot. Clearly, the output process of the $\mathcal{C}$ - ALOHA MUCN, $\{a_j\}_{j\geq 0} = \{a_j(z_{j-1},z_j)\}_{j\geq 0}$, is the arrival process of the i^{th} input stream, according to the terminology of the previous section. The condition that at most one arrival per stream can occur, is also satisfied. The mean time that a packet spends in the central node of the star interconnecting topology is given by the solution of the linear equations given by (7) and (8).

Numerical results for the mean delay in the central

M=2 users per network									
λ_{in}	λ_{out}	p	D_{net}	$D_{q,2}$	$D_{q,2-s}$	$D_{q,2-b}$	$D_{q,3}$	$D_{q,3-s}$	$D_{q,3-b}$
.10	.098	.86	1.40	1.06	1.06	1.06	1.14	1.14	1.14
.20	.188	.82	1.64	1.14	1.15	1.15	1.40	1.43	1.43
.30	.266	.78	1.85	1.25	1.27	1.28	2.16	2.20	2.31
.35	.300	.77	1.96	1.32	1.35	1.37	3.56	3.62	3.96
.40	.330	.75	2.06	1.41	1.44	1.49	28.78	29.39	34.00
.50	.381	.73	2.25	1.66	1.70	1.80	***	***	***
.60	.419	.70	2.44	2.07	2.12	2.29	***	***	***
.70	.447	.68	2.61	2.80	2.77	3.11	***	***	***
.80	.480	.64	2.79	3.98	4.00	4.45	***	***	***

Table I.

Results for the mean packet delay in the central node of a star topology of N=2 and N=3 interconnected ALOHA networks under DFT policy; λ_{in} is the per network input rate, λ_{out} is the per network output rate, p is the packet transmission probability, D_{net} is the network induced delay, $D_{q,N}$ is the queueing delay under the developed model, $D_{q,N-s}$ is the queueing delay from the simulations and $D_{q,N-b}$ is the queueing delay under the Bernoulli model.

node of N=2 and N=3 ALOHA networks interconnected according to a star topology and operating under the DFT policy have been obtained by solving the equations in (7) and (8). In Table I, delay results are shown for the simple case of M=2 users per network and for N=2 and N=3 interconnected networks. Similar results are shown for the case of M=10 users per network and for N=2 in Table II and N=3 in Table III. Results are shown under the Bernoulli approximation on the network output processes as well. Both results are compared with simulations. The results show that both the Bernoulli approximation and the developed exact model on the network output processes perform satisfactorily under light traffic. When the traffic increases, the developed model clearly outperforms the Bernoulli approximation.

Notice that as long as the per network packet generation rate is less than .4 (so that the packet rejection probability be small), the induced queueing delay is less than half a packet length in the case of N=2 networks. This was expected since the total output rate from both networks is less than .65, well below the capacity limit of the server which is 1. In the case of N=3 networks, the total packet departure rate from all networks can be as high as the capacity limit of the server. Under such rates the queueing delay introduced by the interconnecting topology can be arbitrarily high as the capacity limit of the server is reached.

M=10 users per network						
λ_{in}	λ_{out}	p	D_{net}	$D_{q,2}$	$D_{q,2-s}$	$D_{q,2-b}$
.10	.099	.51	2.40	1.07	1.06	1.06
.20	.190	.41	3.70	1.16	1.16	1.15
.30	.265	.33	5.41	1.27	1.28	1.28
.40	.320	.29	7.51	1.40	1.42	1.43
.50	.350	.24	9.62	1.49	1.55	1.58
.60	.366	.21	11.63	1.61	1.65	1.69
.70	.375	.18	13.36	1.65	1.70	1.75
.80	.380	.17	14.80	1.74	1.71	1.79

Table II.
Results for the mean packet delay in the central node of a star topology of N=2 interconnected ALOHA networks under DFT policy; λ_{in} is the per network input rate, λ_{out} is the per network output rate, p is the packet transmission probability, D_{net} is the network induced delay, $D_{q,2}$ is the queueing delay under the developed model, $D_{q,2-s}$ is the queueing delay from the simulations and $D_{q,2-b}$ is the queueing delay under the Bernoulli model.

M=10 users per network						
λ_{in}	λ_{out}	p	D_{net}	$D_{q,3}$	$D_{q,3-s}$	$D_{q,3-b}$
.10	.099	.51	2.40	1.15	1.14	1.14
.20	.190	.41	3.70	1.48	1.45	1.44
.30	.265	.33	5.41	2.31	2.27	2.29
.40	.320	.29	7.51	7.14	6.80	7.62
.43	.329	.26	8.12	22.64	19.79	27.64

Table III.
Results for the mean packet delay in the central node of a star topology of N=3 interconnected ALOHA networks under DFT policy; λ_{in} is the per network input rate, λ_{out} is the per network output rate, p is the packet transmission probability, D_{net} is the network induced delay, $D_{q,3}$ is the queueing delay under the developed model, $D_{q,3-s}$ is the queueing delay from the simulations and $D_{q,3-b}$ is the queueing delay under the Bernoulli model.

