Characterization and Multiplexing of Correlated Traffic from Sources with Different Time Constants*

Ioannis Stavrakakis†
Department of Electrical Engineering and Computer Science
University of Vermont
Burlington, Vermont 05405
Tel: (802) 656-8506 FAX: (802) 656-8802

Abstract

Single server, discrete-time queueing systems under independent or Markov arrival processes and deterministic service requirements have been studied widely in the past. The fixed service time is usually set equal to one defining the time unit of the discrete-time system axis. Such queueing systems have been adopted for the modeling of statistical multiplexers in slotted communication networks supporting sources of (fixed size) packetized information.

In this paper, a statistical multiplexer fed by Markov packet sources is considered. Unlike previous considerations, the time-constant of the sources is assumed to be greater than one. As a result, packets can not be generated over consecutive discrete-time instants whose distance is less than the source time-constant, although the source may be active. The interesting case of double-time-constant Markov sources is also considered. The resulting packet traffic is characterized and the formulated queueing model is studied. This work may be useful for the traffic characterization and performance analysis of ATM multiplexers in a high speed networking environment. As an example, numerical results are derived for the case of multiplexing packetized voice sources.

*Research supported by the National Science Foundation under grant NCR-9011962.
†email : stavraka@uvm.edu - 8/92

I. INTRODUCTION

The probabilistic nature of the traffic generated by the users of a networking facility renders inefficient the exclusive allocation of the resources to individual users over a certain time horizon. Some type of statistical multiplexing of the information generated by different sources is usually employed at the access points of a network. In this paper N sources of (fixed size) packetized information are multiplexed before accessing a fixed speed, slotted transmission line, Fig. 1. The constant packet transmission time (slot) is assumed to be equal to one and is called the network time-constant or the network (output) time slot. The slot boundaries of the output transmission line define the discrete-time axis of the system. Packets are temporarily stored in a buffer until the output transmission line is available. Packet arrivals and departures are assumed to occur at the discrete-time instants.

The above statistical multiplexer has been studied in the past under correlated packet arrival processes. The most relevant work can be found can be found in [1-10] (and the references cited there). In [1], the aggregate traffic generated by multiple independent input lines is modeled as a 2-state Markov Modulated Poisson process using moment matching techniques. Then, the resulting statistical multiplexer is studied. In [2], the author considers a single input line and arrivals which depend on an underlying two state Markov chain. In [3-5] N input lines are assumed present. In [3], it is assumed that the packet arrival process in each of the identical input lines depends on an underlying two state Markov chain (active/inactive). In [4] it is assumed that the per line packet arrival process is a first order Markov chain and at most one packet arrival is possible. A closed form solution for the mean packet delay has been derived for the latter case. In [5], a closed form expression for the mean packet delay in the case of Bernoulli per line arrivals can be found. The discrete time system considered in [6,7] corresponds to a statistical multiplexer where packets arrive through a single input line and they form a Markov chain. Finally, statistical multiplexers under discrete-time phase-type arrival processes have been considered in [8-10]. More references on queueing models for statistical multiplexers may be found in [11].

Extensive previous work on similar queueing systems in a continuous time set up can be also found in the literature. In [12], N identical on-off sources with exponentially distributed on and off periods is considered. The case of N identical input lines with exponentially distributed idle periods and arbitrarily distributed active periods is studied in [13,14]. Similar model when the input lines are not identical has been considered in [15]. Other continuous time models in a multi-entry buffer system have been considered in [16,17]. Finally, when the Markov processes describing the per input line packet arrivals is a reversible process, a decomposition method that circumvents the state explosion problem is presented in [18].

The correlated (Markov dependent) discrete-time packet arrival processes considered in the past is assumed to be generated by correlated sources whose time-constant is equal to the network time-constant; the source time constant is defined as the minimum time interval between consecutive packet generation instants. In this paper the packet sources are assumed to be correlated, as determined by a first-order Markov model. Unlike previous considerations, the transition time of the Markov model (source time-constant) is greater than the packet transmission time (network time-constant). The resulting traffic is more complex and defines a queueing system which is different from any considered in the past under correlated traffic. A relevant queueing system has been studied in [19,20] under non-Markov correlated arrivals.

There are many potential applications of the queueing system presented in this paper. It may be adopted for the modeling of a multiplexer at the access points of high speed networks receiving traffic from lower speed networks or from sources with slow packet generation mechanism compared to the network slot. Due to limitations on the information processing speed, the time-constant of most information sources is larger than that of a high speed (fiber optics) networks. Furthermore, earlier developed (low speed) networks behave like slow information sources at the access point of a backbone high speed network. Similar queueing systems appear in a computer processor or a production line where jobs delivered to a server require a pre-processing time greater than that at the server.

In the next section, the packet arrival processes generated by some types of slow sources are described and the resulting queueing systems are formulated. In section III, a relevant queueing model for the study of these systems is presented. Numerical results are presented in the last section, together with a discussion on the impact of the correlation and the source time-constant on the queueing behaviour.

II. SOURCE PACKET TRAFFIC AND THE FORMULATED QUEUEING SYSTEMS

Consider a fixed-speed slotted network line that supports the transmission of packetized information generated by N sources. The network slot (or network time-constant) is equal to the packet transmission time. Packet transmissions are synchronized with the network slot boundaries. The sequence of the slot boundaries defines the discrete-time axis of the system, denoted by $J; J = \{0, 1, 2, ..\}$. The constant length packets generated by the sources are temporarily stored in a buffer of infinite capacity. The formulated queueing system can be modeled in terms of a single server, First-Come First-Served (FCFS), discrete-time queueing system with deterministic and equal to one slot service time, Fig. 1.

Let S^k denote the k^{th} packet source; a superscript k will denote a quantity associated with the k^{th} packet source for $1 \leq k \leq N$. Each of the packet sources is assumed to be either active (state 1) or inactive (state 0). Transitions between the two states occur according to a first-order Markov model, Fig 2. The one step transition time from state 0 and state 1 may or may not be identical, resulting in single- or double-time-constant sources.

II.1 Single-Time-Constant Sources; Asymmetric Case

Consider N single-time-constant sources of packetized information. As it will be shown below, the packet traffic generated by the k^{th} source will be described in terms of the parameters $\{\pi^k(1), \gamma^k, \tau^k, c^k\}$ defined as follows, for $1 \leq k \leq N$.

$\pi^k(i)$: It denotes the steady state probability that the k^{th} Markov chain is in state i, $i \in \{0, 1\}$.

γ^k: It denotes the (source) burstiness coefficient defined by $p^k(1,1) - p^k(0,1)$; $p^k(i,j)$ denotes the transition probability that the k^{th} Markov chain moves from state i to state j, $i, j \in \{0, 1\}$.

τ^k: It is defined as the source time-constant and it is equal to the one-step transition time of the k^{th} Markov chain; $\tau^k \in Z^+$, where Z^+ is the set of the positive integers. A transition is assumed to occur in the chain every τ^k time units, although the its state may not change.

c^k: It is a constant which denotes the time instant when the first transition of the k^{th} Markov chain occurs; $0 \leq c^k \leq \tau^k - 1,\ c^k \in Z^0$, where Z^0 is the set of the nonnegative integers.

Note that the steady state and the transition probabilities associated with the k^{th} source can be obtained from $\pi^k(1)$ and γ^k by incorporating the Markov properties. The sequence of time instants T^k, $T^k \subseteq J$, at which transitions of the k^{th} Markov chain occur is given by the set

$$T^k = \{j \in J : \frac{j - c^k}{\tau^k} \in Z^0\}. \tag{1}$$

The packet traffic $\{A_j^k\}_{j \in J}$ delivered by the k^{th} source is a discrete-time process defined in terms of the (underlying) Markov packet generating mechanism. Packets are assumed to be generated at the end of a one-step transition interval of the Markov chain associated with the source, as a result of the source activity during that interval. That is, they may be generated only at some time instant $t^k,\ t^k \in T^k$. Let i be the state of the k^{th} Markov chain at $t^k - \triangle t,\ t^k \in T^k,\ 0 < \triangle t < 1$. The number of packets, n, generated by the source at t^k is described by the probability mass $\phi^k(i,n),\ i \in \{0,1\},\ n \in Z^0$. It is assumed that no packets are generated when the source is inactive. That is, $\phi^k(0,0) = 1,\ 1 \leq k \leq N$.

