

On the Approximation of the Output Process of Multiuser Random-Access Communication Networks

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Abstract—In this paper, Bernoulli and first-order Markov processes are used to approximate the output process of a class of slotted multiuser random-access communication networks. The output process is defined to be the process of the successfully transmitted packets within the network.

The parameters of the approximating processes are analytically calculated for a network operating under a specific random access algorithm. The applied methods are general and can be used to calculate these parameters in the case of any random access algorithm within a class.

To evaluate the accuracy of the approximations, we consider a star topology of interconnected multiuser random-access communication networks. The mean time that a packet spends in the central node of the star topology is calculated under the proposed approximations of the output processes of the interconnected networks. The results are compared to simulation results of the actual system. It turns out that the memoryless approximation gives satisfactory results up to a certain per network traffic load. Beyond that per network traffic load, the first-order Markov process performs better since it captures some of the strong dependencies, which are introduced by the collision resolution algorithm.

I. INTRODUCTION

CONSIDERABLE work has been done towards the direction of developing communication protocols which determine how a single common resource can be efficiently shared by a large population of users. By now, it is well known that fixed assignment techniques are not appropriate for a system with a large population of bursty users. In this case, random access protocols are more efficient and many such protocols have been proposed, [1], [2]. Usually, the amount of information (in bits) transmitted per time is of fixed length, called a packet. In most of the systems, time is divided into slots of length equal to the time needed for a packet transmission (slotted systems).

The deployment of an ever increasing number of multiuser random access communication networks brings up the question of how packets, whose destination is in another network, should be handled. The issue of network interconnection or multihop packet transmission has been treated in [3], [6], [7]. The basic problem in analyzing interconnected systems is that of characterizing the output process of a multiuser random access communication system, i.e., the departure process of the successfully transmitted packets. Another problem is that of determining how a random access protocol operates in the

presence of a node that forwards exogenous traffic coming from other networks. The latter problem can be avoided by assigning a separate channel to the exogenous traffic. In this case, the operation of the system is not affected by the exogenous traffic but the problem of optimum allocation of the available resources (channels), arises. The latter issue has been discussed in [3] where the objective is to maximize the throughput of the interconnected networks. In [3], delay analysis was not performed and only simulation results were obtained.

The output process of a multiuser random-access communication system depends on the protocol that has been deployed. The description of that process is a difficult task and only approximations based on special assumptions have been attempted, [4]–[7].

In this paper, we model the output process as a Bernoulli and as a discrete-time first-order Markov process. The detailed description and the motivation behind these approximations are given in the next section.

In Section III we introduce the class of random access algorithms whose output processes are approximated. An algorithm from that class is discussed in detail and the parameters of the approximating processes are analytically calculated for the specific algorithm.

In Sections IV and V we describe the experiment to determine the accuracy of the two approximations. Finally, a summary of the work is provided in the last section.

II. THE APPROXIMATION OF THE OUTPUT PROCESS

A slotted multiuser random access communication network is considered. It is assumed that the packet input rate to the system is λ packets per slot and that λ is in the stability region of the system. For such values of λ , all the processes associated with the description of the system are stationary.

In any such system, the communication channel can be in one of the following states: I (idle) if no user is using the channel at that time; S (success) if only one user is transmitting; C (collision) if more than one users are transmitting at that time. In the above channel description, we have made the assumption that if only one user transmits, then his packet is considered as a successful one. Successfully transmitted packets appear in the output process of the network, while I and C states of the channel do not result in an output from the network. It is assumed that capture events are not present, [18], and that channel errors cannot occur.

We define the output process, $\{a^j\}_{j \geq 0}$, to be a discrete-time binary process associated with the end of the slots of a slotted multiuser random-access communication system. The random variable a^j takes the value 1 if a successful packet transmission takes place in the j th slot, and takes the value 0 otherwise. It is clear that the output process can be interpreted as a two-state channel-status process $\{x^j\}_{j \geq 0}$ where $x^j \in \{S, NS\}$; by NS we denote the union of the states I and C . The purpose of this interpretation of the output process is to relate it to the channel-status, which is used by the random-access protocol. The latter is true since the evolution of the channel-status process (which determines completely the output process) depends on the current (and possibly the past) channel state and the employed

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protocol. This occurs because the state of the channel is fed back to the users, who determine their action based on this feedback and the protocol. A type of feedback information is needed by almost any random-access algorithm.