REFERENCES

1. R. T. Dumm, "Local Area Gateways to Wide Area Networks", British Telecommunications Technology Journal, Vol. 2, No. 3, pp. 28-32, 1984.
2. K. S. Heard, "Local Area Networks and Practical Aspects of Internetworking", Computer Networks, Vol. 7, pp. 343-348, 1983.
3. N. F. Schneidewind, "Interconnecting Local Networks to Long-Distance Networks", IEEE Computer, Vol. 16, pp. 15-24, Sept. 1983.
4. G. Juanole, A. Onod, "On Architectures for Internetworking", Proceeding of Computer Networking Symposium, Washington D.C., pp. 214-223, Nov. 1986.
5. L. H. Landweber, D. M. Jennings, I. Fuchs, "Research Computer Networks and their Interconnection", IEEE Communications Magazine, Vol 24, June 1986.
6. J. F. Shoch, L. Stewart, "Interconnecting Local Networks via Packet Radio Network", Proceedings of Data Communications Symposium, Pacific Grove, CA, pp. 153-158, Nov. 1979.
7. Y. C. Cheng, T. G. Robertazzi, "Annotated Bibliography of Local Communication System Interconnection", IEEE Journal on Selected Ereas in Communications, Vol. SAC-5, No. 9, Dec. 1987.

8. I. Stavrakakis, D. Kazakos, "Interconnected Multi-user Packet Radio Networks", Symposium on C3 Research, Jun. 16-19, 1987, National Defense University, Fort Lesley J. McNair, Washington D.C.

9. C. Ko, W. Wong, K. Lye, "Performance of CSMA/CD Networks Connected by Bridges", Proceedings of International Conference in Communications ICC88, Philadelphia, June 1988.

10. H. Takagi, L. Kleinrock, "Output Processes in Contention Packet Broadcasting Systems", IEEE Transactions on Communications, Vol. COM-33, No. 11, Nov. 1985.

11. F. A. Tobagi, "Analysis of a Two-Hop Centralized Packet Radio Network-Part I: Slotted ALOHA", IEEE Transactions on Communications, Vol. COM-28, No. 2, Feb. 1980.

12. F. A. Tobagi, "Analysis of a Two-Hop Packet Radio Network-Part II: Carrier Sense Multiple Access", IEEE Transactions on Communications, Vol. COM-28, No. 2, Feb. 1980.

13. H. Takagi, L. Kleinrock, "Throughput-Delay Characteristics of Some Slotted ALOHA Multihop Packet Radio Network", IEEE Transactions in Communications, Vol. COM-33, No. 11, Nov. 1985.

14. F. A. Tobagi, "Packet Delay and Interdeparture Time", Journal of the Association for Computing Machinery, Vol. 23, No. 4, Oct. 1982.

15. I. Stavrakakis, D. Kazakos, "On the Approximation of the Output Process of Multi-user Random Access Communication Networks", IEEE INFOCOM'88, Mar. 28-31, 1988, N. Orleans (to appear in IEEE Transactions on Communications).

16. I. Stavrakakis, D. Kazakos, "2^{nd}-order Markovian Approximation on the Output Process of a Multi-user Communication Network", 22^{nd} Annual Conference on Information Sciences and Systems, Princeton University, March 16-18, 1988, Princeton (also submitted for publication).

17. R. Klessig, "Overview of Metropolital Area Networks", IEEE Communications Magazine, Vol. 24, No.1, Jan. 1986.

18. O. Yang, J. Mark, "Design Issues in Metropolitan Area Networks", Proceedings of International Conference in Communications ICC86, Toronto, Canada, June 1986.

19. N. Abramson, "The Throughput of Packet Broadcasting Channels", IEEE Transactions on Communications, Vol. COM-25, pp. 117-128, April 1977.

20. F. A. Tobagi, "Multiaccess Protocols in Packet Communication Systems", IEEE Transactions on Communications, Vol. COM-23, No. 4, April 1980.

21. L. Kleinrock, S. Lam, "Packet Switching in a Multiaccess Broadcast Channel: Performance Evaluation", IEEE Transactions on Communications, Vol. COM-23, pp. 410-423, April 1975.

22. D. Bertsekas, R. Gallager, "Data Networks", Prentice Hall, 1987.

23. D. Heyman, M. Sobel, "Stochastic Models in Operations Research", McGraw-Hill, 1982.

24. H. Stark, J. Woods, "Probability, Random Processes and Estimation Theory for Engineers", pp. 20, Prentice Hall, 1986.

25. H. Heffes, D. Lucantoni, "A Markov Modulated Characterization of Packetized Voice and Data Traffic and Related Statistical Multiplexer Analysis", IEEE Journal in Selected Areas in Communications, Vol. SAC-4, No. 6, Sept. 1986.

26. H. Bruneel, "Queueing Behavior of Statistical Multiplexers with Correlated Inputs", IEEE Transactions on Communications, Vol. 36, No. 12, Dec. 1988.

27. D. Towsley, "The Analysis of a Statistical Multiplexer with Nonindependent Arrivals and Errors", IEEE Transactions on Communications, Vol. 28, No. 1, Jan. 1980.

28. A. Viterbi, "Approximate Analysis of Time Synchronous Packet Networks", IEEE Journal on Selected Areas in Communications, Vol. SAC-4, No. 6, Sept. 1986.

29. M. Reiser, "Performance Evaluation of Data Communication Systems", Proc. IEEE, Vol. 70, Feb. 1982.

30. I. Stavrakakis, "Analysis of a Statistical Multiplexer under a General Input Traffic Model", INFOCOM'90, June 5-7 1990, San Francisco.