# DATA COMMUNICATION SYSTEMS

## and Their Performance

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# PREFACE

As in previous editions, the 4th International Conference on Data Communication systems and their Performance has intended accept papers describing recent and original developments on techniques, tools and applications in the area of performance of communication systems.

The thirty one accepted papers plus two invited papers chosen for this edition have been organized in ten sessions devoted to:

- Polling Systems
- ISDN Switch
- Modeling of Switching Techniques
- ATM Switching
- Performance Studies
- Acces Methods
- Network Management
- Workload
- Protocols
- Tools and Measurements

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# STATISTICAL MULTIPLEXING UNDER NON-I.I.D. PACKET ARRIVAL PROCESSES AND DIFFERENT PRIORITY POLICIES

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An Integrated Services Digital Network (ISDN) accommodates packets of information generated by sources with different statistical characteristics and packet delivery requirements. Efficient multiplexing of packets coming from different sources would require an appropriate unified description of the packet traffics and a service policy based on both the statistical characteristics of the sources and the packet delivery requirements. In this paper, the per source packet traffic is modeled as a Markov Modulated Generalized Bernoulli Process (MMGBP), which is a non-i.i.d. process. Various packet multiplexing policies with priorities are proposed to introduce fairness in the service process, accommodate different packet delay requirements and avoid monopolization of the transmission media by some sources.

## 1. INTRODUCTION

Integrated Services Digital Networks (ISDN's) should not be seen as a simple evolution of Data networks (DN's) which have been developed over the last two decades. The significant differences among the sources of information involved in ISDN's, regarding, for instance, the packet generation processes and the packet delivery requirements, create a more complex environment compared to that found in Data Networks.

Although the unit of information is a fixed size packet for all potential users of the system, to facilitate the integrating operation of an ISDN, the characteristics of the various packet processes of interest can be dramatically different from those present in a traditional Data Network. Poisson, Bernoulli, or general i.i.d. processes, widely incorporated in the analysis of Data Networks, are rather inappropriate for the description of the packet processes in an ISDN. For instance, packetized voice traffic can be modeled as blocks of packets arriving over consecutive time slots with geometrically distributed length (talkspurt) followed by periods of silence with geometrically distributed length. Other kinds of packetized information (such as long files, video traffic, etc) may be described as blocks of packets whose length follows a general distribution. The output of a computer over a slot may contain more than one packets of information; fast transmission lines may also deliver more than one packets per slot. The packet traffic generated by a concentrator / transmitter and being delivered through a slotted line is constant (one packet per slot), whenever its buffer is non-empty and it is zero otherwise. Packet traffics generated by various sources in an ISDN or by network components in both an ISDN or a DN cannot be described with the memoryless models mentioned before.

In a discrete time slotted network, the packet traffics for the cases described above (among other ones) can be appropriately described by a Markov Modulated Generalized Bernoulli Process (MMGBP). That is, it is assumed that the source of information (i.e. network component or user) visits M states of an underlying Markov chain. Given the current state, the number of packets generated follows a general distribution. Clearly this packet process is a non-i.i.d. one. It is easy to establish that the cases of packet traffics described before may be described (or approximated) by a MMGBP. For instance, the packetized voice traffic is a MMGBP with two states, "talkspurt" and "silence". The probability that the voice source generates one packet when in state "talkspurt" is one; the probability that it generates zero packets when in state "silence" is one. The packet process of blocks of packets arriving over consecutive time slots may be described by a MMGBP, [1], as well.

The other important issue in a packet network accommodating packets from sources with different characteristics is that of the allocation of the common facility among the sources. The allocation policy should take into consideration the time constraints imposed on certain packets and the possible monopolization of the common resource by certain sources over long periods; the latter could introduce unacceptable delays to short messages (e.g. consisted of single packets) of interactive communication or control information.

In this paper, we analyze a number of statistical multiplexing schemes under packet arrival processes described by a MMGBP and under various priority policies. The non.i.i.d. MMGBP may be appropriate for the description of complex packet processes while the prioritization may introduce fairness and increased efficiency in the system.