As a consequence of the above discussion the output process, $\{a^j\}_{j \geq 0}$, and the two-state channel-status process, $\{x^j\}_{j \geq 0}$, are identical. The problem of characterizing the output process of a multiuser random-access communication network is identical to that of characterizing the channel-status process, $\{x^j\}_{j \geq 0}$. From now on, we will be referring to the process $\{x^j\}_{j \geq 0}$ rather than to the process $\{a^j\}_{j \geq 0}$, to emphasize the dependence of the output process on the employed random access algorithm and to understand qualitatively the implications of the proposed approximations.

Let $\{y^j\}_{j \geq 0}$ be a Bernoulli-type process with probability of success λ ; S is the successful event and NS is its complement. By approximating $\{x^j\}_{j \geq 0}$ by the process $\{y^j\}_{j \geq 0}$, we actually assume that the state of the channel at the current slot is independent from the channel state in the previous slot. In a random-access algorithm operating under light traffic, the collision resolution algorithm is "idle" most of the time, since packet collisions are extremely rare. Since it is the collision resolution algorithm that introduces the dependencies among channel states in successive slots, it is implied that the Bernoulli process is a reasonable approximation of the channel-status (output) process. Under moderate or heavy traffic load, the collision-resolution algorithm is in effect. In this case, at least intuitively, the Bernoulli-type approximation is not pleasing.

Under moderate and, especially, under heavy-traffic load, the dependencies introduced by the collision resolution algorithm are strong and extend beyond successive slots. We will try to capture some of these dependencies by proposing a discrete-time first-order Markov process, $\{z^j\}_{j \geq 0}$ to approximate the channel-status (output) process $\{x^j\}_{j \geq 0}$; the state space of the proposed Markov process is $\{S, NS\}$. We expect that the Markov approximation will perform better than the Bernoulli-type one, under heavy-traffic load.

So far we have not made any assumptions on the type of random-access algorithm which is used in the network. Thus, the previous discussion concerning the characterization of the output process makes sense for any multiuser random-access communication network. The single parameter of the Bernoulli-type process, i.e., the probability of having a successful packet transmission, is trivially calculated for any random-access algorithm; its value equals the value of the packet input traffic rate (over one slot) λ under stable operation of the network. The steady-state probabilities of the discrete-time Markov process $\{z^j\}_{j \geq 0}$ are also trivially calculated. If $\pi(S)$ is the steady-state probability of the channel process being in state S , and $\pi(NS)$ is the corresponding steady-state probability for the state NS , then it is obvious that $\pi(S) = \lambda$ and $\pi(NS) = 1 - \lambda$.

The method to analytically calculate the transition probabilities of the discrete-time Markov process $\{z^j\}_{j \geq 0}$ depends on the random-access algorithm being employed. In the next section, a specific random-access algorithm is considered and the transition probabilities are calculated. The same method for calculating the transition probabilities can be applied to most of the limited-sensing random access algorithms. We speculate that the method can be applied to any algorithm whose analysis is based on the concept of the session (explained in the next section) and which operates in statistically identical cycles of finite length. The class of such algorithms is large and includes many well-known random-access algorithms, [12], [14], [19], [20].

III. TRANSITION PROBABILITIES FOR A LIMITED-SENSING RANDOM-ACCESS ALGORITHM

We consider multiuser random-access slotted communication networks in which a binary-feedback, (collision/noncollision, C/NC), limited-sensing collision-resolution algorithm is deployed. The input traffic to the network is assumed to be Poisson with intensity λ packets per slot. This algorithm has been developed and analyzed in [11] and [10]. There, the analysis was limited to the derivation of the

maximum stable throughput and the average packet delay. In [21], the analysis was extended to the calculation of other quantities of interest as well. The characterization of the process of the successfully transmitted packets, i.e., the output process of the network, is still an open problem.

A brief description of the collision-resolution algorithm is provided at this point. Each user is assigned a counter whose initial value is zero (no packet to be transmitted). This counter is updated according to the steps of the algorithm and the feedback from the channel. Upon packet arrival, the counter content increases to one. Users whose counter content is equal to one at the beginning of a slot, transmit in that slot. If the channel feedback is collision (C), the counters whose content is greater than one increase it by one; the counters whose content is one maintain this value with probability p (splitting probability) or increase it to two with probability $1-p$. If the channel feedback is noncollision (NC), all nonzero counters decrease their content by one. A detailed description of the algorithm can be found in [10], [11].