In view of the above discussion, $\{A_j^k\}_{j \in J}$ is completely determined in terms of the parameters

$$\{\pi^k(1), \gamma^k, \tau^k, c^k, \phi^k(1,n) \text{ for } n \in Z^0\}. \tag{2}$$

Notice that the sequence of time instants of potential packet arrivals from the k^{th} source, T^k, is completely determined from (2). The same holds true for the set of time instants L^k at which it is guaranteed that no packet will be generated by the k^{th} source. L^k is given by

$$L^k = J - T^k = \{j \in J \ : \ \frac{j^k - c^k}{\tau^k} \notin Z^0\} \tag{3}$$

Note that if $j \in L^k$ then $j + n\tau^k \in L^k$ as well, for $n \in Z^0$

The packet arrival process $\{A_j^k\}_{j \in J}$ determined by (2) is different from those associated with similar queueing systems that have been studied in the past. When $\tau^k = 1$ (which implies that $c^k = 0$), $\{A_j^k\}_{j\in J}$ becomes the standard Markov Modulated Generalized Bernoulli (MMGB) process; here, the term standard implies that the one-step transition time is equal to one time unit. Of course, $\{A_j^k\}_{j\in J}$ for $\tau^k > 1$ may be seen as a standard MMGB process provided that the time unit of the system be equal to τ^k. When packet sources with different time-constants are involved, the selection of the network time-constant for the construction of the system time axis J results in convenient characterization of both the arrival and the departure (from the buffer) processes. One departure occurs at every time instant $j \in J$, provided that the buffer is non-empty. In this paper it is assumed that the time-constant of the sources are integer multiples of the network time-constant. Thus, all arrivals will occur at some time instant in J. A realization of an arrival process $\{A_j^k\}_{j\in J}$ for $\tau^k = 2$ and $c^k = 1$ is shown in Fig. 3.

A packet source that generates an arrival process described by (2) for some $\tau^k > 1$ is considered to be slow (with respect to the network time constant) and correlated. Such packet sources may be adopted for the modeling of packet arrival processes appearing in communication networks. For instance, the packet traffic delivered by a (slow) transmission line to a multiplexer feeding a network line which is τ^k times faster, can be described in terms of a process determined by (2). Another example is the traffic delivered to a high speed line by a slow processor. In this case τ^k, $\tau^k > 1$, describes the processing time required before an output (possibly multi-packet) is generated.

It is important to note that the packet arrival process $\{A_j^k\}_{j\in J}$ has a structure which affects significantly the buffer behavior. This process presents a sequence of periodic time instants (contained in L^k) at which no packet is delivered. The existence of these periodic sequences has a positive impact on the intensity of the queueing problems, as it will be illustrated. Furthermore, the larger the value of τ^k, the less significant will be the impact of the correlation in the source on the queueing intensity. When $\tau^k > 1$, the correlation in the arrival process, as seen by the network (server), is said to have been reduced, compared to that present in the corresponding source. A large value of τ^k will allow the network to transmit a large number (up to τ^k) of packets before the next packet(s) is delivered by the correlated source. The special case of the arrival process $\{A_j^k\}_{j\in J}$ which is based on an uncorrelated packet generating mechanism is also interesting and different from an independent and identically distributed packet arrival process. The latter process may deliver packets over consecutive time instants in J; the former may not if $\tau^k > 1$.

In the rest of this section, the cumulative packet arrival process, $\{A_j\}_{j\in J}$ where

$$A_j = \left[\sum_{k=1}^{N} A_j^k\right] \tag{4}$$

is described in terms of an appropriate MMGB model. Let

$$\Delta_j = \{S^k : j \in T^k\}, \quad j \in J. \tag{5}$$

That is Δ_j is the set of all sources whose underlying Markov mechanism undergoes transition at time instant j. These sources potentially generate packets at that time instant. From the periodicity of the set T^k (see (3)) turns out that Δ_j is also periodic. Let

$$M = LCM\{\tau^1, \tau^2, \cdots, \tau^N\} \tag{6}$$

where $M = LCM\{\cdots\}$ denotes the least common multiplier of the argument. It is easily shown that

$$\Delta_j = \Delta_{j+nM} \ for\ n \in Z^0 \quad or \quad \Delta_j = \Delta_{jmodM} \ \ for\ j \in J \tag{7}$$

For example for $N = 2$ and $\tau^1 = 2,\ \tau^2 = 3,\ c^1 = 0,\ c^2 = 0$ turns out that $M = 6$ and $\Delta_0 = \{S^1, S^2\}$, $\Delta_1 = \{\phi\}$, $\Delta_2 = \{S^1\}$, $\Delta_3 = \{S^2\}$, $\Delta_4 = \{S^1\}$, $\Delta_5 = \{\phi\}$.

Let $\{X_j\}_{j\in J} = \{(I_j^1, I_j^2, ..., I_j^N, M_j)\}_{j\in J}$ be an $(N+1)$-dimensional discrete-time process; $I_j^k,\ I_j^k \in \{0,1\},\ 1 \le k \le N$, is a random variable describing the state of S^k at time instant j; $\{M_j\}_{j\in J}$ is a periodic Markov chain with state space $\{0, 1, \cdots, M-1\}$ which evolves as described below.

$$M_{j+1} = (M_j + 1) modM \ \text{ with probability } 1 \tag{8}$$

The state of $\{M_j\}_{j\in J}$ determines which of the sources undergo a state transition at the current time. The latter information together with that provided by the process $\{(I_j^1, I_j^2, \cdots I_j^n)\}_{j\in J}$ make $\{X_j\}_{j\in J}$ a Markov process. Let $\Omega,\ \Omega = \{0,1\}^N \times \{0, 1, \cdots, M-1\}$, denote the state space of $\{X_j\}_{j\in J}$ and let $\bar{\Omega},\ \bar{\Omega} = 2^N M$, denote its cardinality. The transition probabilities of $\{X_j\}_{j\in J}$ are given by

$$p\{(i^1,i^2,\cdots,i^N,n),(\tilde{i}^1,\tilde{i}^2,\cdots,\tilde{i}^N,\tilde{n})\} = \prod_{k:S^k\in\Delta_{(n+1)\bmod M}} p^k(i^k,\tilde{i}^k)1_{\{\tilde{i}^k=i^k \text{ if } s^k\notin\Delta_{(n+1)\mathrm{mod}M}\}}$$

$$1_{\{\tilde{n}=(n+1)\mathrm{mod}M\}}, \quad (i^1,i^2,\cdots,i^n,n),\ (\tilde{i}^1,\ \tilde{i}^2,\cdots,\tilde{i}^N,\tilde{n})\in\Omega \tag{9}$$

where it is defined that

$$\prod_{k\,:\,S^k\in\Delta_{(n+1)\bmod M}} p^k(i^k,\tilde{i}^k)=1 \ \text{ if } \Delta_{(n+1)\mathrm{mod}M}=\{\phi\}.$$

Notice that only the transition probabilities associated with the Markov chains of the sources in $\Delta_{(n+1)modM}$ are considered, since only these chains undergo a state transition at $j=(n+1)modM$. The first indicator function imposes the condition that the state of the sources not contained in $\Delta_{(n+1)modM}$ remain unchanged. The second indicator function imposes the requirement that the Markov chain $\{M_j\}_{j\in J}$ moves to the next state as determined by (8).

