Controlling Smoothness and Loss Rate for Elastic Continuous Media Flows

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The class of rate adaptation schemes based on linear *Loss Dependent Decrease* (LDD) policies is considered. A particular LDD policy, referred to as the *history independent* (hi)-LDD policy, is identified and studied here and some interesting properties are shown. Based on these properties it is possible to estimate the fair share and use it to adjust dynamically the control parameters so that a given targeted level of smoothness and loss rate be achieved; this policy is referred to as the *Dynamic hi-LDD* policy. Rate adaptation schemes based on the introduced Dynamic hi-LDD policy are suitable for elastic Continuous Media (CM) flows since they provide for a smooth rate adaptation and low packet loss rates. Numerical results illustrate the good properties and intrinsic advantages of the investigated schemes.

1. Introduction

Internet's robustness and stability is due to the congestion control and avoidance algorithm [1,2] implemented in its mainly employed protocol, Transport Control Protocol (TCP), which belongs in the class of the Additive Increase / Multiplicative Decrease (AI/MD) algorithms [3]. TCP has been designed and successfully used for unicast reliable data transfer but is unsuitable for the continuous media (CM) streaming applications [16], since its window-based congestion control scheme halves the transmission rate, when losses occur, affecting TCP flow's smoothness, and its retransmission mechanism introduces typically significant end-to-end delays and delay variations. The CM streaming applications require smooth flows' rate adaptation, low end-to-end delays and delays variation. Although, in contrast to data transferring services, they may tolerate packet losses, the induced losses should be controlled and kept low since they impact on the perceptual quality of the CM stream: the requirement for the timely delivery either prohibits the recovery of the lost packets through a retransmission scheme or the recovery mechanism fails to reach the decoding deadlines [16,17]. The User Datagram Protocol (UDP) is mainly used by the CM streaming services as the base transport protocol, in conjunction with the Real Time Protocol (RTP) and Real Time Control Protocol (RTCP) [4]. Applications that use the UDP transport protocol should also implement end-to-end congestion control to retain the stability of the Internet otherwise all supported applications will suffer (UDP flows will get most of the bandwidth) and eventually the network will collapse [5].

In the current best effort Internet individual flows of different requirements, e.g., data flows (TCP) and CM (RTP/UDP) flows compete with each other, whereas in the forthcoming network environment based of the DiffServ architecture [14] flows of common requirements, e.g., CM flows, will probably be isolated from flows with other requirements, e.g., data flows. Even in the context of a DiffServ environment the CM flows should be elastic (rate adaptive) since the number of CM flows may be increased and the total load may exceed the available network capacity [8].

Examples of RTP/RTCP based rate control algorithms are [6]-[11], of which the schemes presented in [9]-[11] are TCP-friendly. Rate control protocols, such as RAP [12] and TFRCP [13], that are TCP-friendly and do not rely on RTP/RTCP/UDP, have been proposed in the literature as well. TCP-friendly congestion control schemes try to prevent CM streaming applications from getting more bandwidth than that of a TCP flow. In the RTCP-based mechanisms the feedback frequency is about 1 every 5 sec ($\pm$ 1.5 randomly determined), whereas in the ACK-based it is equal to one every round trip time (plus the delay variation). Most congestion control schemes exploit the feedback only in the decision function, as is the case with the binary feedback of the basic AI/MD scheme. Some congestion control schemes (ACK- and RTCP-based) try to estimate the "effective transmission rate" of a flow based on the packet loss rate.

The latter may be derived explicitly by the feedback, e.g., for RTCP Receiver Reports in the case of RTCP-based congestion control schemes or implicitly by combining the feedback and local information available at the source in the case of ACK-based congestion control schemes. Examples of such congestion control schemes are presented in [9,15]. Such decrease policies could be referred to as belonging to the class of *Loss Dependent Decrease (LDD) policies*. It has been observed that schemes based on LDD policies present a smoother adaptation behavior and, thus, they may be more appropriate for applications sensitive to a large rate variation, such as the CM applications. This observation has motivated the work presented here.

In this paper, a particular policy of the LDD class, referred to as history-independent LDD (hi-LDD), is identified - the feedback is considered to be *non binary* and its frequency has a larger time scale than the round-trip-time (RTT) - and some interesting properties are shown: (a) the total load after a decrease step is independent from the prior values of the total load; (b) the policy immediately responds to congestion and the total load falls below the efficiency line in a single rate decrease; (c) the flows adapt their rates in a regular and periodic manner. Based on the aforementioned properties it is possible to estimate the fair share and use it to adjust dynamically the multiplicative control parameters so that a given targeted level of smoothness and loss can be achieved. The latter scheme is referred to as the *Dynamic hi-LDD* policy.

This paper is organized as follows. In Section 2 the conditions under which linear control functions converge to fairness are shown. In Section 3, the rate adaptation behavior of the hi-LDD policy is analyzed. In Section 4, the Dynamic hi-LDD policy which provides for controlling smoothness and packet loss rate is introduced and studied. The behavior of the presented schemes is illustrated through a set of simulation results in Section 5. Finally, the conclusions are presented in the last section.

2. Linear Rate Control Functions and Convergence Issues

A discrete time network model that consists of *n* users (flows) is considered ; Let $\{\ldots, t-1, t, t+1, \ldots\}$ denote the discrete time instants which are assumed to coincide with the times at which all users receive feedback from their peer entities (synchronized feedback assumption). Let $f_i(t)$ denote the feedback received by user i at time instant t; $f_i(t)$ is *non binary* and specifies the packet loss rate that the corresponding flow i experienced over the preceding time interval $(t-1, t)$. Under the Assumptions shown below either all flows will experience packet losses or none, and consequently, all flows will react in the same manner: they will all either increase (under zero losses) or decrease (under non-zero losses) their rate.

Assumptions 1.

- *(i) The flows are initiated (terminated) at discrete time instants with possibly different initial rates, and the round trip times and the delays required by the flows to adjust their rate are zero.*
- *(ii) The packet losses are distributed to all flows in proportion to their rate.*

The above assumptions imply that the feedbacks $f_i(t)$ are synchronously received and the rate adaptations are synchronized.

Let $x_i(t)$ denote the transmission rate of user i over the interval $(t-1, t)$. Under a linear rate control function the next rate $(x_i(t+1))$ is determined by the current rate $(x_i(t))$ and the reported packet losses $f_i(t)$, as follows

$$x_i(t+1) = \begin{cases} x_i(t)b_{\mathrm{I}} + a_{\mathrm{I}} & \text{if } f_i(t) = 0\,; \\ x_i(t)b_{\mathrm{D}} + a_{\mathrm{D}} & \text{if } f_i(t) > 0\,; \end{cases} \quad (1)$$

where a_{I}, a_{D} are responsible for the additive increase and decrease steps, respectively, and *are independent of* $x_i(t)$, and b_{I}, b_{D} are responsible for the multiplicative increase and decrease steps, respectively. These parameters are selected in such a way that they implement an increase (if $f_i(t) = 0$) or a decrease (if $f_i(t) > 0$) step. Typically, $b_{\mathrm{I}} = 1$ (pure additive increase policy) and $0 < b_{\mathrm{D}} < 1$. In this paper, the most general case of linear control functions considered allows for b_{D} to be equal to 1 (pure additive decrease, in which case a_{D} has to have a negative value for the function to implement a decrease step) and b_{I} to be less than 1 (in which case a_{I} will need to be properly selected for the function to implement an increase step); an example of a policy with $b_{\mathrm{I}} < 1$ is given in section 4.2. When $b_\mathrm{I} < 1$ the convergence speed to fairness is enhanced, as it is shown in Proposition 1. In this paper it is assumed that b_I, $b_\mathrm{D} \in (0,1]$.

Let $\vec{x}(t) = \{x_1(t), x_2(t), \ldots, x_n(t)\}$ denote the rate vector at time instant t associated with n supported users (flows) and let $X(t)$ denote the total network load associated with $\vec{x}(t)$. That is, $X(t) = \sum_{i=1}^{n} x_i(t)$.

Let X_eff denote the targeted maximum total load which is typically the network capacity. Let $L(t)$ and $l_i(t)$ denote the number of packet losses for the network and user (flow) i, respectively, associated with the time instant t (that is, occurred over $(t-1,t)$). Since the length of the interval $(t-1,t)$ is equal to 1, $L(t) = |X(t) - X_\mathrm{eff}|^+$, where $|w|^+ = w$ if $w > 0$ and 0 otherwise. Let $f(t)$ and $f_i(t)$ denote the packet loss rate for the network and user (flow) i, respectively, associated with time instant t. That is

$$f(t) = \frac{L(t)}{X(t)} = \frac{|X(t) - X_\mathrm{eff}|^+}{X(t)} \text{ and } f_i(t) = \frac{l_i(t)}{x_i(t)},$$

Since $L(t) = f(t)X(t)$ and $L(t) = \sum_{i=1}^{n} l_i(t) = \sum_{i=1}^{n} f_i(t)x_i(t)$, we have the following equation

$$f(t)X(t) = \sum_{i=1}^{n} f_i(t)x_i(t). \tag{2}$$

In the sequel, a necessary and sufficient condition under which a linear control scheme as in (1) with b_I, $b_\mathrm{D} \in (0,1]$ converges to the fair share is presented. This condition holds for the schemes presented here and therefore is useful for showing that the schemes convergence to fairness.

