STATISTICAL MULTIPLEXING OF CORRELATED SLOW TRAFFIC SOURCES

IOANNIS STAVRAKAKIS

University of Vermont
CS/EE Dept., Burlington, Vermont 05405 U.S.A.

ABSTRACT A statistical multiplexer supporting Markov, fixed-packet sources is studied in this paper. Unlike previous considerations, the time-constant of the sources is assumed to be greater than system time-unit (packet service time). As a result, the resulting arrival process is correlated but packets can not be generated over consecutive discrete-time instants whose distance is less than the source time-constant, although the source may be active. The interesting case of double-time-constant Markov sources is also considered. As an example, numerical results are derived for the case of multiplexing packetized voice sources.

I. INTRODUCTION

In this paper N sources of (fixed size) packetized information are multiplexed before accessing a fixed speed, slotted transmission line. The constant packet transmission time (slot) is assumed to be equal to one and is called the network time-constant or the network (output) time slot. The slot boundaries of the output transmission line define the discrete-time axis of the system. Packets are temporarily stored in a buffer until the output transmission line is available. Packet arrivals and departures are assumed to occur at the discrete-time instants.

The above statistical multiplexer has been studied in the past - in both discrete and continuous time - under correlated packet arrival processes. The most relevant work can be found can be found in the references in [1].

The correlated (Markov dependent) discrete-time packet arrival processes considered in the past are assumed to be generated by correlated sources whose time-constant is equal to the network time-constant; the source time constant is defined as the minimum time interval between consecutive packet generation instants. In this paper the packet sources are assumed to be correlated, as determined by a first-order Markov model. Unlike previous considerations, the transition time of the Markov model (source time-constant) is greater than the packet transmission time (network time-constant). The resulting traffic is more complex and defines a queueing system which is different from those considered in the past under correlated traffic. A relevant queueing system has been studied in [2,3] under non-Markov correlated arrivals.

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There are many potential applications of the queueing system presented in this paper. It may be adopted for the modeling of a multiplexer at the access points of high speed networks receiving traffic from lower speed networks or from sources with slow packet generation mechanism compared to the network slot. Due to limitations on the information processing speed, the time-constant of most information sources is larger than that of a high speed (fiber optics) networks. Furthermore, earlier developed (low speed) networks behave like slow information sources at the access point of a backbone high speed network.

The packet arrival processes generated by some types of slow sources are described in the next section and the general queueing model is formulated. The queueing analysis is then performed by invoking the study presented in [4], [5]. Numerical results are presented in the last section, together with a discussion on the impact of the correlation and the source time-constant on the queueing behaviour.

II. SOURCE TRAFFIC DESCRIPTION IN TERMS OF A COMMON MODEL

Consider the multiplexing of N sources of (fixed size) packetized information as described at the beginning of section I. Let the sequence of the slot boundaries define the discrete-time axis of the system, denoted by $J; J = \{0, 1, 2, ..\}$. The constant length packets generated by the sources are temporarily stored in a buffer of infinite capacity. The formulated queueing system can be modeled in terms of a single server, First-Come First-Served (FCFS), discrete-time queueing system with deterministic and equal to one slot service time. Let S^k denote the k^{th} packet source; a superscript k will denote a quantity associated with the k^{th} packet source for $1 \leq k \leq N$. Each of the packet sources is assumed to be either active (state 1) or inactive (state 0). Transitions between the two states occur according to a first-order Markov model. The one step transition time from state 0 and state 1 may or may not be identical, resulting in single- or double-time-constant sources; let $\pi^k(i)$ and $p^k(i,j)$ denote the steady state and the the transition probabilities, respectively, of the k^{th} Markov chain, $i, j \in \{0, 1\}$.

II.1 Single-Time-Constant Sources.