The cumulative arrival process $\{A^j\}_{j\in J}$ can be determined by the packet generation probabilities

$$\phi(i^1,i^2,\cdots,i^N,n;m)=\left[\bigotimes_{S^k\in\Delta_{n+1}}\phi^k(i^k;\cdot)\right](m),\quad (i^1,i^2,\ldots,i^N,n)\in\Omega,\ m\in Z^0 \tag{10}$$

where

$$\left[\bigotimes_{S^k\in\Delta_{n+1}}\phi^k(i^k;\cdot)\right](m)\equiv\begin{cases}0 \ \ for \ \ m\neq 0 & \text{if } \Delta_{n+1}=\{\phi\}\\ 1 \ \ for \ \ m=0 & \text{if } \Delta_{n+1}=\{\phi\}\\ \text{k-fold convolution of } \phi^k(i^k;\cdot), & \text{for all } k:s^k\in\Delta_{n+1}, \text{ at } m.\end{cases}$$

Note that packets may be generated only at the transition instants of the corresponding Markov chain and not at every discrete-time instant $j, j\in J$, at which the source is active.

The original queueing system may now be modeled in terms of an equivalent system whose packet arrival process is described by a MMGB process $\{A_j\}_{j\in J}$; $\{A_j\}_{j\in J}$ is described in terms of the Markov chain $\{X_j\}_{j\in J}$ and the packet generating probabilities $\phi(x;m)$, $x\in\Omega$ given by (10). This queueing system is studied in the next section.

II.2 Single-Time-Constant Sources; Symmetric Case.

When the N packet arrival processes are identical (and synchronized) - that is, when the parameters in (2) are identical for all processes - then the $(N+1)$-dimensional Markov chain $\{X_j\}_{j\in J}$ can be replaced by the 2-dimensional Markov chain

$$\{X_j^{id}\}_{j\in J} = \{(I_j^{id}, M_j^{id})\}_{j\in J}.$$

$\{M_j^{id}\}_{j\in J}$ is a periodic Markov chain defined as $\{M_j\}_{j\in J}$, with parameter $M = \tau$; I_j^{id} is the random variable which describes the number of active sources at time instant j, $j \in J$. Let Ω^{id}, $\Omega^{id} = \{0,1,\ldots,N\}\times\{0,1,\ldots,\tau-1\}$, denote the state space of $\{X^{id}\}_{j\in J}$; its cardinality $\bar{\Omega}^{id}$ is equal to $(N+1)\tau$. The sets $T^k(L^k)$ defined by (1) ((3)) are identical for all k, $1 \le k \le N$, and they are denoted by $T^{id}(L^{id})$. The set of active sources, Δ^{id}, is given by

$$\Delta_j^{id} = \begin{cases} \{\phi\} & \text{if } j \in L \\ \{S^1, S^2, \cdots, S^n\} & \text{if } j \in T \end{cases} \tag{11}$$

For simplicity it may be assumed that $c^k = 0$ for all k, $1 \le k \le N$. In this case (11) becomes

$$\Delta_j^{id} = \begin{cases} \{\phi\} & \text{if } j mod \tau = 0 \\ \{S^1, S^2, \cdots, S^N\} & \text{otherwise} \end{cases}$$

The transition probabilities of $\{X_j^{id}\}_{j\in J}$ are given by

$$p^{id}\{(i,n),(\tilde{i},\tilde{n})\} = [\, q(i,\tilde{i})1_{\{\tilde{n}=0\}} + 1_{\{\tilde{n}\neq 0,\tilde{i}=i\}}\,]1_{\{\tilde{n}=(n+1)mod\tau\}} \tag{12}$$

where the transition probabilities $q(i\tilde{i})$ $0 \le i, \le N$, of the Markov chain $\{I_k^{id}\}_{k\in T^{id}}$ - defined on the transition time instants in T^{id} - with unit-step transition time τ are given by

$$q(i,\tilde{i}) = \sum_{k=\max\{0,i+j-N\}}^{\min\{i,\tilde{i}\}} \begin{pmatrix} i \\ k \end{pmatrix} p^k(1,1)p^{i-k}(1,0) \begin{pmatrix} N-i \\ \tilde{i}-k \end{pmatrix} p^{\tilde{i}-k}(0,1)p(0,0)^{N-i-(\tilde{i}-k)}. \tag{13}$$

The last indicator function in (12) imposes the condition that the Markov chain $\{M_j\}_{j\in J}$ moves to the next state as determined by (8). The first indicator function imposes the requirement that a transition occurs at a time instant in T^{id} $(\tilde{n}=0)$ in which case the transition probability is given by (13). The second indicator function imposes the requirement that the state of $\{I_j^{id}\}$ remain unchanged if $(j+1) \notin T^{id}$ $(\tilde{n} \neq 0)$. If $\tilde{n} \neq 0$ and $\tilde{i} = i$, the transition probability is equal to one, provided that $\tilde{n} = (n+1) mod M$.

The cumulative packet arrival process $\{A_j^{id}\}_{j \in J}$ can be described in terms of a MMGB process based on the underlying Markov chain $\{X_j^{id}\}_{j \in j}$ and the packet generating probabilities given by

$$\phi^{id}(i,n;m) = \begin{cases} 0 \ \ for \ \ m \neq 0 & \text{if } n+1 \neq 0 \ or \ i = 0 \\ 1 \ \ for \ \ m = 0 & \text{if } n+1 \neq 0 \ or \ i = 0 \\ \left[\bigotimes_i \phi^{id}(1;\cdot)\right](m) & \text{if } n+1 = 0 \end{cases} \tag{14}$$

where $\bigotimes_i$ denotes an i-fold convolution and $\phi^{id}(1;\cdot)$ denotes the common packet generating probability associated with a single source, as employed in (2).

Although the cumulative packet arrival process in the case of identical packet sources may be described in terms of $\{X_j\}_{j \in J}$ (and the corresponding probabilities be given by (10)), the formulation of $\{X_j^{id}\}_{j \in J}$ results in a state space Ω^{id} whose cardinality $\bar{\Omega}^{id}$ increases linearly with respect to N; on the other hand, $\bar{\Omega}$ increases exponentially with respect to N. As it will be seen in the next section, the cardinality at the underlying Markov chain affects significantly the complexity of the solution.

II.3 Double-Time-Constant Sources.

In the packet arrival processes considered so far, it has been assumed that the underlying Markov mechanism of the source makes a transition every τ^k time units. As a result, the set T^k determines the sequence of time instants in J at which activation of the source may occur. In some applications it is reasonable to assume that it takes τ^k time units for the source to generate (prepare) a packet, when active. On the other hand, the re-activation of the source may occur at any time instant and not necessarily at some integer multiple of τ^k. A correlated sourse with this behavior may be described in terms of the Markov model shown in Fig. 2, with two different one-step transition times (double-time-constant source). A transition from state 1 (active) occurs every τ^k time units while a transition from state 0 (inactive) occurs every one time unit. The packet arrival process generated by such a source is described by the parameters in (2) with the understanding that transition from a state 0 may occur in the next slot.

Under this modeling, the re-activation of the source may occur at any time instant in J and not necessarily at the time instants contained in T_k. The length of an active period is geometrically distributed with mean $1/p^k(1,0)$ in time units of length τ^k; the length of an inactive period is geometrically distributed with mean $1/p^k(0,1)$ in system time units.

The double-time-constant Markov source introduced above may be alternatively described in terms of the Markov chain $\left\{Y_j^k\right\}_{j\in J}$ shown in Fig. 4. Let Ω_y^k, $\Omega_y^k = \left\{0, 1, \cdots, \tau^k\right\}$ denote its state space whose cardinality $\bar{\Omega}_y^k$ is equal to $\tau^k + 1$. Let $p_y^k(i,j), i,j \in \Omega_y^k$, denote the transition probabilities of $\left\{Y_j^k\right\}_{j\in J}$; the nonzero such probabilities are given by

$$p_y^k(i,j) = 1,\ j = i+1,\ 1 \leq i < \tau^k \quad , \quad p_y^k(\tau^k, 1) = 1 - p^k(1,0) \quad , \tag{15}$$

$$p_y^k(\tau^k, 0) = p^k(1,0) \quad , \quad p_y^k(0,1) = p^k(0,1) \quad , \quad p_y^k(0,0) = 1 - p^k(0,1) \tag{15}$$

Note that state 0 in Fig. 4 corresponds to the inactive state 0 (Fig. 2). A visit to state τ^k determines a time instant at which the active source will undergo a state transition. The source moves from state τ^k to state 0 (becomes inactive) in the next time instant with probability $p_y^k(\tau^k, 0) = p^k(1,0)$; the source moves from state 0 to state 1 with probability $p_y^k(0,1) = p^k(0,1)$. The source remains in the active state for τ^k time units. Upon visit to state τ^k packets are generated according to the probabilities

$$\phi_y^k(\tau^k; m) = \phi^k(i; m),\ m \in Z^0. \tag{16}$$

No packets are generated from the other states. That is,

$$\phi_y^k(i; 0) = 1,\ 0 \leq i < \tau^k \tag{16}$$

In view of the above discussion, the packet arrival process $\{B_j^k\}_{j\in J}$, generated by the k^{th} double-time-constant source, $1 \leq k \leq N$, can be described as a MMGB process based on the underlying Markov chain $\{Y_j^k\}_{j\in J}$ and the packet generating probabilities given by (16).