At this point we calculate the transition probabilities of the discrete-time Markov process $\{z^j\}_{j \geq 0}$, which approximates the two-state channel-status process of the previously described random-access algorithm. The procedure to be followed can be applied to any limited-sensing stack-type random-access algorithm, [12], [13]. The authors believe that the method can also be applied in the case of other limited-sensing or continuous-sensing random-access algorithms, [14], [19], [20], which operate in statistically identical cycles of finite length (under stability).

Most of this section is devoted to the calculation of the transition probability $p(S/NS)$. The other transition probabilities are, then, trivially calculated at the end of this section. The calculation of the transition probability $p(S/NS)$ is simplified by computing the joint probability $p(NS, S)$ first, i.e., the probability of having an NS slot followed by an S slot in the approximating process $\{z^j\}_{j \geq 0}$. The transition probability $p(S/NS)$ is then calculated from the joint probability $p(NS, S)$ and the steady-state probability $\pi(NS)$. The joint probability is calculated as the probability that a pair (NS, S) of consecutive slots occurs in the channel-status process, $\{x^j\}_{j \geq 0}$, under stable operation of the network.

An important concept used in the analysis of most of the limited and continuous-sensing random-access algorithms is the session. A session is defined as the time interval between two renewal points of the operation of the system. For the particular algorithm under consideration, the renewal points of interest are determined by an imaginary marker. This marker is essentially an imaginary counter whose content b varies according to the channel feedback. Its value is originally zero ($b = 0$). If $b = 0$ and the feedback is C , then $b = 2$; if $b = 0$ and the feedback is NC , then $b = 0$. If $b > 0$ and the feedback is C , then $b = b + 1$; if $b > 0$ and the feedback is NC then $b = b - 1$. The slots in which $b = 0$ are renewal points of the system. Due to the statistical splitting among collided users, the system might be empty and the marker still be positive. This implies that the marker determines only a subsequence of renewal points. The session is determined as the time interval between successive renewal points, as determined by the marker. The length of such a session is easy to describe via recursive equations. The multiplicity of a session is defined as the number of packet transmission attempts in the first slot of the session. The following quantities are useful in the analysis that is presented in this section.

(NS, S) Pair: A pair of consecutive slots with the first slot being in state NS and the second in state S .

Internal (NS, S) Pair: An (NS, S) pair is internal if both slots belong to the same session.

l_K : Length of a session of multiplicity K (in slots).

L_K : Expected value of l_K .

L : Expected value of L_K with respect to K .

$\tau_K^{NS,S}$: Number of internal (NS, S) pairs in a session of multiplicity K .

$T_K^{NS,S}$: Expected value of $\tau_K^{NS,S}$.

$T^{NS,S}$: Expected value of $T_K^{NS,S}$ with respect to K .

i_K : A random variable associated with the last slot of a session of

multiplicity K ; $i_K = 1$ if that slot is idle; $i_K = 0$ if that slot is involved in a successful transmission.

I_K : Expected value of i_K .

I : Expected value of I_K with respect to K .

As will become clear later, an important quantity for the calculation of the joint probability $p(NS, S)$ is the mean session length L . This quantity can be calculated by following procedures similar to those that appear in [11], [12], [13], [14]. In fact, for the specific algorithm under consideration, L has been calculated in [10] and [11]. We believe that the recursive equations with respect to I_K which describe the operation of the system will be very helpful for the better understanding of the procedure for the calculation of $p(NS, S)$. For this reason we start by calculating L .