A statistical multiplexer with N packet input processes, each of which is described by a MMGBP has been analyzed in [1], under the first-in first-out (FIFO) service policy. The analysis of the system in [1] (the results of which are presented in the next section) is the ground on which the methodology for the analysis of the multiplexing schemes with priorities will be built.

Packets are assumed to arrive through slotted synchronous lines. That is, all packet arrivals are declared at common time instants which coincide with the end of the slots (slot boundaries). Discrete time queueing models for statistical multiplexing schemes under non-i.i.d. inputs and without priorities have been analyzed in the past, [1]-[5]. Previous work on statistical multiplexing where packets with different priorities are involved, is heavily based on the assumption of a memoryless packet arrival process (e.g. Poisson), [6]-[8]. Notice that the proposed MMGBP includes simpler processes such as the Bernoulli or the generalized Bernoulli (more than one packet arrivals may occur over the same slot) processes and the first-order Markov process (arrival / no arrival), approximating packet arrivals in bursts or describing the packetized voice traffic. Even under these simple arrival processes and for the priority policies considered in this paper, the corresponding multiplexing schemes have not been analyzed before.

The rest of the paper is organized as follows. In the next section the statistical multiplexer presented in [1] is briefly described and the results from the analysis in [1] are presented. In section III, four different multiplexing schemes are considered and the methodology, based on the construction of systems equivalent to the one in [1], is presented. The mean buffer occupancy and the mean packet delay for all packet categories are derived for all cases considered. In section IV, some numerical results on the mean packet delay are presented for the cases considered in section III. Finally, the conclusions of this work appear in the last section.

## 2. THE FIFO STATISTICAL MULTIPLEXER

In this section we describe the statistical multiplexer analyzed in [1] and present the equations derived for the calculations of the mean buffer occupancy and the mean packet delay. This system will be modified to accommodate the priority policies in the next section. By establishing equivalent systems with the one presented briefly in this section, similar equations will be used for the derivation of the queueing results of interest in the next section.

A statistical multiplexer which is fed by N input lines is shown in Fig. 1. The input lines (which are mutually independent) are assumed to be slotted and packet arrivals and service completions are synchronized with the end of the slots. A slot is defined to be the fixed service (transmission) time required by a packet. At most one packet can be served in one slot. The first-in first-out (FIFO) service discipline is adopted. Packets arriving at the same slot are served in a randomly chosen order. The buffer capacity is assumed to be infinite. The packet arrival process associated with line i is defined to be the discrete time process $\{a_j^i\}_{j\geq 0}$, $i=1,2,\ldots,N$, of the number of packets arriving at the end of the $j^{th}$ slot; $a_j^i=k$ , $0\leq k<\infty$, if k packets arrive at the end of the $j^{th}$ slot through input line i.

Let $\{z_j^i\}_{j\geq 0}$, be a finite state Markov chain imbedded at the end of the slots, which describes the state of the input line i. Let $S^i=\{x_0^i, x_1^i, \cdots, x_{M^i-1}^i\}$, $M^i<\infty$, be the state space of $\{z_j^i\}_{j\geq 0}$. It is assumed that the state of the underlying Markov chain determines (probabilistically) the packet arrival process of the corresponding line. That is, if $a^i(x^i) : S^i \to Z_0$, is a probabilistic mapping from $S^i$ into the non-negative finite integers, $Z_0$, then the probability that k packets arrive at the buffer at the end of the $j^{th}$ slot is given by $\phi(z_j^i,k)=Pr\{a^i(z_j^i)=k\}$. Furthermore, it is assumed that there is at most one state, $x_0^i$ such that $\phi(x_0^i,0)>0$ and that the rest of the states of the underlying Markov chain result in at least one (but a finite number of) packet arrivals, i.e. $\phi(x_k^i,0)=0$, for $1\leq k\leq M^i-1$. All packet arrivals are assumed to occur at the end of the slots. To avoid instability of the buffer queue it is assumed that there is always one state $x_0^i$, such as described above.