The cummulative packet arrival process $\{A_j^r\}_{j\in J}$, generated by the arrival processes $\{B_j^k\}_{j\in J}$, $1 \leq k \leq N$, can be described as a MMGB process based on the N-dimensional Markov chain $\{X_j^r\}_{j\in J}$=$\{(Y_j^1, \cdots Y_j^N)\}_{j\in J}$

with state space $\Omega^r = \Omega_y^1 \times \Omega_y^2 \times \Omega_y^N$, transition probabilities

$$p^r((i^1, i^2, \cdots, i^N), (\tilde{i}^1, \tilde{i}^2, \cdots, \tilde{i}^N)) = \prod_{k=1}^{N} p_y^k(i^i, \tilde{i}^k) \tag{17}$$

and packet generating probabilities

$$\phi^r(y^1, y^2, \cdots y^N; k) = [\bigotimes_k \phi_y^k(y^k; \cdot)](k),\ k \in Z^0,\ \ (y^1, y^2, \cdots y^N) \in \Omega_r \tag{18}$$

In the next section the analysis of the queueing model formulated above is presented.

III. THE QUEUEING SYSTEM UNDER MMGB ARRIVALS

III. 1. The General Case

Consider the FCFS, single server, discrete-time queueing systems with cumulative packet arrivals described by a MMGB process $\{A_j\}_{j\epsilon J}$; J denotes the system axis, as defined in the previous section. Let $\{X_j\}_{j\epsilon J}$ denote the underlying Markov process associated with $\{A_j\}_{j\epsilon J}$; let Ω denote its state space with cardinality $\bar{\Omega}$; let $\pi(x)$ and $p(x, y)$ denote its steady state transition probabilities, $x, y \epsilon \Omega$. Let $\phi(x; k)$ denote the probabilistic mapping which determines the probability that k packets arrive when $\{X_j\}_{j\epsilon J}$ is in state $x, x\epsilon\Omega$, $k\epsilon Z^0$ and $k \leq R$, $0 < R \leq \infty$; let $\mu(x)$ and $\sigma(x)$ denote the first and the second moments, respectively, of the probability mass function $\phi(x; .)$.

Let $p^m(j; y)$ denote the joint probability that there are j packets in the (queueing) system at the m^{th} time instant (packet arrivals at this time instant are included) and the state of $\{X_j\}_{j\epsilon J}$ is y; the packet arrivals generated due to the visit to state y have not been accounted for. The evolution of the buffer occupancy process can be described by an $(\bar{\Omega} + 1)$-dimensional Markov chain with state space $\bar{\Omega} \times \{0, 1, 2, \cdots\}$ and state probabilities described by the equations:

$$p^m(j; y) = \sum_{x\epsilon\Omega} \sum_{k=0}^{R} p^{m-1}(j + 1 - k; x)p(x, y)\phi(x; k) \quad , \quad j \geq R + 1 \tag{19a}$$

$$p^m(j; y) = \sum_{x\epsilon\Omega} p(x, y) \left[p^{m-1}(k; x)\phi(x; j + 1 - k) + p^{m-1}(0; x)\phi(x; j)\right] \quad , \quad 0 \leq j \leq R \tag{19b}$$

Under the stability condition for the queueing system, given by

$$\lambda = \sum_{x \epsilon \Omega} \mu(x)\pi(x) < 1 \tag{20}$$

the $(\bar{\Omega}+1)$-dimensional Markov chain described in (19) is ergodic. By considering the limiting probabilities as $m \rightarrow \infty$ and their generating function, the following equations are derived.

$$P(z;y) = \sum_{k=0}^{R} \sum_{x \epsilon \Omega} z^{k-1} \left[P(z;x) + (z-1)p(0;x) \right] p(x,y)\phi(x;k) \tag{21}$$

$p(0;x)$ denotes the boundary probability that the system is empty and the state of the packet arrival process is x; its computation is discussed in Appendix A. $P(z;y)$ denotes the probability generating function of $p(j;y)$, $0 \leq j \leq \infty$. Note that

$$P(z) = \sum_{y \epsilon \Omega} P(z;y) \tag{22}$$

where $P(z)$ denotes the probability generating function of the buffer occupancy process. Since the k^{th} derivative of $P(z)$, evaluated at $z = 1$, is equal to the k^{th} factorial moment of the number of packets in the system, [21], it turns out that the k^{th} moment can be evaluated by differentiating (21) k times, setting $z = 1$ and solving the resulting system of equations. These equations are not linearly independent. An additional, linearly independent equation is derived by following a procedure similar to that shown in Appendix B (for the first moment) and invoking L' Hospital's rule. By differentiating (21) and setting $z = 1$ the following system of $\bar{\Omega}$ linear equations is obtained with respect to $P'(1;x)$, $x \epsilon \Omega$.

$$P'(1;y) = \sum_{x \epsilon \Omega} p(x,y) \left[P(1;x) + (\mu(x)-1)\pi(x) + p(0;x) \right] \quad , \quad y \epsilon \Omega \tag{23a}$$

The linearly independent equation (Appendix A) is given by

$$\sum_{x \epsilon \Omega} \left[2(\mu(x)-1)P'(1;x) + 2(\mu(x)-1)p(0;x) + (2+\sigma(x)-3\mu(x))\pi(x) \right] = 0 \tag{23b}$$

The mean buffer occupancy, $Q_1 = P(1)$, and the mean packet delay, D, (by using Little's theorem) are then obtained from

$$Q_1 = \bar{I}\bar{P}_1^T \quad , \quad D = \frac{Q}{\lambda} \tag{24}$$

where $\bar{I}$ is the $\bar{\Omega}$-dimensional identity row vector, $\bar{P}_1^T$ is the transpose of the row vector of $P'(1;x)$, $x \epsilon \Omega$, and λ is the mean packet arrival rate given by (20).

If $\{A_j\}_{j\epsilon J}$ is an independent and identically distributed process, it may be considered to be a MMGB process based on an underlying Markov chain with a single state, x_0. In this case $\Omega = \{x_0\}$, $\pi(x_0) = 1$, $p(x_0, x_0) = 1$, $\mu(x_0) = \lambda$ and $\sigma(x_0) = \sigma$. From (23), (24) it is easily obtained that

$$Q_1^{ind} = \lambda + \frac{\sigma - \lambda}{2(1-\lambda)} \quad and \quad D^{ind} = 1 + \frac{\sigma/\lambda - 1}{2(1-\lambda)} \tag{25}$$

Equations (23) and (24) provide for the mean packet delay induced by the queueing system under all source models considered in Section II. The dimensionality of this system of equations is equal to the cardinality of the underlying Markov chain $\{X_j\}_{j\epsilon J}$. Under single-time-constant sources and asymmetry (Section II.1), the underlying Markov chain $\{X_j\}_{j\epsilon J}$ has a state space of cardinality $2^N M$; its transition probabilities are given by (9) and the associated packet generating probabilities by (10). Under single-time-constant sources and symmetry (section II.2), the underlying Markov chain $\{X_j^{id}\}_{j\epsilon J}$ has a state space of cardinality $(N+1)\tau$; its transition probabilities are given by (12), (13) and the associated packet generating probabilities by (14). Notice that the number of equations which need to be solved increases exponentially for the asymmetric case and linearly for the symmetric case, with respect to the number of sources N. Under double-time-constant sources (section II.3), the underlying Markov chain $\{X_j^{\tau}\}$ has a state space of cardinality $\prod_{k=1}^{n}(\tau^k + 1)$; its transition probabilities are given by (17) and the associated packet generating probabilities by (18). When the double-time-constant sources are identical with time-constant τ, the number of linear equations which need to be solved can be significantly smaller than $K_{asym} = (\tau + 1)^N$. In Appendix C it is shown that the number of equations in this case is given by

$$K_{sym} = \frac{(N+\tau)!}{N\,!\tau\,!} \tag{26}$$

The reduced set of linear equations and a discussion on the numerical complexity may be found in Appendix C.