Let γ^k denote the (source) burstiness coefficient defined by $p^k(1,1)-p^k(0,1)$. Let τ^k denote the source time-constant which is defined to be equal to the one-step transition time of the k^{th} Markov chain; $\tau^k \in Z^+$, where Z^+ is the set of the positive integers. A transition is assumed to occur in the chain every τ^k time units, although its state may not change. Finally, let c^k be a constant which denotes the time instant when the first transition of the k^{th} Markov chain occurs; $0 \leq c^k \leq \tau^k - 1$, $c^k \in Z^0$, where Z^0 is the set of the nonnegative integers. Note that the steady state and the transition probabilities associated with the k^{th} source can be obtained from $\pi^k(1)$ and γ^k by incorporating the Markov properties. It is easy to establish that the sequence of time instants T^k, $T^k \subseteq J$, at which transitions of the k^{th} Markov chain occur is given by the set

$$T^k = \{j \in J : \frac{j - c^k}{\tau^k} \in Z^0\}. \tag{1}$$

The packet traffic $\{A_j^k\}_{j \in J}$ delivered by the k^{th} source is a discrete-time process defined in terms of the (underlying) Markov packet generating mechanism. Packets are assumed to be generated at the end of a one-step transition interval of the Markov chain associated with the source, as a result of the source activity during that interval. That is, they may be generated only at some time instant t^k, $t^k \in T^k$. Let i be the state of the k^{th} Markov chain at $t^k - \Delta t$, $t^k \in T^k$, $0 < \Delta t < 1$. The number of packets, n, generated by the source at t^k is described by the probability mass $\phi^k(i,n)$, $i \in \{0,1\}$, $n \in Z^0$. It is assumed that no packets are generated when the source is inactive. That is, $\phi^k(0,0) = 1$, $1 \leq k \leq N$.

In view of the above discussion, $\{A_j^k\}_{j \in J}$ is completely determined in terms of the parameters

$$\{\pi^k(1), \gamma^k, \tau^k, c^k, \phi^k(1,n) \text{ for } n \in Z^0\}. \tag{2}$$

Notice that the sequence of time instants of potential packet arrivals from the k^{th} source, T^k, is completely determined from (2). The same holds true for the set of time instants L^k at which it is guaranteed that no packet will be generated by the k^{th} source. L^k is given by

$$L^k = J - T^k = \{j \in J \ : \ \frac{j^k - c^k}{\tau^k} \notin Z^0\}. \tag{3}$$

Note that if $j \in L^k$ then $j + n\tau^k \in L^k$ as well, for $n \in Z^0$.

The packet arrival process $\{A_j^k\}_{j \in J}$ determined by (2) is different from those associated with similar queueing systems that have been studied in the past. When $\tau^k = 1$ (which implies that $c^k = 0$), $\{A_j^k\}_{j \in J}$ becomes the standard Markov Modulated Generalized Bernoulli (MMGB) process; here, the term standard implies that the one-step transition time is equal to one time unit. Of course, $\{A_j^k\}_{j \in J}$ for $\tau^k > 1$ may be seen as a standard MMGB process provided that the time unit of the system be equal to τ^k. When packet sources with different time-constants are involved, the selection of the network time-constant for the construction of the system time axis J results in convenient characterization of both the arrival and the departure (from the buffer) processes. One departure occurs at every time instant $j \in J$, provided that the buffer is non-empty. In this paper it is assumed that the time-constant of the sources are integer multiples of the network time-constant. Thus, all arrivals will occur at some time instant in J. A realization of an arrival process $\{A_j^k\}_{j \in J}$ for $\tau^k = 2$ and $c^k = 1$ is shown in Fig. 1.

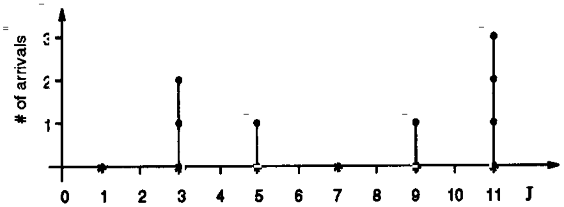

Fig. 1: A realization of the arrival process $\{A_j^k\}_{\{j \in J\}}$ for $\tau^k = 2$, $c^k = 1$; * denotes potential arrival points.