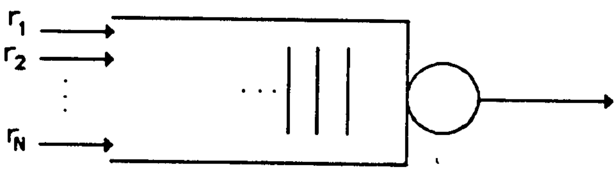


Figure 1.
The FIFO statistical multiplexer with N inputs.

The expected number of packets in the system is given by, [1],

$$Q = \sum_{\bar{y}\in\bar{S}} W(\bar{y}) \tag{1}$$

where $\bar{S}=S^1xS^2x\cdots xS^N$ and $W(\bar{y})$, $\bar{y}\in\bar{S}$, are the solutions of any $M^1xM^2x\cdots xM^N-1$ linear equations given by

$$W(\bar{y})=\sum_{\bar{x}\in\bar{S}} W(\bar{x})p(\bar{x},\bar{y})+\sum_{\bar{x}\in\bar{S}}(\mu_{\bar{x}}-1)p(\bar{x},\bar{y})\pi(\bar{x})+\sum_{\bar{x}\in\bar{S}} q_0(\bar{x})p(\bar{x},\bar{y}) \ , \ \bar{y}\in\bar{S} \tag{2a}$$

and the linearly independent equation

$$\sum_{\bar{x}\in\bar{S}}\left[2(\mu_{\bar{x}}-1)W(\bar{x})+2(\mu_{\bar{x}}-1)q_0(\bar{x})+(2+\sigma_{\bar{x}}-3\mu_{\bar{x}})\pi(\bar{x})\right]=0 \tag{2b}$$

where

$$\pi(\bar{x})=\prod_{i=1}^{N}\pi^i(x^i), \quad p(\bar{x},\bar{y})=\prod_{i=1}^{N}p^i(x^i,y^i), \quad q_0(\bar{x})=(1-\lambda)p(\bar{x}_0,\bar{x})$$

$$\mu_{\bar{x}} = \sum_{\nu=1}^{R} \nu g_{\bar{x}}(\nu), \quad \sigma_{\bar{x}} = \sum_{\nu=1}^{R} \nu^2 g_{\bar{x}}(\nu), \quad g_{\bar{x}}(\nu) = \Pr\left\{\sum_{i=1}^{N} a^i(x^i) = \nu\right\}$$

and

$$\lambda = \sum_{\bar{x} \in s} \mu_{\bar{x}}\, \pi(\bar{x}) < 1$$

is the total input traffic which is less than 1 for stability. R is the maximum number of packets which may arrive at the same slot from all N lines; $\pi^i(x^i)$ and $p^i(x^i,y^i)$ are the steady state and the transition probabilities of the Markov chain associated with the $i^{th}$ input line. The mean packet delay is given by using Little's formula, i.e.

$$D = \frac{Q}{\lambda} \tag{3}$$

## 3. STATISTICAL MULTIPLEXING WITH PRIORITIES

In this section we consider various multiplexing schemes under different priority policies. The per slot and line packet arrival processes are described by the MMGBP introduced in section II.

### 3.1. Case 1

Consider the statistical multiplexer shown in Fig.2; the input lines, $r_1$ and $r_2$, are assumed to carry synchronous packet traffic. The packet arrival processes $\{a_j^1\}_{j\geq 0}$ and $\{a_j^2\}_{j\geq 0}$ are assumed to be two MMGBP's. In particular, $\{a_j^1\}_{j\geq 0}$ is assumed