IV. <u>RESULTS AND CONCLUSIONS</u>

In this section, numerical results are presented for the mean packet delay induced by a statistical multiplexer under the packet arrival processes described in Section II.

In Fig. 5 some numerical results are presented, as a function of the source time-constant τ, under various packet traffic models. The packet sources are assumed to be identical with parameters

$$\{\pi^k(1) = .35,\ \gamma^k = .93,\ \tau,\ c^k = 0,\ \phi^k(1,1) = 1\}\ ,\ 1 \leq k \leq N$$

where the number of sources, N, is set to be equal to 2τ. The resulting cumulative packet traffic rate is equal to $N\pi^k(1)/\tau = .70$ packets/time units. Notice these packet sources have the characteristics of packetized voice sources delivering one packet every τ time units, when active.

Curve (1) in Fig. 5 presents the induced packet delay. Notice that, for the same total traffic rate, the induced packet delay decreases as the time-constant increases. This decrement in the delay is attributed to the reduction of the source correlation ($\gamma^k = .93$) as seen by the server (as τ increases), as well as to the increased independence in the cumulative traffic due to the increased number of (mutually independent) contributing sources ($N = 2\tau$). The cumulative arrival process $\{A_j^{id}\}_{j \in J}$ is determined as described in section II.2. In this case, $(2\tau + 1)\tau$ linear equations are contained in (23).

Curve (2) presents the induced packet delay when c^k, $1 \leq k \leq N$, are not identical for all sources but they are spread nearly uniformly over τ. When $\tau = 1$, no such spreading is possible and $c^k = 0, 1 \leq k \leq N$. In this case curves (1) and (2) coincide. Notice that the larger the value of τ, the larger the smoothing of the arrival process that can be accomplished through the spreading of c^k. The intensity of the queueing problems is decreased as τ increases, as a result of the distribution of the potential packet arrival instants associated with the various sources. The cumulative arrival process $\{A_j\}_{j \in J}$ is determined as described in section II.1. In this case, $2^{2\tau}\tau$ linear equations are contained in (23).

Curve (3) presents the mean packet delay results when the packets are delivered by each of the $N = 2\tau$ sources according to a Bernoulli process with rate $\pi^k(1)/\tau$ packets per time unit. The delay results, in this case, are obtained from (25). The resulting cumulative packet arrival process is a binomial with parameters 2τ and $\pi^k(1)/\tau$. The second moment of this process is given by

$$\sigma = N\frac{\pi^k(1)}{\tau}\left[1-\frac{\pi^k(1)}{\tau}\right]+\left[N\frac{\pi^k(1)}{\tau}\right]^2 = 2\pi^k(1)\left[1-\frac{\pi^k(1)}{\tau}\right]+\left[2\pi^k(1)\right]^2$$

Notice that σ increases as τ increases and thus the mean packet delay also increases, according to equation (25). This trend is clearly observed in curve (3) of Fig. 5. Notice that as τ increases the difference between curves (1) (or (2)) and curve (3) decreases, as expected.

Curve (4) presents the delay results under the traffic modeling considered for curve (1) but assuming that $\gamma^k = 0$, $1 \leq k \leq N$. That is, the sources are assumed to be uncorrelated. Notice that the resulting cumulative packet arrival process is different from the binomial process which was considered for the derivation of curve (3). For $\gamma^k = 0$, $1 \leq k \leq N$, and $\tau > 1$, the source can generate packets only at time instants in T^k (separated by τ time units) and not at any time instant in J, which is the case under the underlying Bernoulli model assumed for curve (3). Notice that the delay results have smaller values in this case compared to those under curve (3), possibly due to the positive effect of the constraint that packets can not be generated at time instants separated by less than τ time units under the model for curve (4). The cumulative arrival process $\{A_j^{id}\}_{j\in J}$ is determined as described in section II.2. In this case, τ linear equations are contained in (23), since the underlying Markov chain $\{X_j^{id}\}_{j\in J}$ becomes one dimensional and it is described by $\{M_j^{id}\}_{j\in J}$.

Finally, curve (5) presents the delay results under the model considered for curve (4), but under the assumption that the values of c^k, $1 \leq k \leq N$, are near-uniformly spread over τ. As it was explained before, this spreading of the potential packet arrival instants has a positive impact on the queueing intensity; this effect is clearly observed by comparing curves (4) and (5) of Fig. 5. The cumulative arrival process $\{A_j\}_{j\in J}$ is determined as described in section II.2. In this case, $M = LCM\{\tau, \tau, \cdots, \tau\} = \tau$ linear equations are contained in (23), since the underlying Markov chain $\{X_j\}_{j\in J}$ becomes one dimensional and it is described by $\{M_j\}_{j\in J}$.

Notice that when $\tau = 1$ the arrival processes may change state at any slot. In this case, the results may also be obtained from the closed form expression given by, [4],

$$\hat{D} = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda^n \ \lambda^m \Big(1 + \frac{\gamma^n}{1-\gamma^n} + \frac{\gamma^m}{1-\gamma^m}\Big)}{(1 - \sum_{n=1}^{N} \lambda^n) \ \sum_{n=1}^{N} \lambda^n} \right] \tag{26}$$

Fig. 6 presents the delay results obtained by multiplexing $N = 3$ packet sources with different time-constants given by $\tau^1 = 2$, $\tau^2 = 4$ and $\tau^3 = 4$. The rest of the parameters are assumed to be identical. The source burstiness coefficient is equal to $\gamma^{id} = .5$ and the packet generating probabilities are given by $\phi^{id}(1,1) = .3$, $\phi^{id}(1,2) = .4$ and $\phi^{id}(1,3) = .3$. The probability that the packet source is in the active state, $\pi^{id}(1)$, is selected so that the resulting cumulative packet traffic rate is equal to some value from the horizontal axis of Fig. 6. Curves (1), (2), (3), (4), and (5) show the mean packet delay under the traffic modeling considered for the derivation of the corresponding curves of Fig. 5. Curve (6) is derived under the assumption that each of the sources may be re-activated at any time instant and not at integer multiples of τ^k, $1 \leq k \leq 3$. Thus, the double-time-constant source model, discussed in Section II.3, has been assumed. $\prod_{k=1}^{N}(\tau^k + 1)$ linear equations are contained in (23), in this case. By expoiting the partial symmetry ($S^2 = S^3$) and properly formulating the equations in (23) (see Appendix C) a reduced number of linear equations will need to be solved. Curve (7) presents results under the source modeling considered for curve (6) under the assumption that $\gamma^k = 0$, $1 \leq k \leq 3$. Notice that states 0 and 1 may be merged in this case (Fig. 4), resulting in a reduced number of $\prod_{k=1}^{N} \tau^k$ linear equations contained in (23). Further reduction may be achieved by expoiting the partial symmetry in the arrival process, as explained before.

Notice that the coincidence of curves (1) and (2) and curves (4) and (5) is due to the specific parameters of the selected example. Finally, notice that curve (6) is always above curves (1) and (2) derived for the same values of γ^k, $1 \leq k \leq 3$. Similarly, curve (7) is always above curves (4) and (5) derived for $\gamma^k = 0$, $1 \leq k \leq 3$. This behavior may be explained in view of the fact that the minimum separation between two consecutive active periods in the traffic for the curves (1), (2), (4) and (5) is equal to τ^k, $1 \leq k \leq 3$, while that under the traffic models for the curves (6) and (7) is equal to one. The latter is expected to have negative effect on the intensity of the resulting queueing problems. Finally, the increased delay results presented under curve (3) may be explained in view of the fact that there is no minimum separation between consecutive packet arrivals, in any case, for the traffic model for curve (3).