A packet source that generates an arrival process described by (2) for some $\tau^k > 1$ is considered to be slow (with respect to the network time constant) and correlated. Such packet sources may be adopted for the modeling of packet arrival processes appearing in communication networks. For instance, the packet traffic delivered by a (slow) transmission line to a multiplexer feeding a network line which is τ^k times faster, can be described in terms of a process determined by (2). Another example is the traffic delivered to a high speed line by a slow processor. In this case τ^k, $\tau^k > 1$, describes the processing time required before an output (possibly multi-packet) is generated.

It is important to note that the packet arrival process $\{A_j^k\}_{j \in J}$ has a structure which affects significantly the buffer behavior. This process presents a sequence of periodic time instants (contained in L^k) at which no packet is delivered. The existence of these periodic sequences has a positive impact on the intensity of the queueing problems, as it will be illustrated. Furthermore, the larger the value of τ^k, the less significant will be the impact of the correlation in the source on the queueing intensity. When $\tau^k > 1$, the correlation in the arrival process, as seen by the network (server), is said to have been reduced, compared to that present in the corresponding source. A large value of τ^k will allow the network to transmit a large number (up to τ^k) of packets before the next packet(s) is delivered by the correlated source. The special case of the arrival process $\{A_j^k\}_{j \in J}$ which is based on an uncorrelated packet generating mechanism is also interesting and different from an independent and identically distributed packet arrival process. The latter process may deliver packets over consecutive time instants in J; the former may not if $\tau^k > 1$.

In the rest of this section, the cumulative packet arrival process, $\{A_j\}_{j \in J}$, where

$$A_j = \left[\sum_{k=1}^{N} A_j^k\right], \tag{4}$$

is described in terms of an appropriate MMGB model. Let

$$\Delta_j = \{S^k : j \in T^k\}, \quad j \in J. \quad (5)$$

That is Δ_j is the set of all sources whose underlying Markov mechanism undergoes transition at time instant j. These sources potentially generate packets at that time instant. From the periodicity of the set T^k (see (1)) turns out that Δ_j is also periodic. Let

$$M = LCM\{\tau^1, \tau^2, \cdots, \tau^N\} \quad (6)$$

where $M = LCM\{\cdots\}$ denotes the least common multiplier of the argument. It is easily shown that

$$\Delta_j = \Delta_{j+nM}, \ n \in Z^0 \ \ or \ \ \Delta_j = \Delta_{j mod M}, j \in J \quad (7)$$

For example for $N = 2$ and $\tau^1 = 2$, $\tau^2 = 3$, $c^1 = 0$, $c^2 = 0$ turns out that $M = 6$ and $\Delta_0 = \{S^1, S^2\}$, $\Delta_1 = \{\phi\}$, $\Delta_2 = \{S^1\}$, $\Delta_3 = \{S^2\}$, $\Delta_4 = \{S^1\}$, $\Delta_5 = \{\phi\}$.

Let $\{X_j\}_{j\in J} = \{(I_j^1, I_j^2, ..., I_j^N, M_j)\}_{j\in J}$ be an $(N+1)$-dimensional discrete-time process; I_j^k, $I_j^k \in \{0,1\}$, $1 \le k \le N$, is a random variable describing the state of S^k at time instant j; $\{M_j\}_{j\in J}$ is a periodic Markov chain with state space $\{0, 1, \cdots, M-1\}$ which evolves as described below.