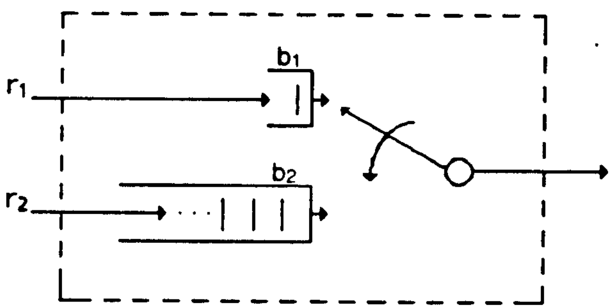


Figure 2.
The statistical multiplexer of Case 1.

to be a MMGBP with two underlying states $x_0^1$ and $x_1^1$ and packet generation probabilities $\phi^1(x_0^1,0)=1$ and $\phi^1(x_1^1,1)=1$. That is, one packet is generated when the line (or the source connected to the line) is in state $x_1^1$ and no packet is generated when in state $x_0^1$. This model may describe the packet traffic generated by a voice source or, in general, blocks of packets of geometrically distributed length, arriving over consecutive slots. The second packet process $\{a_j^2\}_{j\geq 0}$ is assumed to be given by the general MMGBP described in the previous section.

In the statistical multiplexing scheme considered here it is assumed that line $r_1$ carries high priority traffic which has priority over that carried by line $r_2$. That is, it is assumed that the server (which makes decisions at the slot boundaries) moves to line $r_2$ only if the buffer associated with line $r_1$ is empty; it returns to line $r_1$ as soon as the corresponding buffer associated with line $r_1$ becomes non-empty. Since at most one packet arrives through line $r_1$, the service policy implies that a single packet buffer is required for line $r_1$. If the cut-through connection is possible, no buffer is necessary for line $r_1$. An infinite capacity buffer is assigned to line $r_2$.

Clearly, there are two categories of packets, say $C_1$ and $C_2$, with different priorities (a smaller subscript indicates higher priority). Packets in $C_1$ are served (transmitted) right away. Thus, the mean delay of packets in $C_1$, $D_1$, is equal to 1 (the service time). Service of packets in $C_2$ is interrupted whenever a packet arrives through line $r_1$; let $D_2$ be the mean delay of packets in $C_2$.

To compute $D_2$ we consider a FIFO system (shown in Fig. 1) which is equivalent to the one considered here. An *equivalent FIFO system* is defined as a FIFO system whose packet arrival processes are identical to those of the system under consideration; let $D_{12}$ denote the mean packet delay induced by the equivalent FIFO system. Since the queueing system is work conserving and nonpreemptive, the conservation law, [8], [9], implies that $D_{12}$ satisfies the following equation.

$$D_{12}=\frac{\lambda_1 D_1+\lambda_2 D_2}{\lambda_1+\lambda_2} \tag{4}$$

where $\lambda_1$ and $\lambda_2$ are the per slot packet arrival rates through lines $r_1$ and $r_2$, respectively. $D_{12}$ can be computed from equations (1) - (3). Then $D_2$, the mean delay of packets in $C_2$ can be computed from (4) by setting $D_1=1$.

A practical application of the simple priority scheme described here is related to the mixing of voice and data packets; $r_1$ may carry packetized voice ($\lambda_1 < .5$) and $r_2$ may carry blocks of packets of time unconstrained information. The multiplexing scheme provides (in essence) a circuit to the voice traffic which is utilized by data packets when idle. The mean data packet delay, in this case, is given by $D_2$.

## 3.2. Case 2

Consider the statistical multiplexer shown in Fig. 3. Both synchronous traffics $\{\alpha_j^1\}_{j\geq 0}$ and $\{\alpha_j^2\}_{\geq 0}$ are assumed to be modeled as MMGBP's. Case 2 is identical to Case 1 with the only difference being that more than one packets per slot may arrive through line $r_1$, as well. As a result, queueing problems appear in both lines. Line $r_1$ carries high priority traffic (or the source connected to $r_1$ has priority over the one connected to line $r_2$) which has priority over that carried by line $r_2$. To compute $D_1$ and $D_2$, in this case, we proceed as follows.