From the description of the algorithm the following equations can be written with respect to I_k (i.e., the length of a session of multiplicity $K = k$), $k = 1, 2, \dots$.

$$I_0 = 1, \quad I_1 = 1 \quad (1a)$$

$$I_k = 1 + I_{\Phi_1 + F_1} + I_{k - \Phi_1 + F_2}, \quad k \geq 2. \quad (1b)$$

F_1 and F_2 are two independent Poisson distributed random variables over $T = 1$ (length of a slot) with probability function $P_\lambda(\cdot)$ and intensity λ ; Φ_1 is a random variable following the binomial distribution with parameters k and p ($p = 0.5$) and probability function $B_{k,p}(\cdot)$. Equation (1b) can be explained as follows. The length of a session of multiplicity $k \geq 2$ consists of the slot wasted in the collision, plus the length of the subsession of multiplicity $\Phi_1 + F_1$ (which will be initiated in the next slot), plus the length of the subsession of multiplicity $k - \Phi_1 + F_2$ (which will be initiated after the end of the subsession of multiplicity $\Phi_1 + F_1$). Subsessions are statistically identical to the sessions of the same multiplicity. Φ_1 denotes the number of users whose counter content remained one after the splitting; F_1, F_2 denote the number of new users which will be activated (have a packet for transmission) and enter the system in the first slot of the corresponding subsession.

By considering the expected values in (1) with respect to all random variables involved, we obtain

$$L_0 = 1, \quad L_1 = 1 \quad (2a)$$

$$L_k = 1 + \sum_{f_1=0}^{\infty} \sum_{\phi_1=0}^k P_\lambda(f_1) B_{k,p}(\phi_1) L_{\phi_1+f_1} + \sum_{f_2=0}^{\infty} \sum_{\phi_1=0}^k P_\lambda(f_2) B_{k,p}(\phi_1) L_{k-\phi_1+f_2}, \quad k \geq 2. \quad (2b)$$

The infinite dimensional linear system of equations in (2) can be written in the general form

$$L_k = h_k + \sum_{j=0}^{\infty} a_{kj} L_j, \quad k \geq 0. \quad (3)$$

The most widely used definition of stability is the one which relates it with the finiteness of L_k , for $k < \infty$. In [10], [11] it has been found that the system is stable for Poisson input rates $\lambda < S_{\max} = 0.36$ (packets/packet length). The authors in [10], [11] were actually able to find a (linear) upper bound on L_k which is finite for $k < \infty$. S_{\max} is then defined as the supremum over all rates λ for which such a bound L_k^u was possible to obtain.

The existence of $L_k^u < \infty$, for $k < \infty$, implies that (3) has a non-negative solution L_k ; the solution \tilde{L}_k of the finite dimensional system of equations

$$\tilde{L}_k = h_k + \sum_{j=0}^J a_{kj} \tilde{L}_j, \quad 0 \leq k \leq J, \quad (4)$$

is a lower bound on L_k and $\tilde{L}_k \rightarrow L_k$ as $J \rightarrow \infty$, [11], [14], [15].

For sufficiently large J (e.g., 15), L_k is extremely close to \tilde{L}_k and thus, for practical purposes, L_k is considered to be equal to \tilde{L}_k , especially for λ outside the neighborhood of S_{\max} . (The latter can be shown by calculating a tight upper bound on L_k and observing that it almost coincides with \tilde{L}_k , see [11], [13], [14] for the procedure.)

By solving (4) with a_{kj} and h_k given by

$$a_{0j} = a_{1j} = 0, \quad 0 \leq j \leq J \quad (5a)$$

$$a_{kj} = \sum_{\phi_1=0}^{\min\{j,k\}} P_\lambda(j-\phi_1) B_{k,p}(\phi_1) + \sum_{\phi_1=\max\{k-j,0\}}^k P_\lambda(j-k+\phi_1) B_{k,p}(\phi_1), \quad 0 \leq j \leq J \quad (5b)$$

and

$$h_k = 1, \quad 0 \leq k \leq J, \quad (6)$$

we calculate the mean session length of multiplicity k . Since the multiplicities of successive sessions are independent and identically distributed random variables, the mean session length L is calculated by averaging L_k over all k ; k is the number of arrivals in a slot from a Poisson process with intensity λ . In fact, the average for $k \leq J$ is sufficient.

From the description of the algorithm it can be concluded that the last slot of a session can be either idle or involved in a successful transmission. We proceed by calculating the probability that the last slot of a session be idle, since this probability is used in the calculation of $p(NS, S)$.