$$M_{j+1} = (M_j + 1) mod M \ \text{ with probability 1} \quad (8)$$

The state of $\{M_j\}_{j\in J}$ determines which of the sources undergo a state transition at the current time. The latter information together with that provided by the process $\{(I_j^1, I_j^2, \cdots I_j^n)\}_{j\in J}$ make $\{X_j\}_{j\in J}$ a Markov process. Let Ω, $\Omega = \{0,1\}^N \times \{0, 1, \cdots, M-1\}$, denote the state space of $\{X_j\}_{j\in J}$ and let $\bar{\Omega}$, $\bar{\Omega} = 2^N M$, denote its cardinality. The transition probabilities of $\{X_j\}_{j\in J}$ are given by

$$p\{(i^1, i^2, \cdots, i^N, n), (\tilde{i}^1, \tilde{i}^2, \cdots, \tilde{i}^N, \tilde{n})\} = \prod_{k: S^k \in \Delta_{(n+1)\text{mod}M}}$$

$$p^k(i^k, \tilde{i}^k) 1_{\{\tilde{i}^k = i^k \text{ if } s^k \notin \Delta_{(n+1)\text{mod}M}\}} 1_{\{\tilde{n} = (n+1)\text{mod}M\}} \ ,$$

$$(i^1, i^2, \cdots, i^n, n), \ \ (\tilde{i}^1, \ \tilde{i}^2, \cdots, \tilde{i}^N, \tilde{n}) \in \Omega \quad (9)$$

where it is defined that

$$\prod_{k \ : \ S^k \in \Delta_{(n+1)\text{mod}M}} p^k(i^k, \tilde{i}^k) = 1 \ \text{ if } \Delta_{(n+1)\text{mod}M} = \{\phi\}.$$

Notice that only the transition probabilities associated with the Markov chains of the sources in $\Delta_{(n+1)modM}$ are considered, since only these chains undergo a state transition at $j = (n+1)modM$. The first indicator function imposes the condition that the state of the sources not contained in $\Delta_{(n+1)modM}$ remain unchanged. The second indicator function imposes the requirement that the Markov chain $\{M_j\}_{j\in J}$ moves to the next state as determined by (8).

The cumulative arrival process $\{A^j\}_{j\in J}$ can be determined by the packet generation probabilities

$$\phi(i^1, i^2, \ \ \cdot, i^N, n; m) = \left[\bigotimes_{S^k \in \Delta_{n+1}} \phi^k(i^k; \cdot) \right](m)$$

$$(i^1, i^2, \ldots, i^N, n) \in \Omega, \ m \in Z^0 \quad (10)$$

where

$$\left[\bigotimes_{S^k \in \Delta_{n+1}} \phi^k(i^k; \cdot) \right]$$

$$\begin{cases} 0 \ for \ m \neq 0 & \text{if } \Delta_{n+1} = \{\phi\} \\ 1 \ for \ m = 0 & \text{if } \Delta_{n+1} = \{\phi\} \\ \text{k-fold conv. of } \phi^k(i^k; \cdot), & \text{all } k : s^k \in \Delta_{n+1}, \text{ at } m. \end{cases}$$

Note that packets may be generated only at the transition instants of the corresponding Markov chain and not at every discrete-time instant $j, j \in J$, at which the source is active.

The original queueing system may now be modeled in terms of an equivalent system whose packet arrival process is described by a MMGB process $\{A_j\}_{j\in J}$; $\{A_j\}_{j\in J}$ is described in terms of the Markov chain $\{X_j\}_{j\in J}$ and the packet generating probabilities $\phi(x; m)$, $x \in \Omega$ given by (10).

When the N packet arrival processes are identical (and synchronized) - that is, when the parameters in (2) are identical for all processes - then the $(N+1)$-dimensional Markov chain $\{X_j\}_{j\in J}$ can be replaced by the 2-dimensional Markov chain $\{X_j^{id}\}_{j\in J} = \{(I_j^{id}, M_j^{id})\}_{j\in J}$; $\{M_j^{id}\}_{j\in J}$ is a periodic Markov chain defined as $\{M_j\}_{j\in J}$, with parameter $M = \tau$; I_j^{id} is the random variable which describes the number of active sources at time instant j, $j \in J$. Details about the description of the cumulative packet arrival process in terms of a MMGB process, as well as discussion on the achieved reduced numerical complexity in this case, may be found in [1].

II.2 Double-Time-Constant Sources.