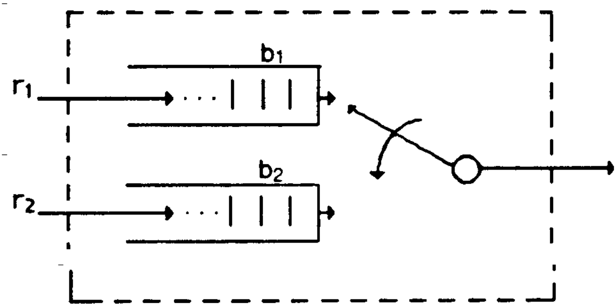


Figure 3.
The statistical multiplexer of Case 2.

Calculation of $D_1$
Consider a FIFO statistical multiplexer with one input line which is identical to $r_1$. By using equations (1)-(3), we compute the mean packet delay induced by this FIFO multiplexer. Clearly, this mean packet delay is equal to $D_1$. The priority of $r_1$ over $r_2$ results in a buffer behavior of line $r_1$ which is not affected by the packet arrival process in $r_2$. Thus, the behavior of the buffer connected to $r_1$ is identical to that of the FIFO multiplexer described above.

Calculation of $D_2$
To compute the mean delay of packets in $C_2$ we use the equivalent FIFO statistical multiplexer. The mean packet delay, $D_{12}$, is obtained from equations (1)-(3). Then $D_2$ is obtained from (4).

3.3. Case 3

Consider the statistical multiplexer shown in Fig. 4. The packet arrival process $\{a_j^1\}_{j\geq 0}$ is assumed to be a MMGBP, as described in section II. To avoid monopolization of the facility by long messages (consisted of many packets) which arrive over a single slot, the following service policy is introduced. The first packet of those arriving during a single slot enters a single packet buffer $b_1$ and it is transmitted in the next slot. The rest of the packets enter an infinite capacity buffer $b_2$. The server moves to buffer $b_2$ only if buffer $b_1$ is empty. This service discipline gives priority to single packets (over a slot); packets other than the first of a slot are served under a FIFO policy interrupted by new arrivals. This service policy introduces some fairness in the service policy and favors single packets.

Clearly, the mean delay of single packets (or of the first packet of a multipacket of a slot) is equal to 1 slot, i.e. $D_1=1$. The mean delay of packets which enter $b_2$ is given by (4), where $\lambda_1$ is equal to $\pi(x\neq x_o)$ (the probability that the line is in any of the packet generating states), $\lambda_2=\lambda_{total}-\lambda_1$ and $D_{12}$ is the mean packet delay of the equivalent FIFO multiplexer of Fig. 1 computed from equations (1)-(3).

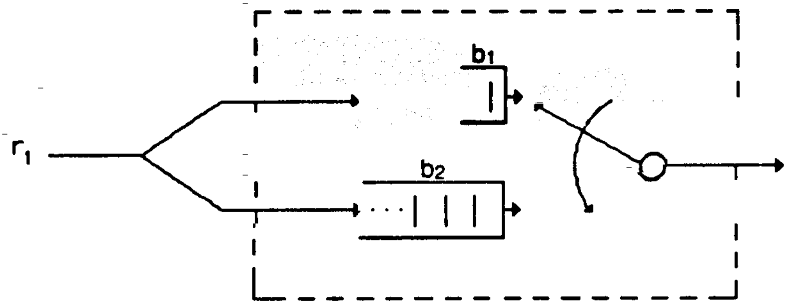


Figure 4.
The statistical multiplexer of Case 3.

### 3.4. Case 4

Consider the statistical multiplexer shown in Fig. 5. The per input line packet arrival process and the service policy are as in Case 3. The first packet per slot arriving in each of the input lines is given priority by being sent to the infinite capacity buffer $b_1$; the rest of the packets arriving over the same slot are sent to the infinite buffer $b_2$. The FIFO service policy is assumed for the packets of the same buffer. Packets in $b_1$ have priority over those in buffer $b_2$. That is, service of the packets in $b_2$ can start only if buffer $b_1$ is empty. This service policy avoids monopolization of the facility by either long messages (independently of the generating source) or certain sources (which by nature generate long messages). To compute $D_1$ and $D_2$ we proceed as follows.