The argument used to derive the recursive equations for I_k , can be used to derive the following equations for i_k ,

$$i_0 = 1, \quad i_1 = 0, \quad i_k = i_{k-\Phi_1+F_2}, \quad k \geq 2. \quad (7)$$

By considering the expected values in (7), we obtain the following infinite dimensional system of linear equations:

$$I_0 = 1, \quad I_1 = 0 \quad (8a)$$

$$I_k = \sum_{f_2=0}^{\infty} \sum_{\phi_1=0}^k P_\lambda(f_2) B_{k,p}(\phi_1) I_{k-\phi_1+f_2}. \quad (8b)$$

Notice that I_k is the probability that the last slot of a session of multiplicity k is idle; $I_k \leq 1 < \infty$, for $k < \infty$. The system in (8) is of the form of that in (3). By using the same arguments as those used in the calculation of L_k we can solve a truncated, up to $J = 15$, version of (8) and obtain a very good approximation of I_k . By averaging the latter over all $k \leq J$, we can approximate I ; I is actually the probability that the last slot of a session is idle.

Up to this point, the average session length L and the probability that the last slot of a session is idle I have been calculated. The objective is to calculate the joint probability $p(NS, S)$. As a last step before the calculation of this probability, we calculate the average number of internal (NS, S) pairs in a session. The following recursive equations are obtained with respect to $\tau_k^{NS,S}$; $\tau_k^{NS,S}$ denotes the number of internal (NS, S) pairs in a session of multiplicity k .

$$\tau_0^{NS,S} = 0, \quad \tau_1^{NS,S} = 0 \quad (9a)$$

$$\tau_k^{NS,S} = \tau_{\Phi_1+F_1}^{NS,S} + \tau_{k-\Phi_1+F_2}^{NS,S} + 1_{\{i_{\Phi_1+F_1}=1, k-\Phi_1+F_2=1\}} + 1_{\{i_{\Phi_1+F_1}=1\}}, \quad k \geq 2. \quad (9b)$$

Observe that the idle slots which are the last of a session and are followed by a session of multiplicity 1 (that would give an (NS, S) pair), are not considered by the expressions in (9).

By considering the expected values in (9), we obtain an infinite dimensional system of linear equations with respect to $T^{NS,S}$. Since

$T_k^{NS,S} \leq L_k < \infty$ for $k < \infty$, the comments that were made in the calculation of L_k apply also to this case. Thus, $T_k^{NS,S}$ can be calculated by solving a truncated version of the system in (9). The resulting finite dimensional system is of the form in (4) with coefficients a_{kj} , $0 \leq j \leq J$, $0 \leq k \leq J$, given by (5) and constants given by

$$h_0^{NS,S} = 0, \quad h_1^{NS,S} = 0 \quad (10a)$$

$$\begin{aligned} h_k^{NS,S} &= P_\lambda(1)B_{k,p}(k) \sum_{f_1=0}^{J-k} P_\lambda(f_1)I_{k+f_1} \\ &+ P_\lambda(0)B_{k,p}(k-1) \sum_{f_1=0}^{J+1-k} P_\lambda(f_1)I_{k-1+f_1} \\ &+ P_\lambda(0)B_{k,p}(1) + P_\lambda(1)B_{k,p}(0), \quad k \geq 2. \end{aligned} \quad (10b)$$

The average number of internal (NS, S) pairs in a session, $T^{NS,S}$, is then approximated by averaging $T_k^{NS,S}$ over all $k \leq J$.

By invoking the strong law of large numbers and the ergodic theorem for stationary processes, we prove in the Appendix that the joint probability $p(NS, S)$ is given by the following expression:

$$p(NS, S) = \frac{T^{NS,S}}{L} + \lambda e^{-\lambda} \frac{I}{L}. \quad (11)$$

The transition probability $p(S/NS)$ is then calculated from the expression:

$$p(S/NS) = \frac{p(NS, S)}{\pi(NS)} = \frac{p(NS, S)}{1-\lambda}.$$

The rest of the transition probabilities are computed from the following expressions:

$$p(NS/NS) = 1 - p(S/NS)$$

$$p(S/S) = 1 - p(S/NS) \frac{\pi(NS)}{\pi(S)}$$

$$p(NS/S) = 1 - p(S/S).$$

The approximating Markov process $\{z^j\}_{j \geq 0}$ is now completely determined since the steady-state and the transition probabilities have been calculated.