In the packet arrival processes considered so far, it has been assumed that the underlying Markov mechanism of the source makes a transition every τ^k time units. As a result, the set T^k determines the sequence of time instants in J at which activation of the source may occur. In some applications it is reasonable to assume that it takes τ^k time units for the source to generate (prepare) a packet, when active. On the other hand, the reactivation of the source may occur at any time instant and not necessarily at some integer multiple of τ^k. A correlated source with this behavior may be described in terms of a Markov model with two different one-step transition times (double-time-constant source). A transition from state 1 (active) occurs every τ^k time units while a transition from state 0 (inactive) occurs every one time unit. The packet arrival process generated by such a source is described by the parameters in (2) with the understanding that transition from a state 0 may occur in the next slot. Under this modeling, the re-activation of the source may occur at any time instant in J and not necessarily at the time instants contained in T_k. The length of an active period is geometrically distributed with mean $1/p^k(1,0)$ in time units of length τ^k; the length of an inactive period is geometrically distributed with mean $1/p^k(0,1)$ in system time units.

The double-time-constant Markov source introduced above may be alternatively described in terms of the Markov chain $\{Y_j^k\}_{j\in J}$ shown in Fig. 2. Let Ω_y^k, $\Omega_y^k = \{0,1,\cdots,\tau^k\}$ denote its state space whose cardinality $\bar{\Omega}_y^k$ is equal to τ^k+1. Let $p_y^k(i,j), i,j \in \Omega_y^k$, denote the transition probabilities of $\{Y_j^k\}_{j\in J}$; the nonzero such probabilities are given by

$$p_y^k(i,j) = 1,\ j = i+1,\ 1 \le i < \tau^k\ , \qquad (11)$$

$$p_y^k(\tau^k,1) = 1 - p^k(1,0)\ \ ,\ \ p_y^k(\tau^k,0) = p^k(1,0)\ , \qquad (11)$$

$$p_y^k(0,1) = p^k(0,1)\ \ ,\ \ p_y^k(0,0) = 1 - p^k(0,1) \qquad (11)$$

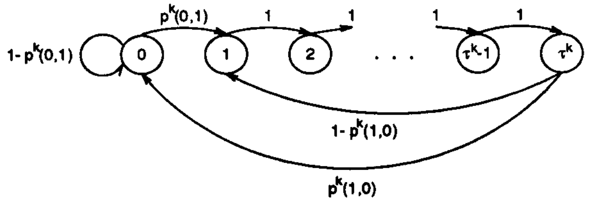

Fig. 2: The Markov model for $\{Y_j^k\}_{\{j\in J\}}$.

Note that state 0 in Fig. 2 corresponds to the inactive state 0 of the original two-state Markov model. A visit to state τ^k determines a time instant at which the active source will undergo a state transition. The source moves from state τ^k to state 0 (becomes inactive) in the next time instant with probability $p_y^k(\tau^k,0) = p^k(1,0)$; the source moves from state 0 to state 1 with probability $p_y^k(0,1) = p^k(0,1)$. The source remains in the active state for τ^k time units. Upon visit to state τ^k packets are generated according to the probabilities

$$\phi_y^k(\tau^k;m) = \phi^k(i;m),\ m \in Z^0. \qquad (12)$$

No packets are generated from the other states. That is,

$$\phi_y^k(i;0) = 1,\ 0 \le i < \tau^k \qquad (12)$$

In view of the above discussion, the packet arrival process $\{B_j^k\}_{j\in J}$, generated by the k^{th} double-time-constant source, $1 \le k \le N$, can be described as a MMGB process based on the underlying Markov chain $\{Y_j^k\}_{j\in J}$ and the packet generating probabilities given by (12). The cummulative packet arrival process $\{A_j^\tau\}_{j\in J}$, generated by the arrival processes $\{B_j^k\}_{j\in J}$, $1 \le k \le N$, can be described as a MMGB process whose parameters are easily described in terms of (11) and (12), [1].