Calculation of $D_1$

Consider a FIFO statistical multiplexer (Fig. 1) whose packet arrival process is given by MMGBP's . The underlying Markov chains of these MMGBP's are identical to those associated with the input lines $r_1, \cdots, r_N$. The probabilistic mapping

$$a(\bar{x})=\sum_{i=1}^{N}a^i(x^i) \quad , \quad \bar{x}\epsilon\bar{S}$$

is modified to describe the packet arrival process to $b_1$. That is,

$$a_1(\bar{x})=\sum_{i=1}^{N}1_{\{x^i\neq x_0^i\}} \quad , \quad \bar{x}\epsilon\bar{S} \tag{5}$$

where $x_o^i$ is the state of line i which generates no packets. Based on (5), the packet generating probabilities $\phi^i(x^i,k)$ are modified to the following

$$\phi^i(x_0^i,0)=1 \quad \text{and} \quad \phi^i(x^i,1)=1 \quad \text{for} \quad x^i\neq x_0^i \tag{6}$$

The mean delay of the packets in $D_1$ is now computed by applying equations (1)-(3) on the FIFO system with packet arrival processes as determined by (6). The total packet arrival rate $\lambda$ (used in (3)) is given by

$$\lambda_1=\sum_{i=1}^{N}\pi(x^i\neq x_0^i). \tag{7}$$

Calculation of $D_2$

The mean delay of packets in buffer $b_2$ is computed from (4), where $\lambda_1$ is given by (7), $\lambda_2=\lambda_{total}-\lambda_1$, and $D_{12}$ is the mean packet delay of the equivalent FIFO multiplexer of Fig. 1, computed from equations (1)-(3).

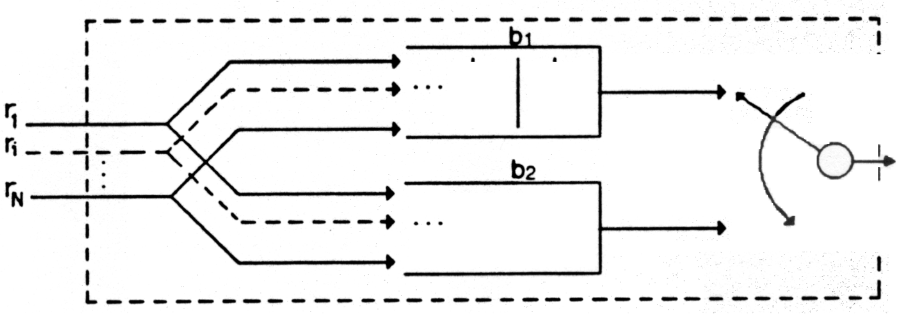


Figure 5.
The statistical multiplexer of Case 4.

## 4. NUMERICAL RESULTS

In this section some numerical results are derived for each of the four priority policies described in the previous section. In the examples considered below it is assumed that the underlying Markov chain associated with any of the input lines has two states, that is $S^i=\{0,1\}$ for the $i^{th}$ line. State 0 is the no-packet generating state (i.e. $a^i(0)=0$); state 1 generates at least one packet, up to a maximum of $K^i$, with probabilities $\phi^i(1,j)$, $1 \leq j \leq K^i$.