The index $DF_{i,j}(t)$ introduced in [18], is exploited to investigate the conditions under which the flows converge to the fair share. This index represents the distance of point $(x_i(t), x_j(t))$ from the fairness line (see Fig. 1 and discussion below) corresponding to equal loads and expresses the *unfairness*, since when the distance is reduced the unfairness is reduced as well. The index $DF_{i,j}(t)$ captures the evolution of unfairness, in a manner closer to the physical meaning of fairness than the fairness index $F(\vec{x}(t)) = \frac{\left(\sum_{i=1}^{n} x_i(t)\right)^2}{n\sum_{i=1}^{n} x_i(t)^2}$, and allows for simpler derivations than the index $F(\vec{x}(t))$. In addition, allows for a simple comparison of the convergence speeds to fairness of different schemes, which is not that simple by exploiting the index $F(\vec{x}(t))$. Nevertheless, the index $F(\vec{x}(t))$ is also used in the rest of the paper for depicting the convergence speed to fairness in the figures, as this index is very common in the research literature.

Fig. 1 depics the index $DF_{i,j}(t)$ associated with rates $x_i(t)$ and $x_j(t)$ (point B) at time t, assuming that $x_j(t) > x_i(t)$; the fairness line, corresponding to equal rates is also shown. The evolution of this index captures the fairness improvement after an adaptation step. From Fig. 1 it is easily derived that the Euclidean distance BC of the point $(x_i(t), x_j(t))$ from the fairness line is equal to $\frac{BD}{2}$. Since $BD^2 = BE^2 + ED^2 = 2|x_j(t) - x_i(t)|^2$ we find that $BC = DF_{i,j}(t) = \frac{|x_j(t)-x_i(t)|}{\sqrt{2}} \geq 0$.

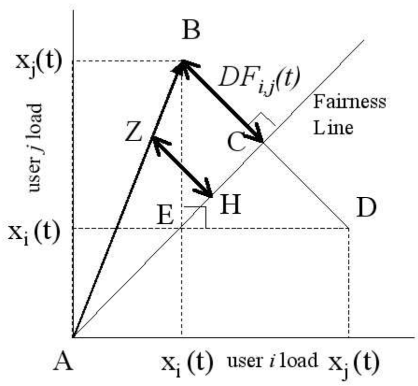

Figure 1. The distance $DF_{i,j}(t)$ of point $(x_i(t), x_j(t))$ from the fairness line.

Proposition 1. *A linear congestion control scheme as in (1) with* b_I, $b_\mathrm{D} \in (0,1]$ *is fair, if and only if* $b_\mathrm{I} \times b_\mathrm{D} < 1$ *(at least one of* b_I, b_D *be less than 1); the lower the value of* b_I *and* b_D *the faster the convergence speed to fairness.*

Proof. Consider any pair of rates $(x_i(t), x_j(t))$ from the rate vector $\vec{x}(t)$ and assume, without loss of generality, that $x_j(t) > x_i(t)$. The index $DF_{i,j}(t)$ at time instant t and the index $DF_{i,j}^{+}(t+1)$ at the next time instant $t+1$ after a load increase action (step), are given by

$$
\begin{aligned}
&DF_{i,j}(t) = \frac{|x_j(t) - x_i(t)|}{\sqrt{2}}\ ,\\
&DF^{+}_{i,j}(t+1) = \frac{|x_j(t+1) - x_i(t+1)|}{\sqrt{2}} = \\
&= \frac{|x_j(t)b_{\mathrm{I}} + a_{\mathrm{I}} - x_i(t)b_{\mathrm{I}} - a_{\mathrm{I}}|}{\sqrt{2}} = \\
&= \frac{b_{\mathrm{I}}|x_j(t) - x_i(t)|}{\sqrt{2}} = b_{\mathrm{I}} DF_{i,j}(t) \qquad (3)
\end{aligned}
$$

Similarly, the index $DF^{-}_{i,j}(t+1)$ at time instant $t+1$, following rate decrease action, is given by

$$DF^{-}_{i,j}(t+1) \quad = \quad b_{\mathrm{D}} DF_{i,j}(t) \qquad (4)$$

Let $k_{\mathrm{I}}(t)$ and $k_{\mathrm{D}}(t)$ denote the number of increases and decreases over the time interval (t_0, t). Then,

$$DF_{i,j}(t_0 + k_{\mathrm{I}}(t) + k_{\mathrm{D}}(t)) = b_{\mathrm{I}}^{k_{\mathrm{I}}(t)} b_{\mathrm{D}}^{k_{\mathrm{D}}(t)} DF_{i,j}(t_0) \quad (5)$$

From the above equation it is easily concluded that $DF_{i,j}(t) \to 0$ as $t \to \infty$, if and only if $b_{\mathrm{I}} \times b_{\mathrm{D}} < 1$ and that the convergence speed to fairness depends on the values of the parameters b_{I} and b_{D}, independently from the values of a_{I} and a_{D}. $\square$

Table 1
Notation

Symbol	Notation
n	Number of flows
X_{eff}	Efficiency line, typical the capacity
t_j	j^{th} overload time instant
$X(t)$	Total load
$X(t_j)$	Total load at j^{th} overload time instant
$X(t_j+1)$	Total load after a decrease step
b_{D}, a_{D}	the decrease parameters
b_{I}, a_{I}	the increase parameters
$f_i(t_j)$	packet loss rate at time instant t_j
k_{T}	number of induced increase steps
x^{n}_{fair}	fair share
f_{c}	fixed packet loss rate

3. The history independent Loss Dependent Decrease (hi-LDD) Policy

In this section the effect of incorporating the *non-binary* packet loss rate $f_i(t)$ in the decrease policy of a linear congestion control scheme is investigated. Suppose that the parameter b_{D} in (1) is a linear function of $f_i(t)$ of the following form:

$$b_{\mathrm{D}}(f_i(t)) = d + g f_i(t) = d(1 + \frac{g}{d} f_i(t)) \qquad (6)$$

where g, d are constants. Such decrease policies may be referred to as *linear Loss Dependent Decrease (LDD)* policies.

To investigate the rate adaptation behavior of the LDD decrease policies, let t_j denote the j^{th} overload time instant, that is the j^{th} time in which the total load $X(t)$ exceeds X_{eff}. Since $X(t_j) > X_{\mathrm{eff}}$ and $f(t_j) > 0$, the total load will decrease at time instant $t_j + 1$. The new total rate at $t_j + 1$ is given by (recall that $f(t)X(t) = \sum_{i=1}^{n} f_i(t) x_i(t)$, see equation (2)):

$$
\begin{aligned}
&X(t_j+1) = \sum_{i=1}^{n} x_i(t_j+1) = \\
&= \sum_{i=1}^{n} (d(1 + \frac{g}{d} f_i(t_j)) x_i(t_j) + a_{\mathrm{D}}) = \\
&= d(1 + \frac{g}{d}\frac{X(t_j) - X_{\mathrm{eff}}}{X(t_j)}) X(t_j) + n a_{\mathrm{D}} = \\
&= d(X(t_j)(1 + \frac{g}{d}) - \frac{g}{d} X_{\mathrm{eff}}) + n a_{\mathrm{D}} \qquad (7)
\end{aligned}
$$

From (7) it is clear that $X(t_j+1)$ depends on $X(t_j)$ for general constants d and g. The following proposition establishes the necessary and sufficient conditions under which $X(t_j+1)$ becomes independent from $X(t_j)$; its proof is straightforward in view of (6) and (7).

Proposition 2. *After an application of a LDD policy, the new total load is independent from its previous value, if and only if* $g = -d$, *that is,* $b_{\mathrm{D}}(f_i(t)) = d(1 - f_i(t))$. *The new total load is given by*

$$X(t_j+1) = d X_{\mathrm{eff}} + n a_{\mathrm{D}} \qquad (8)$$

The independence of $X(t_j+1)$ from $X(t_j)$ is a very interesting property and its consequences are investigated in the sequel. Such decrease policies may be referred to as *history independent Loss Dependent Decrease Policies (hi-LDD).*

Notice from (8) that under fixed capacity $X_{\rm eff}$ and parameters d, $a_{\rm D}$, the total load $X(t_j+1)$ depends only on the number of flows n; in the case of pure multiplicative decrease ($a_{\rm D}=0$), the total load becomes independent of n.