III. NUMERICAL RESULTS

In the previous section the packet arrival processes generated by the correlated sources were described in terms of a MMGB model determined by an appropriate underlying Markov chain and a set of packet generating probabilities. The resulting queueing system has been considered in [4] and [5] and its detailed application may be found in [1]. The moments of the queue occupancy process can be computed through the solution of linear equations whose dimensionality is equal to the cardinality of the underlying Markov chain. In this section, numerical results are presented for the mean packet delay induced by a statistical multiplexer under the packet arrival processes described in Section II.

In Fig. 3 some numerical results are presented, as a function of the source time-constant τ, under various packet traffic models. The packet sources are assumed to be identical with parameters $(1 \le k \le N)$

$$\{\pi^k(1) = .35,\ \gamma^k = .93,\ \tau,\ c^k = 0,\ \phi^k(1,1) = 1\}\ ,$$

where the number of sources, N, is set to be equal to 2τ. The resulting cumulative packet traffic rate is equal to $N\pi^k(1)/\tau = .70$ packets/time units. Notice these packet sources have the characteristics of packetized voice sources delivering one packet every τ time units, when active.

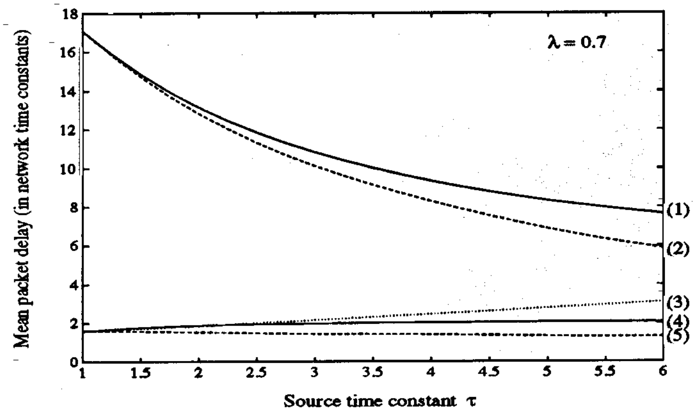

Fig. 3: Mean delay results for voice sources.

Curve (1) in Fig. 3 presents the induced packet delay. Notice that, for the same total traffic rate, the induced packet delay decreases as the time-constant increases. This decrement in the delay is attributed to the reduction of the source correlation ($\gamma^k = .93$) as seen by the server (as τ increases), as well as to the increased independence in the cumulative traffic due to the increased number of (mutually independent) contributing sources ($N = 2\tau$).

Curve (2) presents the induced packet delay when c^k, $1 \le k \le N$, are not identical for all sources but they are spread nearly uniformly over τ. When $\tau = 1$, no such spreading is possible and $c^k = 0$, $1 \le k \le N$. In this case curves (1) and (2) coincide. Notice that the larger the value of τ, the larger the smoothing of the arrival process that can be accomplished through the spreading of c^k. The intensity of the queueing problems is decreased as τ increases, as a result of the distribution of the potential packet arrival instants associated with the various sources.

Curve (3) presents the mean packet delay results when the packets are delivered by each of the $N = 2\tau$

sources according to a Bernoulli process with rate $\pi^k(1)/\tau$ packets per time unit. The second moment of the cumulative arrival process (Binomial) is easily found to increase with τ and, thus, the mean delay also increases, as indicated by the closed form delay formula derived in this case, [1]. This trend is clearly observed in curve (3) of Fig. 3. Notice that as τ increases the difference between curves (1) (or (2)) and curve (3) decreases, as expected.

Curve (4) presents the delay results under the traffic modeling considered for curve (1) but assuming that $\gamma^k = 0$, $1 \leq k \leq N$. That is, the sources are assumed to be uncorrelated. Notice that the resulting cumulative packet arrival process is different from the binomial process which was considered for the derivation of curve (3). For $\gamma^k = 0$, $1 \leq k \leq N$, and $\tau > 1$, the source can generate packets only at time instants in T^k (separated by τ time units) and not at any time instant in J, which is the case under the underlying Bernoulli model assumed for curve (3). Notice that the delay results have smaller values in this case compared to those under curve (3), possibly due to the positive effect of the constraint that packets can not be generated at time instants separated by less than τ time units under the model for curve (4).