IV. PERFORMANCE OF THE APPROXIMATIONS ON THE OUTPUT PROCESS

In the previous sections the Bernoulli-type process $\{y^j\}_{j \geq 0}$ and the discrete-time Markov process $\{z^j\}_{j \geq 0}$, which approximate the two-state channel-status process $\{x^j\}_{j \geq 0}$ (or the output process $\{a^j\}_{j \geq 0}$), were described and their parameters were analytically calculated. In this section, we evaluate the performance of the proposed approximations.

The most interesting, probably, application for which the characterization of the output process of a multiuser random-access communication network is of great importance, is that of analyzing the performance of systems of interconnected multiuser random access communication networks. One such system is a star topology of interconnected networks. In such a topology, the mean time that a packet spends in the central node is an important measure of the performance of the interconnection. Thus, we will compare the accuracy of the two approximations in estimating this quantity.

A star topology of interconnected networks is shown in Fig. 1. Each input stream represents the output process from a multiuser random-access slotted communication system. Let λ_n be the output rate (in packets per slot) of the n th network. A packet arrival in the central node is declared at the end of the slot in which the packet was

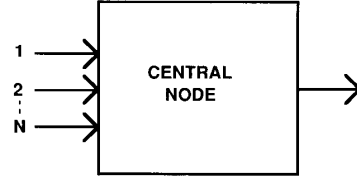


Fig. 1. A star topology of interconnected networks.

successfully transmitted. Thus, the arrival process of each input line is a discrete process. The arrival points in all streams coincide; that is, the networks are assumed to be synchronized and all slots are of the same length.

The service time in the central node is constant and equal to one, which is assumed to be the length of a slot. This implies that arriving and departing packets have the same length. The first in-first out (FIFO) service policy is adopted. More than one arrivals (from different input streams) that occur at the same arrival point are served in a randomly chosen order. The buffer capacity of the central node is assumed to be infinite.

If the output process of a network is approximated by the Bernoulli-type process $\{y^j\}_{j \geq 0}$, then the resulting queueing system in the central node has been studied and the mean time that a packet spends in the central node D_B is given by [8]

$$D_B = \left[1 + \frac{\sum_{n=1}^N \sum_{m>n}^N \lambda_n \lambda_m}{\left(1 - \sum_{n=1}^N \lambda_n\right) \sum_{n=1}^N \lambda_n} \right].$$

If the output process of a network is approximated by the Markov process $\{z^j\}_{j \geq 0}$, then the resulting queueing system in the central node has been studied in [9] and the mean time that a packet spends in the central node D_M is given by

$$D_M = \left[1 + \frac{\sum_{n=1}^N \sum_{m>n}^N \lambda_n \lambda_m \left(1 + \frac{\gamma_n}{1-\gamma_n} + \frac{\gamma_m}{1-\gamma_m}\right)}{\left(1 - \sum_{n=1}^N \lambda_n\right) \sum_{n=1}^N \lambda_n} \right]$$

where $\gamma_n = P(S/S) - P(S/NS)$.

V. RESULTS AND CONCLUSIONS

We consider systems of $N = 2$ and $N = 3$ multiuser random-access communication networks interconnected by the star topology described in the previous section. It is assumed that the limited-sensing collision resolution algorithm, described in Section II, is deployed in each of the networks. The output process of each of the networks is approximated by a Bernoulli-type and a discrete-time Markov process. The parameters of the approximating processes are calculated according to the procedures developed before.

In Table I, the values of the transition probabilities $p(S/NS)$, calculated for various per network input rates λ and according to the procedures developed in Section III, are compared to the corresponding values obtained from the simulation of the actual system. The closeness (up to the third decimal point) between the analytical and the simulation results, shows that the estimation of this probability by solving truncated systems of $J = 15$ linear equations, is extremely good.