The following Corollary - whose proof is obvious in view of (8) - provides the conditions under which the hi-LDD policy ensures an immediate fall - in a single per flow decrease step - of the total load to a fixed level below the efficiency line $X_{\rm eff}$. This results in a fast response to congestion and, thus, to fewer packet losses.

Corollary 1. *$X(t_j+1)$ falls below the efficiency line $X_{\rm eff}$ in a single step under the hi-LDD policy, if and only if the following inequality holds true*

$$na_{\rm D} < (1-d)X_{\rm eff} \quad \textit{or} \quad a_{\rm D} < (1-d)\frac{X_{\rm eff}}{n} \tag{9}$$

The typical case of $a_{\rm D} \le 0$ and $d<1$, fulfils the above inequality.

Notice that in the general case knowledge of the fair share $\frac{X_{\rm eff}}{n}$ is required to determine whether (9) holds. In the case of pure multiplicative decrease ($a_{\rm D}=0$, $d<1$) or multiplicative decrease with negative additive term ($a_{\rm D}<0$, $d<1$) (9) always hold, independently of the value of $\frac{X_{\rm eff}}{n}$. In the remaining of the paper hi-LDD policies with $a_{\rm D}\le 0$ and $d<1$ will be considered.

After a rate decrease, the total load $X(t)$ will start increasing from the fixed level determined by (8) until it exceeds the efficiency line after a number of rate increase steps. Then, it will fall again to the fixed level below the efficiency line, in a single decrease, e.t.c. It is obvious that the overall rate adaptation behavior is *regular and periodic*, which in turn leads to *fixed packet loss rates* and therefore to predictable ones. The following Proposition describes more precisely the aforementioned periodicity of the total load.

Proposition 3. *The hi-LDD policy presents periodic adaptation behavior, as long as its control parameters ($a_{\rm I}$, $a_{\rm D}$, $b_{\rm I}$ and d) and the number of flows n, remain fixed. After the first overload time instant t_1, the total load $X(t)=X(t_j)$, $j>1$ exceeds the efficiency line $X_{\rm eff}$ every $k_{\rm T}+1$ time instants and the total load $X(t_{j+1})$ at the $(j+1)^{th}$ overload time instant is given by*

$$X(t_{j+1}) = X(t_j+k_{\rm T}+1) =$$

$$= \begin{cases} dX_{\rm eff}+na_{\rm D}+k_{\rm T}na_{\rm I} & \textit{if } b_{\rm I}=1 \\ b_{\rm I}^{k_{\rm T}}(dX_{\rm eff}+na_{\rm D})+na_{\rm I}\frac{1-b_{\rm I}^{k_{\rm T}}}{1-b_{\rm I}} & \textit{if } b_{\rm I}<1 \end{cases} \tag{10}$$

where $k_{\rm T}$ is given by

$$k_{\rm T} = \begin{cases} \lceil \frac{(1-d)X_{\rm eff}-na_{\rm D}}{na_{\rm I}} \rceil & \textit{if } b_{\rm I}=1 \\ \lceil \log_{b_{\rm I}} \frac{X_{\rm eff}-\frac{na_{\rm I}}{1-b_{\rm I}}}{dX_{\rm eff}+n(a_{\rm D}-\frac{a_{\rm I}}{1-b_{\rm I}})} \rceil & \textit{if } b_{\rm I}<1 \end{cases} \tag{11}$$

$X(t_j)$ and $X(t_j+1)$ are periodic functions with period $k_{\rm T}+1$; $k_{\rm T}$ and $X(t_j)$ vary with n.

Proof. After the first rate decrease step (see Fig. 2), the total load $X(t)$ will start increasing from an initial load $X(t_1+1)=dX_{\rm eff}+na_{\rm D}$ until it exceeds the efficiency line after a number of rate increase steps; let k_1 denote the required number of increase steps. The total loads $X(t_1+2), X(t_1+3), \ldots X(t_1+i+1)$, $\ldots$, $X(t_1+k_1+1)$, after the first, second, $\ldots$, i^{th}, $\ldots$, ${k_1}^{th}$ load increases will be given by the following equations, provided that $b_{\rm I}$ and $a_{\rm I}$ are <u>fixed</u> during the period $[t_1+1, t_1+k_1+1] = [t_1+1, t_2] = (t_1, t_2]$.

$$\begin{aligned} X(t_1+2) &= b_{\rm I}X(t_1+1)+na_{\rm I} \\ X(t_1+3) &= b_{\rm I}X(t_1+2)+na_{\rm I} \\ &= (b_{\rm I})^2X(t_1+1)+na_{\rm I}(1+b_{\rm I}) \\ &\ldots \end{aligned}$$

$$X(t_1+i) = b_{\rm I}^{i-1}X(t_1+1)+na_{\rm I}(1+b_{\rm I}+\ldots+b_{\rm I}^{i-2})$$

$$= \begin{cases} X(t_1+1)+(i-1)na_{\rm I} & \text{if } b_{\rm I}=1 \\ b_{\rm I}^{i-1}X(t_1+1)+na_{\rm I}\frac{1-b_{\rm I}^{i-1}}{1-b_{\rm I}} & \text{if } b_{\rm I}<1 \end{cases}$$

For $i=k_1+1$, $X(t_1+k_1+1)=X(t_2)$ is obtained,

$$X(t_1+k_1+1) = X(t_2) =$$

$$= \begin{cases} X(t_1+1)+k_1na_{\rm I} & \text{if } b_{\rm I}=1 \\ b_{\rm I}^{k_1}X(t_1+1)+na_{\rm I}\frac{1-b_{\rm I}^{k_1}}{1-b_{\rm I}} & \text{if } b_{\rm I}<1 \end{cases}$$

where $X(t_1+1)=dX_{\rm eff}+na_{\rm D}$.

In view of Proposition 2, the total load $X(t)$ will be reduced again to $dX_{\rm eff}+na_{\rm D}$ in the next time instant t_2+1 and, thus, the total load adaptation behavior will be repeated: the total load will be at the overload level $X(t_j)$ at time instant t_j, $j>1$, then will fall at the level $X(t_j+1)=dX_{\rm eff}+na_{\rm D}$ in the next time instant t_j+1, and, then climb up to the level $X(t_{j+1})$, after $k_{\rm T}$ steps always in the same manner (reaching time instant $t_{j+1}=t_j+1+k_{\rm T}$ from which

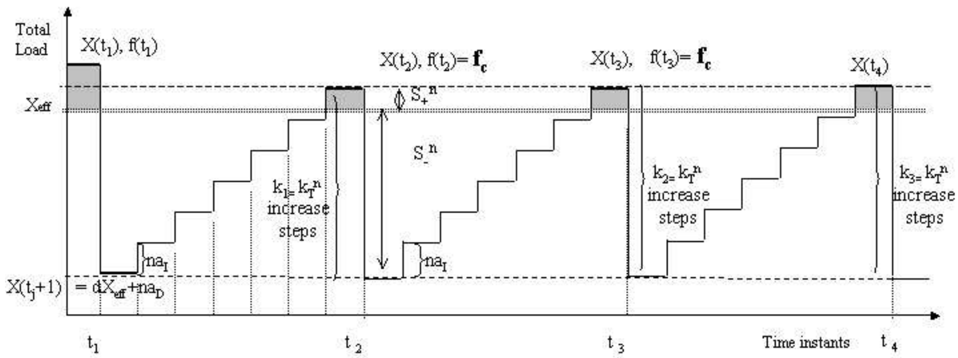

Figure 2. Fixed packet loss rates

the process will be repeated). The total load $X(t_{j+1})$ at time instants t_{j+1} is equal to $X(t_j + k_\mathrm{T} + 1)$ and given by (10), where k_T, is given by (11). $\square$

Regarding $X(t_1)$, that is the total load when the efficiency line is exceeded for the first time, it should be noted that it will - in general - be different than $X(t_j)$, $j > 1$, since $X(t_1)$ is determined by a sequence of increase steps starting from some initial value $X(0)$ and not $dX_\mathrm{eff} + na_\mathrm{D}$, and therefore depends on $X(0)$.

The aforementioned periodic behavior implies that the induced packet losses will be the same during each period. These packet losses are derived in the following proposition by using (10) in the loss computing expression $\frac{X(t_j)-X_\mathrm{eff}}{X(t_j)}$.

Proposition 4. *Following t_1, the packet loss rate $f(t_j)$, $j \geq 2$ under a hi-LDD-based scheme is fixed and given by (12), (see Fig. 2)*

$$f(t_j) \equiv f_\mathrm{c} = 1 - \frac{X_\mathrm{eff}}{X(t_j)} = $$
$$= \begin{cases} 1 - \frac{1}{d + \frac{n}{X_\mathrm{eff}}(a_\mathrm{D} + k_\mathrm{T} a_\mathrm{I})} & \text{if } b_\mathrm{I} = 1 \\ 1 - \frac{1}{b_\mathrm{I}^{k_\mathrm{T}}(d + \frac{n}{X_\mathrm{eff}} a_\mathrm{D}) + \frac{n}{X_\mathrm{eff}} a_\mathrm{I} \frac{1 - b_\mathrm{I}^{k_\mathrm{T}}}{1 - b_\mathrm{I}}} & \text{if } b_\mathrm{I} < 1 \end{cases} \quad (12)$$

By solving (12) with respect to $\frac{X_\mathrm{eff}}{n}$ the following expression for the fair share is derived.