As the delay results illustrate, an input traffic process which generates packets clustered around consecutive slots and followed by a period of inactivity, causes significant queueing problems and the induced packet delay is greater that the one induced under better randomized packet arrivals of the same intensity. Since state 1 generates packets and state 0 does not, it makes sense to use the quantity $\gamma^i$, where,

$$\gamma^i = p^i(1,1) - p^i(0,1) \tag{8}$$

as a measure of the clusterness of the packet arrival traffic; $p^i(k,j)$ is the probability that the Markov chain associated with line i moves from state k to state j. The value of $\gamma^i=0$ corresponds to a per slot independent packet generation process (generalized Bernoulli process). The clusterness coefficient $\gamma^i$ and the packet arrival rate $\lambda^i$ are two important quantities which dramatically affect the delay induced by the multiplexing system. For this reason, each traffic will be characterized by the pair $(\lambda^i, \gamma^i)$ and the distribution $\phi^i(1,j)$, $1 \leq j \leq k^i$. The rest of the parameters of the MMGBP's associated with each input line are computed from the following equations:

$$\pi^i(1) = \frac{\lambda^i}{\sum_{j=1}^{K^i} j\phi^i(1,j)} \quad , \quad \pi^i(0) = 1 - \pi^i(1) \tag{9a}$$

$$p^i(0,1) = (1-\gamma^i)\pi^i(1) \quad ; \quad p^i(1,1) = \gamma^i + p^i(0,1) \tag{9b}$$

$$p^i(1,0) = 1-p^i(1,1) \quad , \quad p^i(0,0) = 1-p^i(0,1) \tag{9c}$$

### 4.1. Case 1

Consider the multiplexing system of Case 1 with distributions $\phi^1(1,1)=1$, $\phi^2(1,1)=.5$, $\phi^2(1,2)=.3$, $\phi^2(1,3)=.2$ and parameters $\lambda^1=\lambda^2=\lambda/2$ and $\gamma^1=\gamma^2=\gamma$. The mean packet delay results $D_1$, $D_2$ and $D_{12}$ are given in Table 1, for various values of $\lambda$ and $\gamma$. It can be easily observed that for a given total input rate $\lambda$, the smallest induced delay is achieved for $\gamma=0$ (independent per slot packet generation process). This is due to the fact that $\gamma=0$ results in the best randomization of the packet arrivals for given $\lambda$ and $\phi^1(1,j)$, $0 \leq j \leq K^i$.

When $\lambda^1=.35$ and $\gamma^1=.93$, line 1 may describe packetized voice traffic with geometrically distributed talkspurt periods (with mean ~22 packets) and geometrically distributed silence periods (with mean ~40 packets), [2]. The distributions of $\phi^1(1,1)$ and $\phi^2(1,j)$, $1 \leq j \leq 3$, are the same as before. The mean delay results are shown in Table 2 for various values of $\lambda^2$ and $\gamma^2$ (Case 1.b). Notice that although the total traffics considered are equal to those in Table 1, the induced mean packet delay $D_2$ is much larger, due to the larger value of the clusterness coefficient $\gamma^1$.

For $\lambda^1=.35$ , $\gamma^1=.93$ and $\phi^2(1,1)=1$, the induced mean packet delay $D_2$ is smaller than that of Case 1.b, for the same values of $\lambda^1$ ,$\gamma^1$ ,$\lambda^2$ and $\gamma^2$ (Case 1.c). This is due to the reduced clusterness resulting from the fact that only single packets arrive through line 2, as well (as opposed to possibly multiple packets arriving under the previous case). These results are shown in Table 2 (Case 1.c).

| $\lambda$ | $\gamma$ | $D_1$ | $D_{12}$ | $D_2$ |
|---|---|---|---|---|
| .90 | .5 | 1.000 | 13.897 | 26.794 |
| .90 | .3 | 1.000 | 9.325 | 17.651 |
| .90 | .0 | 1.000 | 5.897 | 10.794 |
| .70 | .5 | 1.000 | 4.799 | 8.598 |
| .70 | .3 | 1.000 | 3.466 | 5.981 |
| .70 | .0 | 1.000 | 2.466 | 3.931 |

Table 1
Mean packet delay results for Case 1.a

| | | Case 1.b | | Case 1.c | |
|---|---|---|---|---|---|
| $\lambda_2$ | $\gamma_2$ | $D_{12}$ | $D_2$ | $D_{12}$ | $D_2$ |
| .55 | .5 | 41.207 | 66.794 | 33.694 | 54.500 |
| .55 | .3 | 37.541 | 60.794 | 32.472 | 52.500 |
| .55 | .0 | 34.790 | 56.294 | 31.139 | 51.000 |
| .35 | .5 | 11.966 | 22.931 | 9.917 | 18.833 |
| .35 | .3 | 10.966 | 20.931 | 9.583 | 18.167 |
| .35 | .0 | 10.216 | 19.431 | 9.333 | 17.667 |

Table 2
Mean packet delay results for Cases 1.b and 1.c.