The mean time that a packet spends in the central node of the star topology was calculated from the expressions given in the previous section. The results (in slots) are shown in Tables II and III, together with the results obtained from the simulation of the actual system D_S . The simulation results were obtained after the system had operated for 500 000 slots. The criterion was to let the program run for N slots

TABLE I
ANALYTICAL AND SIMULATION RESULTS FOR THE TRANSITION
PROBABILITY $p(S/NS)$

λ	Anal.	Simul.
.01	0.009	0.009
.10	0.095	0.095
.20	0.186	0.186
.30	0.274	0.274
.33	0.300	0.300

TABLE II
RESULTS FOR THE MEAN PACKET DELAY IN THE CENTRAL NODE OF A
STAR TOPOLOGY OF TWO INTERCONNECTED NETWORKS

λ	D_B	D_M	D_S	D_N
.10	1.06	1.06	1.00	1.97
.22	1.20	1.21	1.02	3.85
.25	1.25	1.29	1.05	5.30
.30	1.37	1.44	1.13	11.38
.33	1.48	1.57	1.21	30.00
.35	1.58	1.70	1.30	87.70

TABLE III
RESULTS FOR THE MEAN PACKET DELAY IN THE CENTRAL NODE OF A
STAR TOPOLOGY OF THREE INTERCONNECTED NETWORKS

λ	D_B	D_M	D_S	D_N
.10	1.14	1.15	1.01	1.97
.20	1.50	1.56	1.21	3.33
.25	2.00	2.16	1.70	5.30
.30	4.00	4.54	4.28	11.38
.31	5.42	6.23	6.25	15.00
.32	9.00	10.47	11.37	20.00
.33	34.00	40.14	48.89	30.00

where N was such that $N/2$ runs gave the same results. We considered that as an indication that the quantities of interest had converged to the steady-state values. The network induced mean packet delay D_N is also shown in the last column of these tables; it is the average time between the packet generation instant and the time when this packet is successfully transmitted (and appears in the output process). The results were taken from [10] and are provided to indicate the average total delay (in the network and in the node) that a packet undergoes. The maximum per network output rate under stable operation of the particular algorithm is 0.36 packets per slot. On the other hand, the queueing system of the star topology is stable for total input rates less than 0.99 packets per slot [22].

In the case of $N = 2$, interconnected networks, the maximum total input rate to the central node is 0.72 packets per slot, far from the stability limit of the queueing system. The latter implies that the queueing delay will be low for this system. Indeed, it turns out that the queueing delay is less than 0.5 slots. Both approximations perform satisfactorily in this case in which the queueing problem is not significant.

In the case of $N = 3$, the total input traffic to the central node can be as high as the stability limit of the queueing system (in fact beyond that). Under such conditions the performance of the two approxima-

tions begin to differentiate. As the results in Table III indicate, when the per network traffic is large ($\lambda > 0.3$) and the queueing problem in the central node significant (total input traffic > 0.9), then the Markov approximation performs better than the Bernoulli-type one. This is due to the fact that the Markov model captures some of the strong dependencies introduced by the collision-resolution algorithm. These dependencies affect considerably the mean time that a packet spends in the central node only when there is a significant queueing problem.

As a last comment on the performance of the Bernoulli-type approximation, we note that the good performance of this model under moderate per network traffic (e.g., $N = 3$ and $\lambda = 0.2$) is due to the fact that, although dependencies are introduced by the collision-resolution algorithm, some independence is introduced to the cumulative input traffic to the central node from the mutually independent input streams. We expect that as the number of interconnected networks increases (and the per network output rate decreases, for the stability of the queue) the Bernoulli-type model will perform satisfactorily, with respect to the performance measure under consideration, for increasingly larger cumulative input rates to the central node.

More than three interconnected networks can also be considered.

In such a system, one should make the assumption that only a portion of each network traffic is destined outside the particular network and thus needs to be forwarded to the central node. This assumption is clearly necessary for the stability of the queue. The output process of a network in such a system is not determined by the two-state channel-status process, $\{x^j\}_{j \geq 0}$. It is a combination of the channel-status process and a binomial process, if there is a probability that the destination of a successful packet is outside the particular network. In the latter case, an S -state will result in an output with some probability.

VI. SUMMARY

In this paper, the relation of the output process of multiuser random-access communication networks with the two-state (S - NS) channel-status process was pointed out in an effort to indicate the dependence of the output process on the employed random-access algorithm and the history of the channel-status process. The latter is true because some type of feedback information concerning the state of the channel is provided to the users of any multiuser random-access communication network; thus the evolution of the system (and the output process) depends on the status of the channel. In view of the above, the implications of the Bernoulli-type and the discrete-time first-order Markov approximations on the output process were better understood.