Proposition 5. *The fair share for n flows sharing capacity X_eff is given by*

$$x_\mathrm{fair}^n = \frac{X_\mathrm{eff}}{n} = \begin{cases} \frac{k_\mathrm{T} a_\mathrm{I} + a_\mathrm{D}}{\frac{1}{1-f(t_j)} - d} & \text{if } b_\mathrm{I} = 1 \\ \frac{a_\mathrm{I} \frac{1 - b_\mathrm{I}^{k_\mathrm{T}}}{1 - b_\mathrm{I}} + a_\mathrm{D} b_\mathrm{I}^{k_\mathrm{T}}}{\frac{1}{1-f(t_j)} - d b_\mathrm{I}^{k_\mathrm{T}}} & \text{if } b_\mathrm{I} < 1 \end{cases} \quad (13)$$

3.1. Exploiting x_fair (eqn. (13)), $f(t_j)$ (eqn. (12)) and k_T (eqn. (11)) to improve rate adaptation.

From (13) it is evident that each flow may compute the current fair share without knowledge of X_eff or n, but using only locally available information: the control parameters (a_I,a_D, b_I, d), the reported packet loss rate ($f_i(t)$) and the number of increase steps (k_T) between two consecutive overloads (non-zero loss feedbacks). Knowledge of x_fair can be exploited in improving the rate adaptation process as explained in the sequel.

For instance, after the initiation/termination of a flow, (13) may be used to estimate the new value of x_fair and set all rates to that value. Depending on the next feedback, this rate will then be increased or decreased according to the hi-LDD policy. The immediate setting the current rate to the estimated x_fair is expected to improve the speed of convergence to x_fair after the initiation/termination of a flow.

It should be noted that although the parameters of the hi-LDD policy, the induced losses and the number of increase steps k_T, are involved in the estimation of x_fair (see (13)), x_fair is a quantity that does not depend on those parameters and measured values, but it depends on $X_{\rm eff}$ and n. Assuming that $x_{\rm fair}$ is known (estimated) and that $X_{\rm eff}$ and n do not change, one can set a desired value $\widehat{k}$ and $\widehat{f}$ in the left hand side of equations (11) and (12), respectively, and determine the control parameter four-tuple ($b_{\rm I}$,$a_{\rm I}$,d,$a_{\rm D}$) that makes the right hand side of equations (11) and (12) equal to the desired values $\widehat{k}$ and $\widehat{f}$, respectively. This amounts to using knowledge of $x_{\rm fair}$ to identify the parameters of the hi-LDD policy that will induce a targeted value of $f(t_j)$ and/or $k_{\rm T}$. That is, control the induced packet losses ($f(t_j)$) and/or smoothness ($k_{\rm T}$) of the rate adaptation policy. These ideas are pursued further in the next section.

3.2. Discussion on the hi-LDD and basic MD policies

As already mentioned, the basic MD (with fixed $b_{\rm D}$) policy cannot ensure a self-adjusting adaptation because of its associated static multiplicative decrease factor $b_{\rm D}$. If $b_{\rm D}$ is selected to be relatively large, then the rate decrease under large losses would not be sufficient and thus, more losses would occur in the next time interval(s). On the other hand, if $b_{\rm D}$ is selected to be relatively small, then the rate decrease under small losses would be unnecessarily large with the obvious impact on smoothness and throughput.

The introduced hi-LDD policy takes into consideration the induced packet loss rate when determining the rate decrement: it applies a larger decrease factor under high losses helping to overcome congestion quickly and converge faster to the new fair share if congestion is due to the initiation of a new flow. The hi-LDD policy ensures that after a decrease step the total load will be adapted to a level below the efficiency line, regardless of the number of flows in the network and the level of the packet loss rate (see Corollary 1). In addition, the hi-LDD policy ensures faster convergence to fairness than under the basic MD policy for $d = b_{\rm D}$.

The parameter d could be considered as a *smoothness factor*, since it shapes the value of $k_{\rm T}$. If the value of d is close to 1, then $k_{\rm T} = 1$, and the lower possible shorter-term smoothness - in conjunction with the given increase step $a_{\rm I}$ - is achieved; otherwise, if d is lower, then $k_{\rm T} > 1$, indicating that a larger drop in the rate has preceded and thus a less smooth rate adaptation process. However, the value of d affects the speed of convergence to fairness: the closer the value of d to 1, the slower the convergence to a new fair share rate. Clearly, there is a trade-off between the shorter-term smoothness and the convergence to fairness.

The dynamic hi-LDD policy introduced in the next sections aims to adjust dynamically the value of d in order for the shorter-term smoothness and the convergence speed to fairness to be balanced, as well as the induced losses to be controlled.

4. The Dynamic hi-LDD-based scheme

Consider a hi-LDD-based scheme with fixed parameter $d = d_0$ (to be referred to here as *the basic hi-LDD-based scheme*). Recall that after an overload ($X(t_j) > X_{\rm eff}$), the total load $X(t_j+1)$ falls to the level $d_0 X_{\rm eff} + n a_{\rm D}$ and after $k_{\rm T}$ increase steps it exceeds again the efficiency line ($X(t_{j+1}) > X_{\rm eff}$).

The rate adaptation behavior of the basic hi-LDD-based scheme is shown in Fig. 3 over the time interval [0, t_2+1]. At time t_2 (or any time t_j, $j>1$) $x_{\rm fair}$ is assumed to be known as it can be estimated from k_1 and $f(t_2)$ by using (13). By modifying d_0 to $d_{\rm new}$, the total load will fall to $d_{\rm new} X_{\rm eff} + n a_{\rm D}$ which will result in different values for k_2 and $f(t_3)$ from k_1 and $f(t_2)$ (Fig. 3). In this section it is investigated how to select $d_{\rm new}$ so that the resulting values k_2 and $f(t_3)$ achieve some desirable values $\widehat{k}$ and $\widehat{f}$, respectively. The resulting scheme will be referred to as the *Dynamic hi-LDD-based scheme*.

Proposition 6. *Consider a network with a number of flows whose rate is adapted by employing a basic hi-LDD-based scheme with parameters* $b_{\rm I}$, $a_{\rm I}$, d_0, $a_{\rm D}$*; let* $x_{\rm fair}$ *(assumed to be* >0*) be the corresponding fair share, calculated by (13). Then, some given values of shorter-term smoothness* $\widehat{k}$ *and induced losses* $\widehat{f}$ *can be achieved by adapting* d_0 *to the value* $d_{\rm new}$ *given by*

$$d_{\rm new}(\widehat{k}, \widehat{f}, x_{\rm fair}) = \\ = \begin{cases} \frac{1}{1-\widehat{f}} - \frac{\widehat{k} a_{\rm I} + a_{\rm D}}{x_{\rm fair}} & \text{if } b_{\rm I} = 1 \\ \frac{1}{b_{\rm I}^{\widehat{k}}} \frac{1}{1-\widehat{f}} - \frac{a_{\rm I}(1-b_{\rm I}^{\widehat{k}})}{b_{\rm I}^{\widehat{k}} x_{\rm fair}(1-b_{\rm I})} - \frac{a_{\rm D}}{x_{\rm fair}} & \text{if } b_{\rm I} < 1 \end{cases} \quad (14)$$

Proof. Recall that when $\lceil x \rceil = n$, where $x \in \mathbf{R}$ and $n \in \mathbf{Z}$, then $n-1 < x \leq n$. By substituting $k_{\rm T}$ with k in (11) the following inequalities are obtained

$$k - 1 < \frac{(1-d)X_{\rm eff} - n a_{\rm D}}{n a_{\rm I}} \leq k \quad \text{if } b_{\rm I} = 1$$

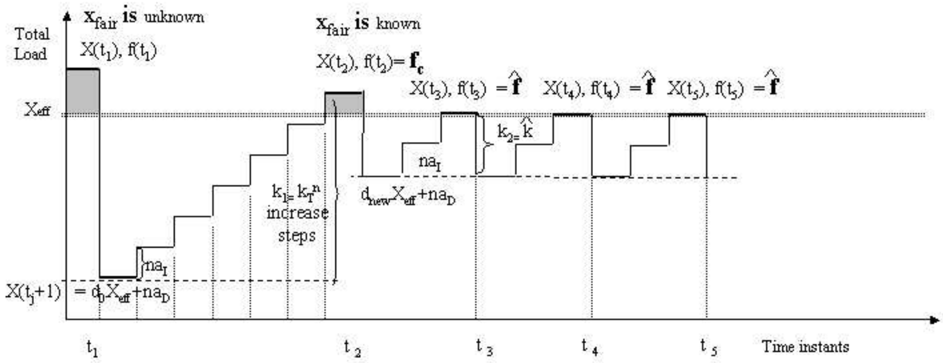

Figure 3. Determination of the new fixed level

$$k-1 < \log_{b_{\mathrm{I}}} \frac{X_{\mathrm{eff}} - \frac{na_{\mathrm{D}}}{1-b_{\mathrm{I}}}}{dX_{\mathrm{eff}} + n(a_{\mathrm{D}} - \frac{a_{\mathrm{I}}}{1-b_{\mathrm{I}}})} \le k \quad \text{if } b_{\mathrm{I}} < 1$$