Appendix A

The boundary probabilities $p(0;x)$ are obtained as follows. When there is at most one state x_0 in Ω such that $\phi(x_0;0) > 0$ (that is, x_0 is the only state which may not deliver a packet to the system), then it is easily established that

$$p(0;x) = q_0 p(x_0, x) \tag{A.1}$$

where $q_0 = 1 - \lambda$ is the probability that the system is empty. Notice that x_0 is the only state at which $\{X_j\}_{j\epsilon J}$ could have been in the previous slot, so that the system is currently empty. When the previous condition is not satisfied, the boundary probabilities may be obtained through the following procedure which may be found in [10].

$$p(0;x) = p(0;i) = q_0 \Pi_w(i) \quad for \quad x\epsilon\Omega \ or \ i\epsilon\{1,2,\cdots,\Omega\} \tag{A.2}$$

where $i,\ 1 \leq i \leq \bar{\Omega}$, denotes an ordering of the states in Ω; $\pi_w(i),\ 1 \leq i \leq \bar{\Omega}$, are computed from

$$\bar{\Pi}_w \bar{e} = 1 \quad , \quad \bar{\Pi}_w[P_w] = \bar{\Pi}_w \tag{A.3}$$

where $\bar{\Pi}_w = [\pi_w(1), \cdots, \pi_w(\bar{\Omega})]$ and the matrix $[P_w]$ is computed from

$$[P_w] = \sum_{k=0}^{R} [G_k][P][P_w]^k \tag{A.4}$$

where $[P]$ is the transition probability matrix of the Markov chain $\{X_j\}_{j\epsilon J}$ and $[G_k]$ is an $\bar{\Omega}x\bar{\Omega}$ diagonal matrix, $[G_k] = diag[\phi(1;k), \phi(2;k), \cdots, \phi(\bar{\Omega};k)]$, $0 \leq k \leq R$; R is the maximum number of packet arrivals over a slot ; $\{\phi(i;k)\}_{i=1}^{\bar{\Omega}} \equiv \{\phi(x;k)\}_{x\epsilon\Omega}$, $0 \leq k \leq R$.

Appendix B

The linearly independent equation shown at (23b) is derived by differentiating (21), adding up the resulting equations and using the derivative of (22).

$$P'(z) = \sum_{k=0}^{R} \sum_{x \epsilon \Omega} \phi(x;k) \left[(k-1) z^{k-2} \left[P(z;x) + (z-1) p(0,x) \right] + z^{k-1} \left[P'(z;x) + p(0;x) \right] \right]$$

By adding

$$ZP'(z) - z \sum_{k=0}^{R} \sum_{x \epsilon \Omega} P'(z;x) \phi(x;k) = 0$$

to the right-hand side of the previous equation, the following expression is obtained

$$P'(z) = \frac{A(z)}{1-z} \tag{B.1}$$

where

$$A(z) = \sum_{k=0}^{R} \sum_{x \epsilon \Omega} \phi(x;k) \left[(k-1) z^{k-2} \left[P(z;x) + (z-1) p(0;x) \right] + z^{k-1} \left[P'(z;x) + p(0;x) \right] - zP'(z;x) \right]$$

Since $P'(1)$ is equal to the mean buffer occupancy, which is finite if (20) holds, and since $1 - z = 0$ for $z = 1$, $P'(1)$ is computed from (B.1) by using L' Hospital's rule. Thus,

$$P'(z) \mid_{z=1} = \frac{dA(z)/d_z}{d(1-z)/d_z} \mid_{z=1} = -dA(z)/d_z \mid_{z=1} \tag{B.2}$$

From (B.2) and the derivative of (22) evaluated at $z = 1$, equation (23b) is easily derived.

Appendix C

Let $\tau + 1$ be the cardinality of the state space of the (assumed identical) underlying Markov chains $\{Y_j^k\}_{j \epsilon J}$ (see section II.3). Let $\bar{x} = (x_1, x_2, \cdots, x_{\tau+1}) \epsilon \left[\Omega_r^k \right]^k$ denote a state of the underlying N dimensional Markov chain $\{X_j^r\}_{j \epsilon J}$. Since the packet arrival processes $\{B_j^k\}_{j \epsilon J}$ are indentical, the resulting cumulative arrival process $\{A_j^r\}_{j \epsilon J}$ will assume (probabilistically) the same values under states $\bar{x} \in \Omega_r$ which correspond to the same number of sources in each state; for instance states $(i_1, i_2, \cdots, i_{\tau+1})$ and $(i_2, i_1, \cdots, i_{\tau+1})$.

If $\bar{v}(\bar{x}) = (v_1(\bar{x}), v_2(\bar{x}), \cdots v_{\tau+1}(\bar{x}))$ is a $(\tau + 1)$-dimensional vector with $v_i(\bar{x})$, $i = 1, 2, \cdots \tau + 1$, denoting the number of input Markov chains at state i, then each such vector $\bar{v}(x)$ with the constraint $\sum_{i=1}^{\tau+1} v_i(\bar{x}) = N$, represents a class of equivalent states $\bar{x}$. The number of equivalent states $\bar{x}$ in a class

$\bar{v}(\bar{x})$ is given by the multinomial coefficient, [22],

$$c(\bar{x}) = \binom{N}{v_1(\bar{x}), v_2(\bar{x}), \cdots v_{\tau+1}(\bar{x})} = \frac{N!}{v_1(\bar{x})!v_1(\bar{x})! \cdots v_{\tau+1}(\bar{x})!} \tag{C.1}$$

where $v_i(\bar{x})$ is the number of input processes in state i, as determined by $\bar{x}$, $i = 1, 2, \cdots, \tau + 1$. This is basically the number of ways in which a population of N elements can be divided into $\tau + 1$ ordered parts of which the first contains $v_1(\bar{x})$ elements, the second contains $v_2(\bar{x})$ elements and so forth. Let F be the set of representative states $\bar{x}$ of the symmetric system (i.e. no two states $\bar{x}\epsilon F$ belong to the same class of equivalent states). Let $v(\bar{x}_0)$ be the class of the equivalent to $\bar{x}_0$ states. For each $\bar{x}_0, \bar{y}_0 \epsilon F$, equations (23a) and (23b) can be written as follows.

$$P'(1; \bar{y}_0) = \sum_{\bar{x}_0 \epsilon F} \left\{ \sum_{\bar{x}\epsilon v(\bar{x}_0)} p(\bar{x}, \bar{y}_0) \right\} P'(1; \bar{x}) + \sum_{\bar{x}\epsilon\Omega^r} p(\bar{x}, \bar{y}_0) \left[(\mu(\bar{x}) - 1)\pi(\bar{x}) + p(0; \bar{x}) \right], \ \bar{y}_0 \epsilon F \tag{C.2a}$$

$$\sum_{x_0 \epsilon F} c(\bar{x}_0) 2(\mu(\bar{x}_0) - 1) P'(1; \bar{x}_0) + \sum_{x\epsilon\Omega^r} \left\{ 2(\mu(\bar{x}) - 1)p(0; \bar{x}) + \left[2 + \sigma - 3\mu(\bar{x}) \right] \pi(\bar{x}) \right\} = 0 \tag{C.2b}$$

where $c(x_0)$ is given by (C.1). The mean buffer occupancy can be obtained from

$$Q_1 = P'(1) = \sum_{x_0 \epsilon F} P'(1; \bar{x}_0) c(\bar{x}_0) \tag{C.3}$$

The induced average packet delay is then obtained from (24). The reduced number of equations in (C.2) are then easily seen to be given by (26), which expresses the number of ways of partitioning N things into $\tau + 1$ groups, [22]. Notice that significant reduction of the number of required equations may be achieved under symmetry. Comparison of k_{sym} (see (26)) and $k_{asym} = (\tau + 1)^N$ for some pairs (N, τ) which result in less then 1,000 linear equations in (C.2), are shown in Table I.