4.2. Case 2

Consider the multiplexing system of Case 2 with probability distributions $\phi^1(1,1)=.6$, $\phi^1(1,2)=.4$, $\phi^2(1,2)=.3$, $\phi^2(1,4)=.5$, $\phi^2(1,6)=.2$ and parameters $\lambda^1=\lambda^2=\lambda/2$ and $\gamma^1=\gamma^2=\gamma$. The mean packet delay results $D_1$, $D_2$ and $D_{12}$ are shown in Table 3 for various values of $\lambda$ and $\gamma$. Notice that $D_1>1$ since more than one packets may arrive over the same slot through line 1.

### 4.3. Case 3

Consider the multiplexing system of Case 3 with probability distribution $\phi^1(1,1)=.4$, $\phi^1(1,2)=.3$, $\phi^1(1,3)=.2$, $\phi^1(1,4)=.1$. The mean packet delay results $D_2$ and $D_{12}$ are shown in Table 3 for various values of $\lambda^1=\lambda$ and $\gamma^1=\gamma$.

### 4.4. Case 4

Consider the multiplexing system of Case 4 with N = 3 input lines, probability distributions as in Case 3 and parameters $\lambda^1=\lambda^2=\lambda^3=\lambda/3$ and $\gamma^1=\gamma^2=\gamma^3=\gamma$. The mean packet delay results are shown in Table 3 for various values of $\lambda$ and $\gamma$.

| | | Case 2 | | | Case 3 | | Case 4 | | |
|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\gamma$ | $D_1$ | $D_{12}$ | $D_2$ | $D_{12}$ | $D_2$ | $D_1$ | $D_{12}$ | $D_2$ |
| .9 | .5 | 2.247 | 33.468 | 64.689 | 18.500 | 36.000 | 1.818 | 27.500 | 53.181 |
| .9 | .3 | 1.831 | 21.754 | 41.676 | 12.786 | 24.571 | 1.506 | 18.357 | 35.208 |
| .9 | .0 | 1.519 | 12.968 | 24.416 | 8.500 | 16.000 | 1.273 | 11.500 | 21.727 |
| .7 | .5 | 2.055 | 11.323 | 20.590 | 6.833 | 12.667 | 1.538 | 9.167 | 16.795 |
| .7 | .3 | 1.703 | 7.608 | 13.513 | 4.928 | 8.857 | 1.333 | 6.373 | 11.413 |
| .7 | .0 | 1.439 | 4.823 | 8.206 | 3.500 | 6.000 | 1.179 | 4.278 | 7.376 |

Table 3
Mean packet delay results for Cases 2, 3 and 4.

## 5. CONCLUSIONS

In this paper some statistical multiplexing schemes under various priority policies have been analyzed. The per input line packet arrival processes are described by the Markov Modulated Generalized Bernoulli Process (MMGBP) defined in section II. The MMGBP can serve as a model for a wide class of complex packet arrival processes and thus, facilitate the appropriate description and the analysis of many practical systems. Furthermore, when certain priority policies are in effect the original MMGBP - describing the per line packet arrival process - can be transformed into another MMGBP where the priority policy is properly incorporated. As a result, auxiliary / equivalent FIFO multiplexing systems can be constructed with inputs described by a MMGBP, as well (see, e.g., Case 4). The previous property of the MMGBP facilitates the analysis of certain multiplexing systems under priorities.

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