Solving the above inequalities for d, the following is obtained

$$d \in [d_{\min}(k, x_{\mathrm{fair}}), d_{\max}(k, x_{\mathrm{fair}})) \qquad \text{where}$$

$$d_{\min}(k, x_{\mathrm{fair}}) = \\ = \begin{cases} 1 - \frac{ka_{\mathrm{I}}+a_{\mathrm{D}}}{x_{\mathrm{fair}}} & \text{if } b_{\mathrm{I}} = 1 \\ \frac{1}{b_{\mathrm{I}}^k} - \frac{a_{\mathrm{I}}(1-b_{\mathrm{I}}^k)}{b_{\mathrm{I}}^k x_{\mathrm{fair}}(1-b_{\mathrm{I}})} - \frac{a_{\mathrm{D}}}{x_{\mathrm{fair}}} & \text{if } b_{\mathrm{I}} < 1 \end{cases} \qquad (15)$$

$$d_{\max}(k, x_{\mathrm{fair}}) = d_{\min}(k-1, x_{\mathrm{fair}})$$

The value of $d_{\min}(k, x_{\mathrm{fair}})$ obtained by equation (15) is the minimum value of d for which $k_{\mathrm{T}} = k$. Consider that the initial value of parameter d_0 is changed at time instant t_2 to $d_{\min}(\widehat{k}, x_{\mathrm{fair}})$, for some given $\widehat{k}$. Then, the total load will fall to the level of $d_{\min}(\widehat{k}, x_{\mathrm{fair}})X_{\mathrm{eff}} + na_{\mathrm{D}}$. It can be easily derived that after $\widehat{k}$ steps $X(t_3)$ (as well as $X(t_j)$, $j > 3$) will be exactly equal to X_{eff}. If the value of d is set to a new value d_{new} larger than $d_{\min}(\widehat{k}, x_{\mathrm{fair}})$ by d_{c}, $d_{\mathrm{c}} > 0$, (that is, $d_{\mathrm{new}} = d_{\min}(\widehat{k}, x_{\mathrm{fair}}) + d_{\mathrm{c}}$, $d_{\mathrm{new}} < 1$) then $X(t_j)$ will exceed X_{eff} and packet losses will occur. Then, $X(t_j)$, $j \geq 3$, and f'_{c} will be given by

$$\begin{aligned} X(t_j) &= d_{\mathrm{new}} X_{\mathrm{eff}} + n(a_{\mathrm{D}} + \widehat{k} a_{\mathrm{I}}) \\ f'_{\mathrm{c}} &= 1 - \frac{X_{\mathrm{eff}}}{X(t_j)} = \frac{b_{\mathrm{I}}^{\widehat{k}} d_{\mathrm{c}}}{1 + b_{\mathrm{I}}^{\widehat{k}} d_{\mathrm{c}}} \end{aligned} \qquad (16)$$

The loss rate f'_{c} is independent from the number of flows, n, in the network. By setting $f'_{\mathrm{c}} \equiv \widehat{f}$ and solving (16) for d_{c}, the proper value of d_{c}, in order to achieve the targeted packet loss rate $\widehat{f}$, is obtained

$$d_{\mathrm{c}}(\widehat{k}, \widehat{f}) = \frac{\widehat{f}}{b_{\mathrm{I}}^{\widehat{k}}(1-\widehat{f})} = \frac{1}{b_{\mathrm{I}}^{\widehat{k}}(\frac{1}{\widehat{f}} - 1)} \qquad (17)$$

By adding (15) and (17) the value of d_{new} that induces $\widehat{k}$ and $\widehat{f}$ is obtained. □

By considering the proof of Proposition 6 and Fig. 3 it is clear that the minimum value of $\widehat{k}$ is 1 (achievable by some d close to 1) and the minimum value of $\widehat{f}$ is very close but not equal to 0 ($\widehat{f} = 0$ is achievable by selecting $d = d_{\min}(\widehat{k}, x_{\mathrm{fair}})$; then, the induced loss rate $f_i(t)$ is equal to 0 resulting in an increase step). Clearly, $\widehat{k}$ and $\widehat{f}$ can only increase beyond 1 and 0, respectively, and this can be happen by decreasing and increasing, respectively, the value of d. Thus, the range of achievable values for $\widehat{k}$ and $\widehat{f}$ would be: $1 \leq \widehat{k} < k_{\mathrm{u}}$ and $0 < \widehat{f} < f_{\mathrm{u}}$. These ranges are determined in the following Corollary.

Corollary 2. *The targeted values* $\widehat{k}$ *and* $\widehat{f}$ *are achievable by a Dynamic hi-LDD-based scheme provided that* $\widehat{k} \in [1, k_{\mathrm{u}})$ *and* $\widehat{f} \in (0, f_{\mathrm{u}})$ *where*

$$k_{\mathrm{u}} \equiv \begin{cases} \frac{x_{\mathrm{fair}}}{a_{\mathrm{I}}} & \textit{if } b_{\mathrm{I}} = 1 \\ \log_{b_{\mathrm{I}}}(1 - \frac{x_{\mathrm{fair}}(1-b_{\mathrm{I}})}{a_{\mathrm{I}}}) & \textit{if } b_{\mathrm{I}} < 1 \end{cases} \qquad (18)$$

$$f_{\mathrm{u}} \equiv \begin{cases} \frac{1}{1+\frac{x_{\mathrm{fair}}}{\widehat{k} a_{\mathrm{I}}}} & \textit{if } b_{\mathrm{I}} = 1 \\ \frac{1}{1+\frac{1}{(b_{\mathrm{I}}^{\widehat{k}}-1)+\frac{a_{\mathrm{I}}(1-b_{\mathrm{I}}^{\widehat{k}})}{x_{\mathrm{fair}}(1-b_{\mathrm{I}})}}} & \textit{if } b_{\mathrm{I}} < 1 \end{cases} \quad (19)$$

Proof. The total load $X(t_j+1)$ after a decrease step is equal to $d_{\mathrm{new}} X_{\mathrm{eff}} + n a_{\mathrm{D}}$ and should be larger than zero. Recall that $d_{\min}(\widehat{k}, x_{\mathrm{fair}})$ is the minimum value that d_{new} could take. Therefore,

$$d_{\min}(\widehat{k}, x_{\mathrm{fair}}) X_{\mathrm{eff}} + n a_{\mathrm{D}} > 0 \Leftrightarrow \widehat{k} < k_{\mathrm{u}}$$

where k_{u} is given by equation(18).

Recall that d_{new} should fulfil (9) or equivalently $d_{\mathrm{new}} < 1 - \frac{a_{\mathrm{D}}}{x_{\mathrm{fair}}}$ in order to fall below the efficiency line in a single decrease step. From this condition and equation (14) $\widehat{f} < f_{\mathrm{u}}$ can easily be derived, where f_{u} is given by equation (19).

□

Corollary 3. *The convergence speed to fairness of a Dynamic hi-LDD-based scheme achieving a targeted* $\widehat{k}$ *increases as* $\widehat{f} \to 0$.