References

(1) H. Heffes, D. Lucantoni, "A Markov Modulated Characterization of Packetized Voice and Data Traffic and Related Statistical Multiplexer Analysis", IEEE J. Select. Areas Commun., (4), 6, Sept. 1986.

(2) D. Towsley, "The Analysis of a Statistical Multiplexer with Nonindependent Arrivals and Errors",

IEEE Trans. Commun., (28), 1, Jan. 1980.

(3) H. Bruneel, "Queueing Behavior of Statistical Multiplexers with Correlated Inputs", IEEE Trans. Commun., (36), 12, Dec. 1988.

(4) A. Viterbi, "Approximate Analysis of Time Synchronous Packet Networks", IEEE J. Select. Areas Commun., (4), 6, Sept. 1986.

(5) M. Reiser, "Performance Evaluation of Data Communication Systems", Proc. IEEE, (70), Feb. 1982.

(6) M. S. Ali Khan, "Infinite Dams with Inputs Forming a Markov Chain", J. Appl. Probab., (5), pp. 72-84, 1968.

(7) M. S. Ali Khan, "Finite Dams with Inputs Forming a Markov Chain", J. Appl. Probab., (7), pp. 291-303, 1970.

(8) J. Daigle, Y. Lee, N. Magalhaes, "Discrete Time Queues with Phase Dependent Arrivals", IEEE INFOCOM'90 Conf., June 5-7, 1990, San Francisco.

(9) I. Stavrakakis, "Analysis of a Statistical Multiplexer under a General Input Traffic Model", IEEE INFOCOM'90 Conf., June 5-7, 1990, San Francisco.

(10) I. Stavrakakis, "Efficient Modeling of Merging and Splitting Processes in Large Networking Structures", IEEE J. Select. Areas Commun., (9), 8, Oct. 1991.

(11) J. Daigle, "Queueing Theory for Telecommunications", Addison Wesley, 1992.

(12) D. Anick, D. Mitra, M. Sondhi, "Stochastic Theory of Data-Handling System with Multiple Sources", Bell Syst. Tech. J., (61), 8, Oct. 1982.

(13) M. Rubinovitch, "The Output of a Buffered Data Communication System", Stoch. Proces. and Their Applic., (1), pp. 375-382, 1973.

(14) J. Cohen, "Superimposed Renewal Processes and Storage with Gradual Input", Stoch. Proces. and Their Applic., (2), pp. 31-58, 1974.

(15) H. Kaspi, M. Rubinovitch, "The Stochastic Behavior of a Buffer with Non-identical Input Lines", Stoch. Proces. and Their Applic., (23), pp. 73-88, 1975.

(16) L. Kosten, "Stochastic Theory of a Multi-Entry Buffer (I)", Delft Progress Rep., Math. Engin., Math. and Inf. Engin., (1), pp. 10-18, 1974.

(17) L. Kosten, "Stochastic Theory of a Multi-Entry Buffer (II)", Delft Progress Rep., Math. Engin., Math. and Inf. Engin., (1), pp. 44-50, 1974.

(18) T. Stern, "Analysis of Separable Markov-Modulated Queueing Processes", Tech. Rep. of CTR, No. CU/CTR/TR/-113-88-41, Columbia University, 1988.

(19) B. Gopinath, J. Morrison, "Discrete-Time Single Server Queues with Correlated Inputs", Bell Syst. Tech. J., (56), 9, Nov. 1977.

(20) A. Fraser, B. Gopinath, J. Morrison, "Buffering of Slow terminals", Bell Syst. Tech. J., (57), 8, Oct. 1978.

(21) D. Heyman, M. Sobel, "Stochastic Models in Operations Research", McGraw Hill, 1982.

(22) H. Stark, J. Woods, "Probability, Random Processes and Estimation Theory for Engineers", pp. 20, Prentice Hall, 1986.

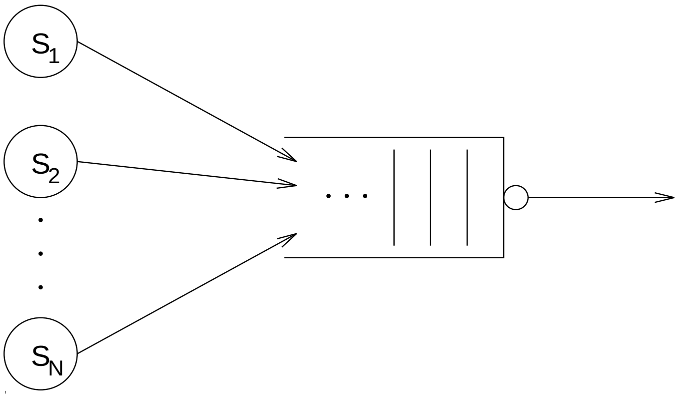

Fig. 1: The statistical multiplexer.

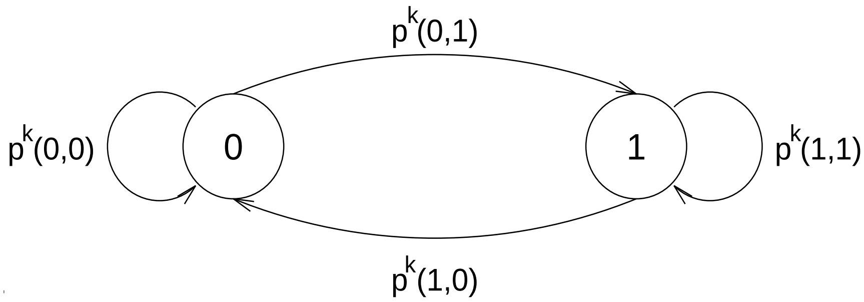

Fig. 2: The first-order Markov model for the packet source.

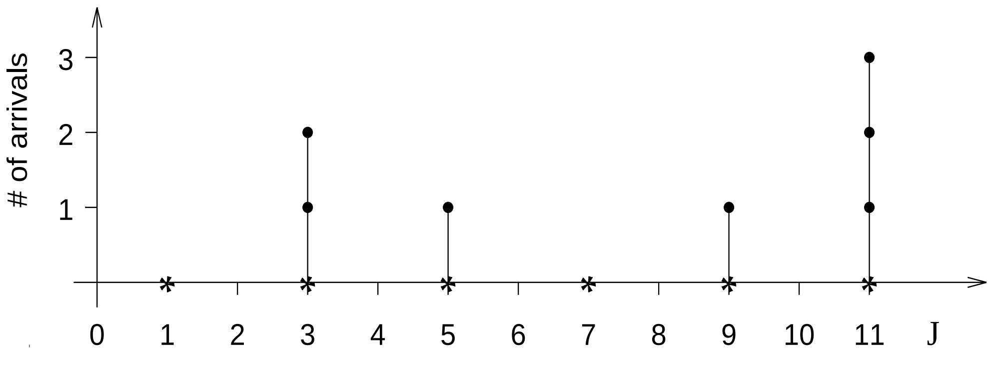

Fig. 3: A realization of the arrival process $\{A_j^k\}_{\{j\in J\}}$ for $\tau^k = 2,\ c^k = 1$; * denotes potential arrival points.

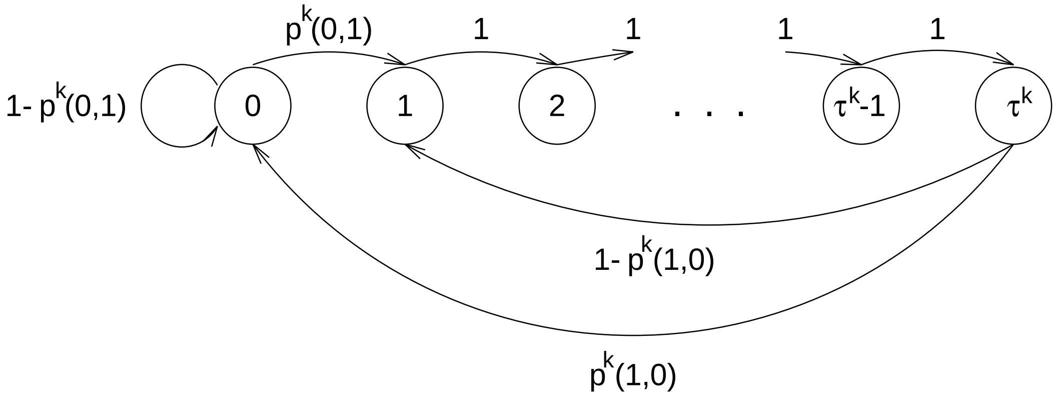

Fig. 4: The Markov model for $\{Y_j^k\}_{\{j \in J\}}$.