Finally, curve (5) presents the delay results under the model considered for curve (4), but under the assumption that the values of c^k, $1 \leq k \leq N$, are near-uniformly spread over τ. As it was explained before, this spreading of the potential packet arrival instants has a positive impact on the queueing intensity; this effect is clearly observed by comparing curves (4) and (5) of Fig. 3.

Notice that when $\tau = 1$ the arrival processes may change state at any slot. In this case, the results may also be obtained from the closed form expression derived in [6].

Fig. 4 presents the delay results obtained by multiplexing $N = 3$ packet sources with different time-constants given by $\tau^1 = 2$, $\tau^2 = 4$ and $\tau^3 = 4$. The rest of the parameters are assumed to be identical. The source burstiness coefficient is equal to $\gamma^{id} = .5$ and the packet generating probabilities are given by $\phi^{id}(1,1) = .3$, $\phi^{id}(1,2) = .4$ and $\phi^{id}(1,3) = .3$. The probability that the packet source is in the active state, $\pi^{id}(1)$, is selected so that the resulting cumulative packet traffic rate is equal to some value from the horizontal axis of Fig. 4. Curves (1), (2), (3), (4), and (5) show the mean packet delay under the traffic modeling considered for the derivation of the corresponding curves of Fig. 3. Curve (6) is derived under the assumption that each of the sources may be re-activated at any time instant and not at integer multiples of τ^k, $1 \leq k \leq 3$. Thus, the double-time-constant source model, discussed in Section II.2, has been assumed. Curve (7) presents results under the source modeling considered for curve (6) under the assumption that $\gamma^k = 0$, $1 \leq k \leq 3$. Notice that states 0 and 1 may be merged in this case (Fig. 2), resulting in a reduced number of linear equations. Further reduction may be achieved by expoiting the partial symmetry in the arrival process.

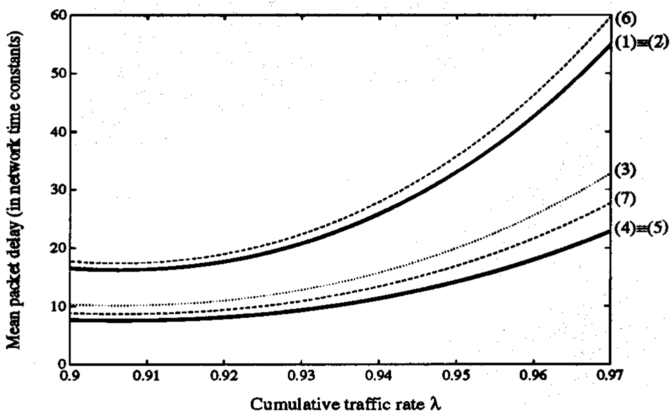

Fig. 4: Mean delay results for the N=3 packet sources.

Notice that the coincidence of curves (1) and (2) and curves (4) and (5) is due to the specific parameters of the selected example. Finally, notice that curve (6) is always above curves (1) and (2) derived for the same values of γ^k, $1 \leq k \leq 3$. Similarly, curve (7) is always above curves (4) and (5) derived for $\gamma^k = 0$, $1 \leq k \leq 3$. This behavior may be explained in view of the fact that the minimum separation between two consecutive active periods in the traffic for the curves (1), (2), (4) and (5) is equal to τ^k, $1 \leq k \leq 3$, while that under the traffic models for the curves (6) and (7) is equal to one. The latter is expected to have negative effect on the intensity of the resulting queueing problems. Finally, the increased delay results presented under curve (3) may be explained in view of the fact that there is no minimum separation between consecutive packet arrivals, in any case, for the traffic model for curve (3).

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