Proof. Consider two flows i and j, that adjust their rates under a Dynamic hi-LDD-based scheme, with an initial distance $DF_{i,j}$ from the fairness line equal to δ_2 at the time instant t_2. After ξ decrease steps, that is $\xi \times (\widehat{k}+1)$ total increase and decrease steps, the distance from the fairness line, denoted by δ_ξ, under the Dynamic hi-LDD-based scheme, is given by (see above results and Proposition 1)

$$\delta_\xi = \delta_2 [d_{\mathrm{new}} (1-\widehat{f}) b_{\mathrm{I}}^{(\widehat{k}+1)}]^\xi \quad (20)$$

Solving the above equation for ξ, equation (21) is obtained.

$$\xi = \log_{[b_{\mathrm{I}}^{\widehat{k}+1} d_{\mathrm{new}}(1-\widehat{f})]} \frac{\delta_\xi}{\delta_2} \quad (21)$$

Consider now a given targeted distance $\widehat{\delta_\xi}$ from the fairness line and let $\widehat{\xi}$ denote the number of decrease steps required to achieve it; $\widehat{\xi}$ is derived from (21) by setting $\delta_\xi = \widehat{\delta_\xi}$. For a given $\widehat{k}$, $\widehat{\xi}$ is minimized (and thus the convergence speed is maximized) when $d_{\mathrm{new}}(1-\widehat{f}) b_{\mathrm{I}}^{\widehat{k}+1}$ is minimized. The latter is written as (see (14) and (17))

$$d_{\mathrm{new}}(1-\widehat{f}) = (d_{\min}(\widehat{k}, x_{\mathrm{fair}}) + d_{\mathrm{c}}(\widehat{k}, \widehat{f}))(1-\widehat{f})$$

$$= d_{\min}(\widehat{k}, x_{\mathrm{fair}}) + \widehat{f}\left(\frac{1}{b_{\mathrm{I}}^{\widehat{k}}} - d_{\min}(\widehat{k}, x_{\mathrm{fair}})\right) \quad (22)$$

where $\left(\frac{1}{b_{\mathrm{I}}^{\widehat{k}}} - d_{\min}(\widehat{k}, x_{\mathrm{fair}})\right) > 0$, since $b_{\mathrm{I}} \leq 1$ and $d_{\min}(\widehat{k}) < 1$. It is concluded from equation (22) that when $\widehat{f} \to 0$, the aforementioned product $d_{\mathrm{new}}(1-\widehat{f}) b_{\mathrm{I}}^{\widehat{k}+1}$ reaches its minimum value for a given $\widehat{k}$, $d_{\min}(\widehat{k}, x_{\mathrm{fair}})$, resulting in the fastest convergence speed to fairness for a given $\widehat{k}$. □

In view of Proposition 6 and the preceding Corollary it is clear that the Dynamic hi-LDD-based scheme will outperform any basic hi-LDD-based one in a dynamic environment (where flows are initiated and terminated) as a new value of d, d_{new}, may be applied when the environment changes to improve on a desirable performance metric. Even if the most effective value of d is selected under the basic hi-LDD-based scheme, this value is likely not to be as effective when the environment changes.

5. Simulation Results

In the sequel a set of simulations is carried out to illustrate some of the results derived in this paper. Assumption Set 1 is assumed to hold in the simulations, where a single-hop network model is considered in Matlab. In all simulations the network capacity is set to 8 Mbps and the number of flows n is equal to 12, 13, 14 and 13 for the time periods [0, 450), [450, 600), [600, 750) and [750, 1000], respectively. The initial rate vector for the first 12 flows is given by $\vec{x} = \{m+1*(M-m)/n, m+2(M-m)/n, \cdots, M\}$ where $m = 56$ Kbps, $M = 1.2$ Mbps, $n = 12$ and, thus, the initial total load is $X(1) = 8.108$ Mbps Flows 13 and 14 are initiated at time instants 450 and 600 with initial loads $x_{13}(450) = x_{14}(600) = 600$ Kbps . Flow 14 is terminated at time instant 750.

In the figures that follow the behavior of flow 1 (which has an initial rate of 151.3 Kbps) is shown. The figures (that follow) illustrate the convergence of flow 1 to the current fair share rate, the flow's oscillatory behavior both during the transition period (e.g., [0, ∼350] in Fig. 6) as well as during the "steady state" period (e.g., [∼350, 450] in Fig. 6), and the response of flow 1 to (a) the introduction of flow 13 ([450, 600]) and flow 14 ([600, 750]) and (b) the termination of flow 14 ([750, 1000]). In all simulations the value of the additive increase and decrease parameters $a_{\rm I}$, $a_{\rm D}$ is set to 22 [1] Kbps and 0, respectively.

Fig. 4 and 5 show the results under two basic AI/MD schemes with different decrease factors $b_{\rm D} = 0.99$ and $b_{\rm D} = 0.94$, respectively; these values have been chosen close to 1 to lead to a smooth rate adaptation. The smaller decrease factor ($b_{\rm D} = 0.94$) would lead to a larger diversification of the decrements that would be applied to flows of different rates (larger decrement to higher rates) and thus improve the convergence speed to fairness (compared to $b_{\rm D} = 0.99$). This indeed is observed to be the case by comparing Fig. 4 and 5. The price paid for this improvement (compared to the case of $b_{\rm D} = 0.99$) is a larger number of consecutive rate increase steps (less smooth rate in the shorter-term). In Fig. 4 and 5 the irregular adaptation behavior of the flows under the basic AI/MD schemes is also illustrated.

In the sequel, the simulation results of the basic and Dynamic hi-LDD-based schemes are presented. All schemes achieve convergence to fairness, since either $b_{\rm D}$ or $b_{\rm I}$ or both are less than 1 (see Proposition 1). The schemes present different convergence speeds to fairness depending on the values of $b_{\rm I}$, d and the induced packet loss rate $f(t)$. All schemes present *regular and periodic adaptation behavior* due to the employed hi-LDD policy, as expected (see Proposition 3). The schemes present different periods and number of increase steps $k_{\rm T}$ depending on the value of the smoothness parameter d. Basic hi-LDD-based schemes with a constant value of d, lower than 1 (e.g., 0.94), present a greater number of increase steps, $k_{\rm T}$, compared to schemes with a value of d close to 1 (e.g., 0.99), that induce a value of $k_{\rm T}$ close or equal to 1 (better shorter-term smoothness).

5.1. $b_{\rm I} = 1$

In the sequel simulation results concerning hi-LDD-based schemes with multiplicative increase parameter $b_{\rm I}$ equal to 1, are presented. Recall that the parameters $a_{\rm I}$, $a_{\rm D}$ are set to 22 Kbps and 0, respectively. Fig. 6 and 7 show the results under the AI/hi-LDD schemes with parameter d set arbitrarily to 0.99 and 0.94, respectively. Both adaptation behaviors are *regular* and *periodic* as established in section 2, (the same period over the entire duration regardless the number of flows); this behavior may be contrasted against the *irregular* behavior of the flows under the basic AI/MD schemes shown in Fig. 4 and 5. The *convergence speed to fairness* under the AI/hi-LDD schemes with $d = 0.99$ and $d = 0.94$, *is faster* than that under the basic AI/MD schemes with $b_{\rm D} = 0.99$ and $b_{\rm D} = 0.94$, respectively. The parameter d affects the number of increase steps $k_{\rm T}$, that is, the shorter-term smoothness ($k_{\rm T} = 1$ for $d = 0.99$ and $k_{\rm T} = 2$ for $d = 0.94$). Under the lower value of d the shorter-term smoothness is negatively affected in absolute terms (larger size of oscillation), as expected.

Under all schemes considered here, the amount of induced losses depends on the difference between the total load achieved after the last increase step (that results in a total load above $X_{\rm eff}$) and $X_{\rm eff}$. This total load does not have a monotonic relation to d or $b_{\rm D}$ but it is shaped by other factors such as the actual value of the total load before the first increase step, the size of the increase step and the number of flows. Thus, it is expected that the losses induced due to the bandwidth probing process (without considering any flow initiation/termination) may or may not be improved under the AI/hi-LDD scheme.

The aforementioned inability of the AI/hi-LDD scheme to guarantee better loss performance than the basic AI/MD scheme can be addressed by considering the introduced AI/Dynamic hi-LDD scheme that allows us to shape d so that a targeted loss rate can be achieved (as well as a given level of shorter-term smoothness -$k_{\rm T}$).

Fig. 8 shows the result under the AI/Dynamic hi-LDD scheme with targeted values $\widehat{f} = 0.01\%, \widehat{k} = 1$, and initial value of d_0 equal to 0.94. The values of $d_{\rm new}$ computed according to Proposition 6 are given by 0.9671, 0.9644, 0.9616 and 0.9644 over the periods [0, 450], [450, 600], [600, 750] and [750, 1000], respectively. The resulting scheme induces a value of $k_{\rm T}$ equal to 1 (see Fig. 8) and losses measured to be equal to 0.01%. It may also be noted that the convergence speed to fairness (Fig. 8) is not worse than that under the AI/hi-LDD scheme (Fig. 6) for which $k_{\rm T} = 1$ as well, while its induced losses are measured to be much higher; in fact the measured convergence speed to fairness is slightly better under the

[1] The mean size of a single frame of a video encoded as MPEG-4 single layer at mean bitrate of 660 Kbps (fair share for 12 flows) is 22 Kbit (660 Kbps = 30 frames/second x 22 Kbit/frame) [19].

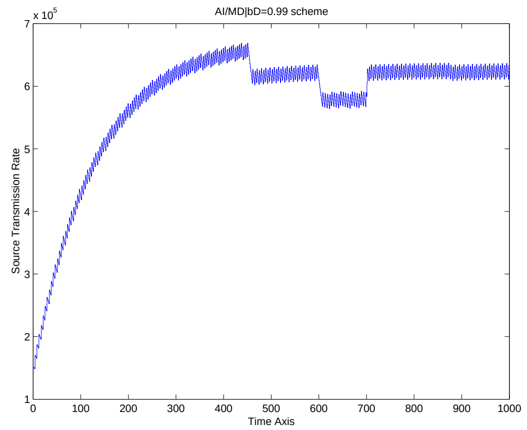

Figure 4. Adaptation of the basic AI/MD$|b_{\mathrm{D}} = 0.99$ scheme.