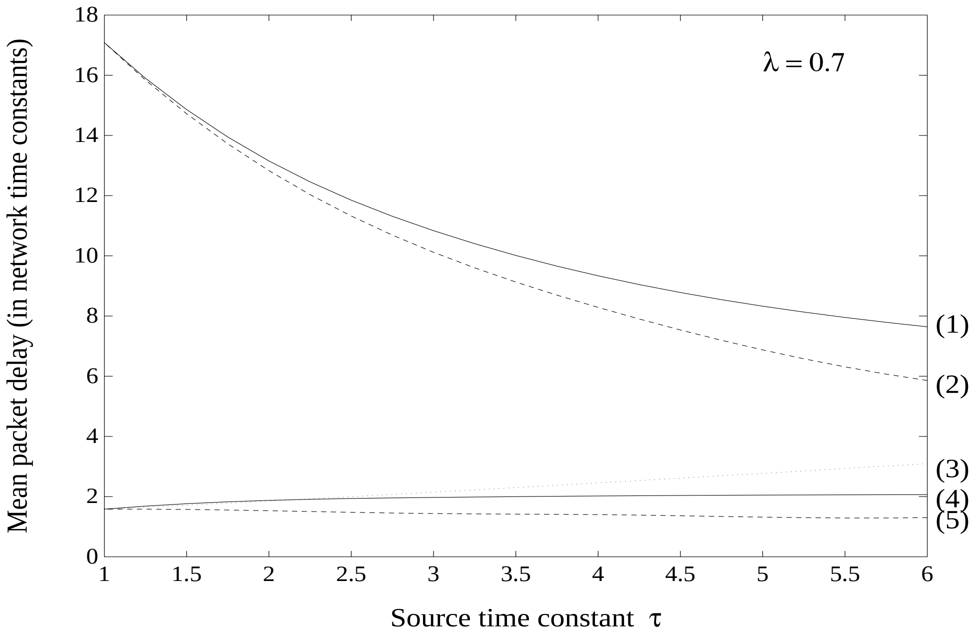

Fig. 5: Mean delay results for the packetized voice sources.

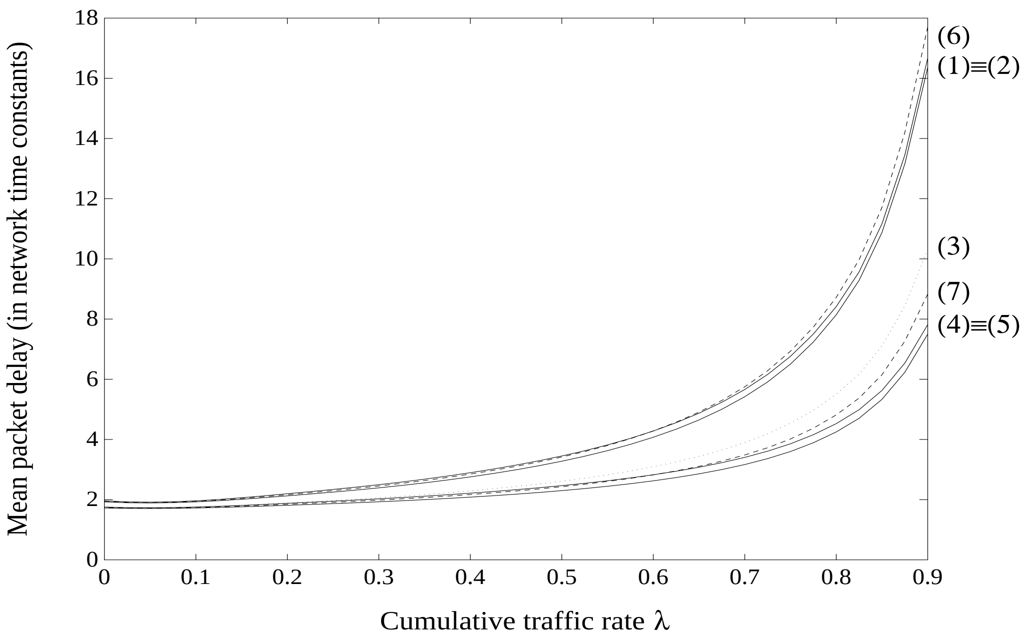

Fig. 6-(a): Mean delay results for the N=3 packet sources under various traffic models; $0 \leq \lambda \leq .9$.

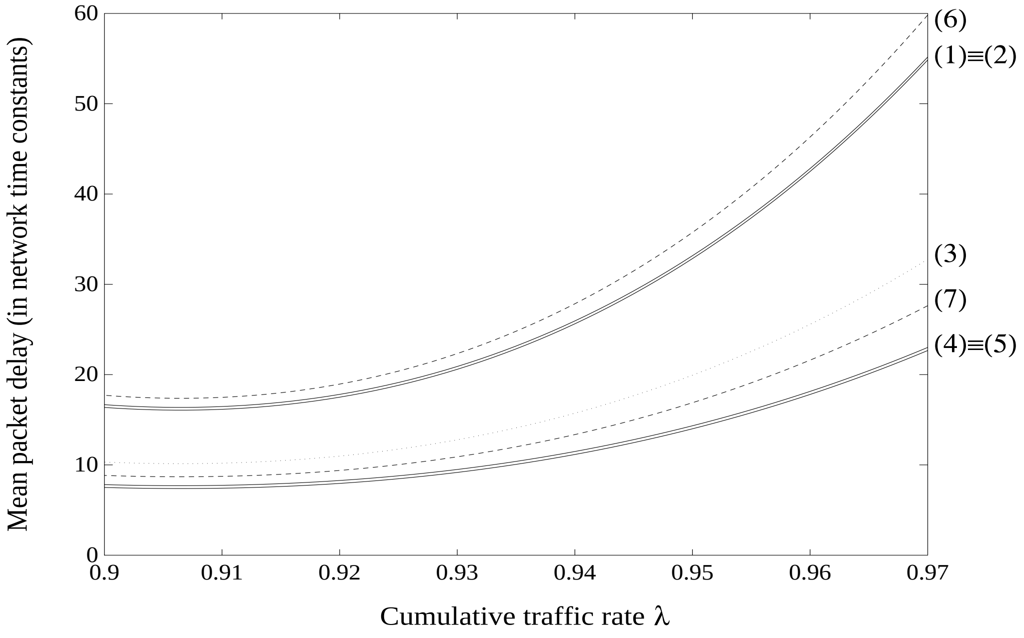

Fig. 6-(b): Mean delay results for the N=3 packet sources under various traffic models; $.9 \leq \lambda \leq .97$.

N	$\tau+1$	K_{sym}	K_{asym}
2	31	496	961
	44	990	$1,936$
3	10	220	$1,000$
	17	816	$4,913$
5	4	56	$1,024$
	8	792	$32,768$
10	2	11	$1,024$
	3	66	$59,049$
	4	286	$1,048,576$
	5	$1,001$	$9,765,625$
15	2	16	$32,768$
	3	136	$*$
	4	816	$*$
20	2	21	$1,048,576$
	3	213	$*$
30	2	31	$*$
	3	496	$*$
43	2	44	$*$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
N	2	$N+1$	2^N

Tab. I: Numerical complexity for the symmetric (K_{sym}) and the asymmetric (K_{asym}) cases as a function of N and $\tau+1$(* indicates a number greater than 9,999,999).