We developed a procedure to calculate the transition probabilities of the Markov model based on the average such transitions of the actual channel-status process, as they are determined by the operation of a specific random-access algorithm. The suggested method can also be applied to any random-access algorithm from that class. The authors believe that the same procedure can also be applied in the case of almost any random-access algorithm whose analysis is based on the concept of the session. The method can also be used for the calculation of the steady-state and the transition probabilities of the three-state (I , S , C) channel-status process, if such quantities are of interest.

Finally, the accuracy of the two approximations was evaluated by applying them to random-access networks interconnected according to a star topology. The results and the conclusions about the accuracy of the proposed approximations are given in the previous section.

APPENDIX

In this Appendix, we prove (11). Let us define the following random variables.

External (NS , S) Pair: It is a pair whose slots belong to different sessions; it is assumed that the first slot determines the session which this pair is assumed to belong to the following.

l_n : Length of the n th session from the time origin (in slots).

$\tau_n^{NS,S}$: Number of internal (NS , S) pairs of the n th session.

$$J_v^{NS,S} = \begin{cases} 1 & \text{if the } v\text{th slot of a session is the first slot} \\ & \text{of an } (NS, S) \text{ pair} \\ 0 & \text{otherwise;} \end{cases}$$

$$\omega_n^{NS,S} = \sum_{v=1}^{l_n} J_v^{NS,S}; \text{ Number of } (NS, S) \text{ pairs in the } n\text{th session;}$$

$$\xi_n^{NS,S} = \begin{cases} 1 & \text{if the } n\text{th session has an external } (NS, S) \text{ pair} \\ 0 & \text{otherwise.} \end{cases}$$

Notice that $\xi_n^{NS,S}$ is associated with the last slot of the n th session.

The joint probability $p(NS, S)$ can be calculated, by using the law of large numbers, from the following expression:

$$p(NS, S) = \lim_{N \rightarrow \infty} \frac{\sum_{n=1}^N \omega_n^{NS,S}}{\sum_{n=1}^N l_n}.$$

The above expression can be written as

$$\begin{aligned} p(NS, S) &= \lim_{N \rightarrow \infty} \frac{\sum_{n=1}^N (\omega_n^{NS,S} - \xi_n^{NS,S})}{\sum_{n=1}^N l_n} + \lim_{N \rightarrow \infty} \frac{\sum_{n=1}^N \xi_n^{NS,S}}{\sum_{n=1}^N l_n} \\ &= \lim_{N \rightarrow \infty} \frac{\frac{1}{N} \sum_{n=1}^N \tau_n^{NS,S}}{\frac{1}{N} \sum_{n=1}^N l_n} + \lim_{N \rightarrow \infty} \frac{\frac{1}{N} \sum_{n=1}^N \xi_n^{NS,S}}{\frac{1}{N} \sum_{n=1}^N l_n}. \end{aligned}$$

Clearly, the random variables l_n , $\tau_n^{NS,S}$, $n \geq 1$, are independent and identically distributed with mean value $L < \infty$ and $T^{NS,S} < \infty$ (since $T^{NS,S} < L$), respectively. Thus, the strong law of large numbers, [16], asserts that

$$p(NS, S) = \frac{T^{NS,S}}{L} + \frac{\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N \xi_n^{NS,S}}{L}. \quad (A1)$$

The random variables $\xi_n^{NS,S}$, $n \geq 1$, are not independent but $\{\xi_n^{NS,S}\}_{n \geq 1}$ is a stationary process and $\xi_n^{NS,S}$ has expected value given by

$$E\{\xi_n^{NS,S}\} = P\{(I\text{-slot}, k' = 1)/\text{last slot of the } n\text{th session}\}$$

where k' denotes the multiplicity of the next session. Since the two events, "last slot of a session is idle" and "multiplicity of next session is 1," are independent, we can write that

$$E\{\xi_n^{NS,S}\} = I \cdot \lambda e^{-\lambda} < \infty. \quad (A2)$$

The term $\lambda e^{-\lambda}$ of the above product is the probability of having a session of multiplicity 1. By applying the ergodic theorem for stationary processes to (A1), [17], and by considering (A2), we obtain the expression in (11).

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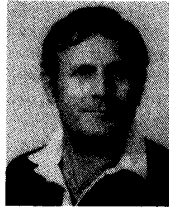
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