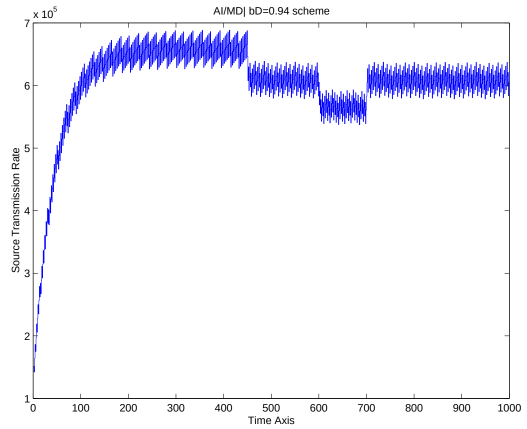

Figure 5. Adaptation of the basic AI/MD$|b_{\mathrm{D}} = 0.94$ scheme.

AI/Dynamic hi-LDD scheme.

5.2. $b_{\mathrm{I}} < 1$

In this section simulation results concerning hi-LDD-based schemes with multiplicative increase parameter b_{I} less than 1, are presented. The results under $b_{\mathrm{I}} < 1$ are in line with the results of the previous section, where $b_{\mathrm{I}} = 1$. The *Distance Weighted Additive Increase* (DWAI) policy [18], in which $b_{\mathrm{I}} < 1$ is used as the increase policy. Recall that in this policy the rate is shaped by $x_i(t+1) = \min\{M, x_i(t) + \frac{M - x_i(t)}{M-m} I\}$, where M and m are the upper and lower bounds of the rate, and I is a base increase step. The rate increment under the DWAI policy is a linear function of the distance of the current rate from the maximum allowable; the rate increment $\frac{M-x_i(t)}{M-m} I \to I\ (0)$ as $x_i(t) \to m\ (M)$, enabling larger increments for the lower rate flows and resulting in faster convergence to fairness. The DWAI policy could be rewritten as $x_i(t+1) = b'_{\mathrm{I}} x_i(t) + a'_{\mathrm{I}}$, where $b'_{\mathrm{I}} = 1 - \frac{I}{M-m} < 1$ and $a'_{\mathrm{I}} = M \frac{I}{M-m}$. $x_i(t+1) > x_i(t)$ is equivalent to

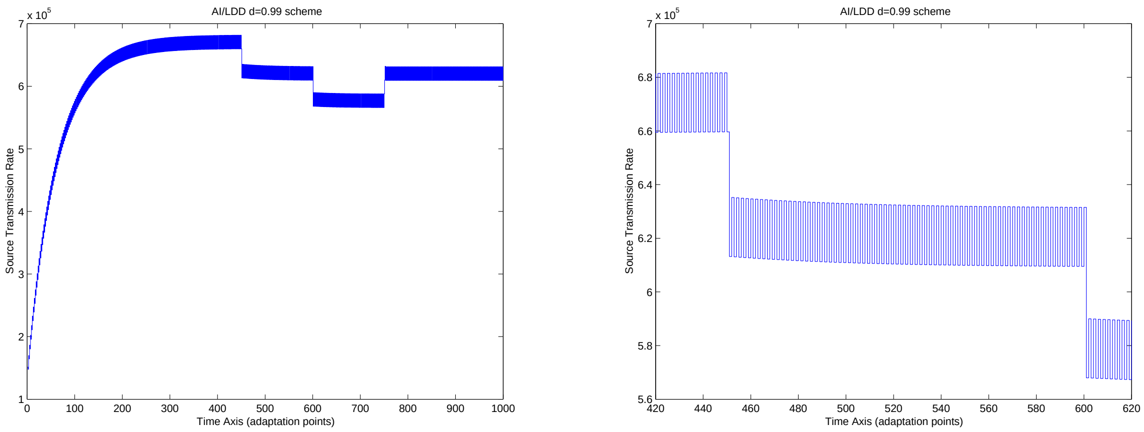

Figure 6. Adaptation of the AI/hi-LDD $|d = 0.99$ scheme.

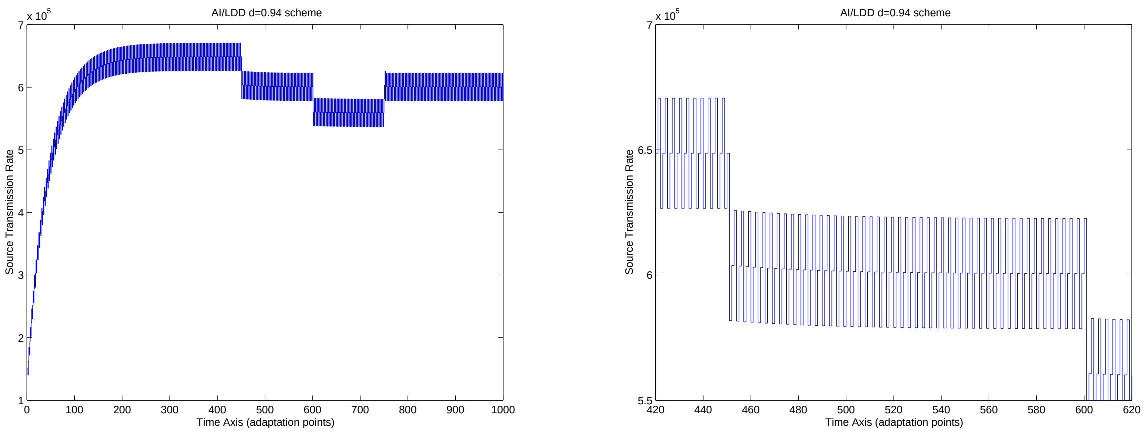

Figure 7. Adaptation of the AI/hi-LDD $|d = 0.94$ scheme.

$\frac{a'_{\rm I}}{1-b'_{\rm I}} > x_i(t)$. Under the DWAI policy, $\frac{a'_{\rm I}}{1-b'_{\rm I}} = M$ which, as mentioned before is the upper bound of the rate, and therefore $x_i(t+1)$ always is larger than $x_i(t)$.

The shorter-term smoothness for $k_{\rm T} = 1$ is given by $\Delta x_i = x_i(t+1) - x_i(t) = \frac{M - x_i(t)}{M-m} I$. Note that this increment is not fixed as in the case of pure AI policy, but varies with $x_i(t)$. In these simulations, the base increase step I is set to the values 22 Kbps and 47.2 Kbps, for which values the value of Δx_i is equal to 10.26 Kbps and 22 Kbps, respectively, when the flows have reached the fair share level, that is $x_i(t) = x_{\rm fair}$ (666.66 Kbps).

Fig. 9 shows the results under the DWAI/Dynamic hi-LDD scheme with $I = 47.2$ Kbps [2] , $a_{\rm D} = 0$, targeted values $\widehat{f} = 0.01\%$, $\widehat{k} = 1$ and initial value of d equal to $d_0 = 0.94$. The values of $d_{\rm new}$ computed according to Proposition 6 are given by

[2] When the fbws have reached the fairness line, the increment (Δx_i) under the DWAI policy is equal to 22 Kbps, the same with the increase step under the AI/Dynamic hi-LDD scheme.

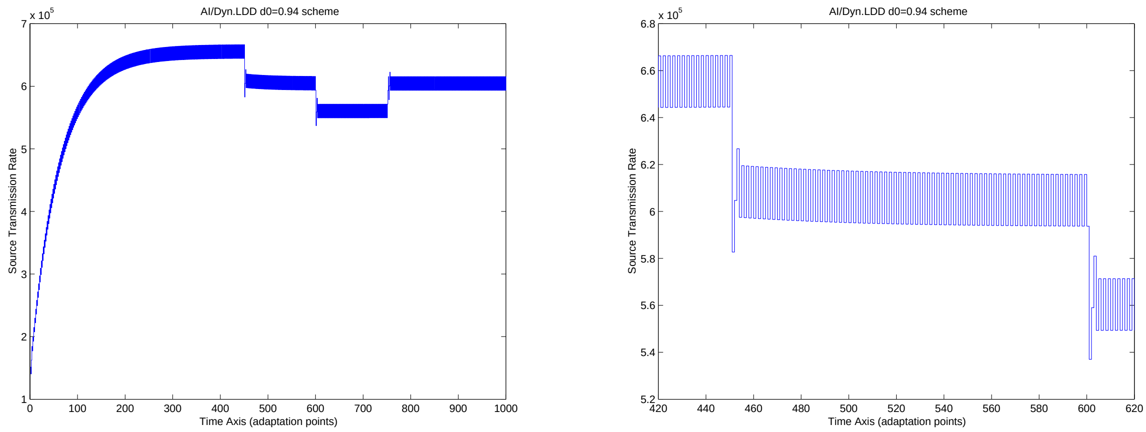

Figure 8. Adaptation of the AI/Dynamic hi-LDD$|d_0 = 0.94$ scheme with $\widehat{f} = 0.01\%$, $\widehat{k} = 1$.

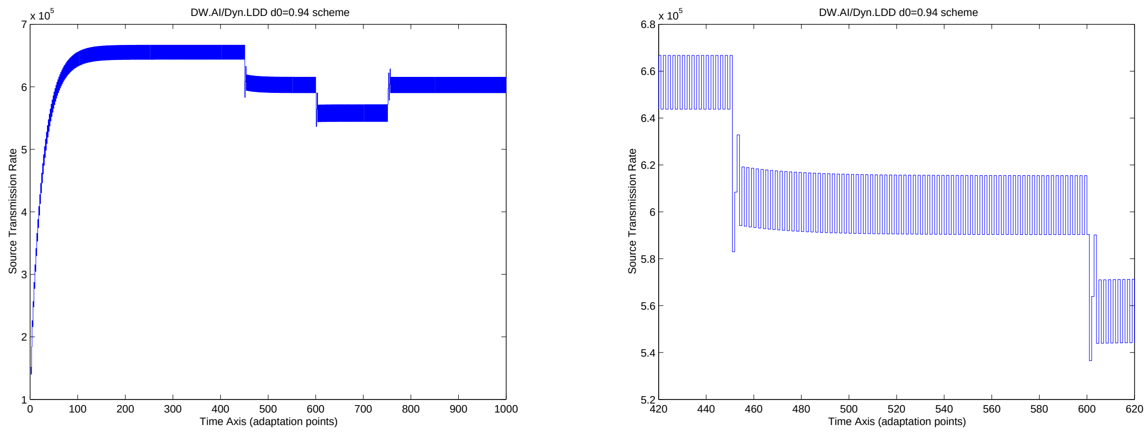

Figure 9. Adaptation of the DWAI/Dynamic hi-LDD$|d_0 = 0.94$ scheme, $I = 47.2$ Kbps.

0.9657, 0.9592, 0.9528 and 0.9592 over the periods [0, 450], [450, 600], [600, 750] and [750, 1000], respectively. The resulting scheme induces a value of k_{T} equal to 1 (see Fig. 9) and losses measured to be equal to 0.01%. The convergence speed to fairness is dramatically faster under this scheme than under the AI/Dynamic hi-LDD scheme for $d_0 = 0.94$ (see the corresponding fairness indices F($\vec{x}(t)$) in Fig. 11), due to the fact that $b_{\mathrm{I}} < 1$ and d_{new} are lower under the DWAI/Dynamic hi-LDD scheme than the corresponding parameters under the AI/Dynamic hi-LDD scheme (0.9671, 0.9644, 0.9616 and 0.9644). This result demonstrates that for similar shorter-term smoothness ($k_{\mathrm{T}} = 1$, $\Delta x_i = 22$ Kbps) when the rates are close to the fair share and loss rates are similar, the DWAI/Dynamic hi-LDD scheme outperforms the AI/Dynamic hi-LDD scheme with respect to convergence speed to fairness.

Fig. 10 shows the results under the DWAI/Dynamic hi-LDD scheme with $I = 22$ Kbps, $a_{\mathrm{D}} = 0$, targeted

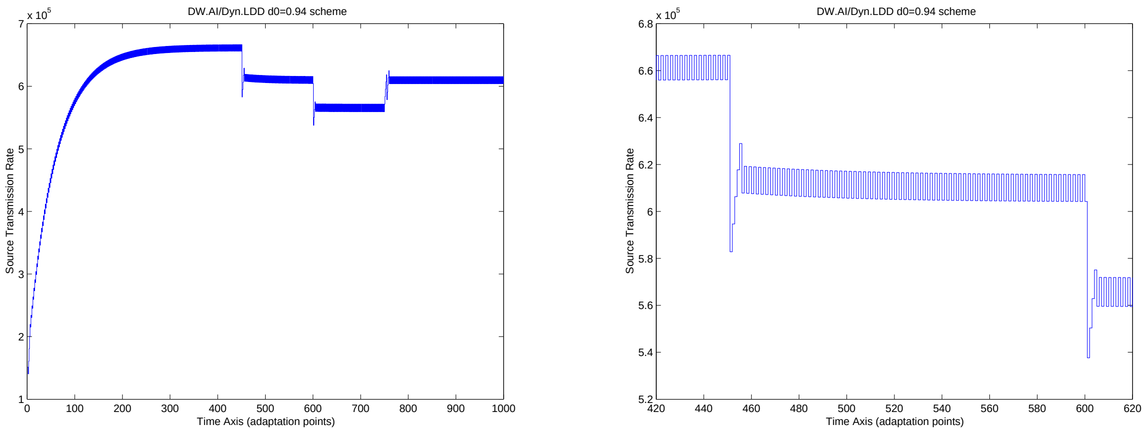

Figure 10. Adaptation of the DWAI/Dynamic hi-LDD$|d_0 = 0.94$ scheme, $I = 22$ Kbps.

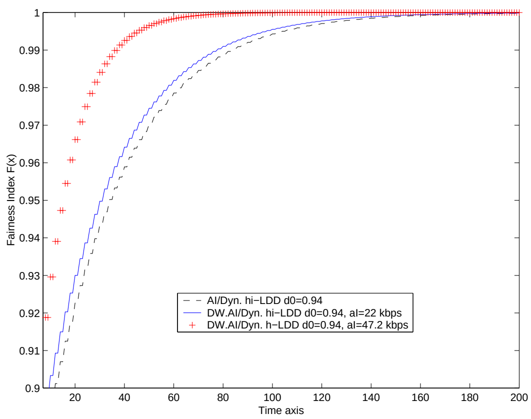

Figure 11. Fairness indices.

values $\widehat{f} = 0.01\%$, $\widehat{k} = 1$ and initial value of d equal to $d_0 = 0.94$. The values of d_{new} computed according to Proposition 6 are given by (0.9844, 0.9815, 0.9785 and 0.9815) over the periods [0, 450], [450, 600], [600, 750] and [750, 1000], respectively. The resulting scheme induces a value of k_{T} equal to 1 (see Fig. 10) and losses measured to be equal to 0.01%. It may be noted that the convergence speed to fairness is not dramatically but only slightly faster than that under the AI/Dynamic hi-LDD scheme (see the corresponding fairness indices F($\vec{x}(t)$) in Fig. 11) and that the size of oscillations is lower (equal to $\Delta x_i = 10.26$ Kbps) (see Fig. 10 and 8). The non-dramatic improvement is attributed to the fact that although the convergence speed is improved during the increase steps due to the DWAI policy (as opposed to the neutral impact of the basic AI policy), the values of d_{new} under the DWAI/Dynamic hi-LDD scheme with $I = 22$ Kbps (0.9844, 0.9815, 0.9785 and 0.9815) are higher than the values of d_{new} under the AI/Dynamic hi-LDD scheme with $a_{\text{I}} = 22$ Kbps (0.9671, 0.9644, 0.9616 and 0.9644). The higher values of d_{new} (than those under the DWAI/Dynamic hi-LDD with $I = 47.2$ Kbps) are attributed to the lower value of I (22 Kbps). This result demonstrates that for similar shorter-term smoothness ($k_{\text{T}} = 1$), loss rates and convergence speed to fairness, the DWAI/Dynamic hi-LDD scheme outperforms the AI/hi-LDD with respect to the size of oscillation (10.26 versus 22 Kbps) (see Fig. 10 and 6).

6. Conclusions

In this paper, the class of *Loss Dependent Decrease* (LDD) policies is considered, and a particular policy, the *history-independent LDD* (hi-LDD), is derived. Under a hi-LDD-based scheme: (a) the total load after a decrease step is independent from its previous values; (b) the policy immediately responds to congestion and the total load falls below the efficiency line in a single rate decrease step; (c) the flows adapt their rates in a regular and periodic manner.

Based on the aforementioned properties a hi-LDD-based scheme can estimate the fair share and use it to adjust dynamically the multiplicative control parameters so that a targeted level of smoothness and loss can be achieved. This novel scheme is referred to as the *Dynamic hi-LDD*-based scheme. The scheme retains the aforementioned properties of the basic hi-LDD-based schemes and, in addition, determines dynamically the parameter d so that the desired level of smoothness and loss rate achieved without requiring prior proper configuration of the parameter d.

The proposed schemes have been studied under the assumption of synchronized feedback and packet losses that are proportional to the flow's rate; and the detailed oscillatory adaptation behavior has been investigated. Meaningful comparisons with the behavior of pure Multiplicative Decrease (MD)-based schemes have also been presented.

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