Fractal Interpolation Surfaces. Theory and Applications in Image Compression.

Pantelis Bouboulis*

National and Kapodistrian University of Athens
Department of Informatics and Telecommunications
macaddic@otenet.gr

Abstract. In this dissertation, the problem of the construction of Fractal Interpolation Surfaces and their application in image compression is studied. We give exact conditions, so that this construction is valid and we introduce some free parameters that make our model as flexible as possible. In addition, we compute the box-counting dimension of a Fractal Interpolation Surface. Finally, we give a new image compression algorithm, based on fractal interpolation that differs from other fractal compression techniques.

1 Introduction

Fractal theory has been drawing considerable attention of researchers in various scientific areas. The application of fractals created by *iterated function systems* (IFSs) in the area of image compression is probably the most known one. Applications of fractal surfaces have been also found in computer graphics, metallurgy, geology, chemistry, medical sciences and several other areas where there is great need to construct extremely complicated objects; see for example [10], [14].

Mazel and Hayes (see [12]) use Fractal Interpolation Functions (FIFs) (introduced by Barnsley in [1]) to approximate discrete sequences of data (like one-dimensional signals). They demonstrated the effectiveness of their method by modelling seismic and electrocardiogram data. Recently, Navascues and Sebastian (in [13]) gave a generalisation of Hermite functions using FIFs.

Fractal Interpolation Surfaces (FISs) were used to approximate surfaces of rocks, metals, terrains, planets and to compress images. Self-affine FISs were first introduced in [11] in the case where the domains are triangular and the interpolation points on the boundary of the domain are coplanar. A few years later Geronimo and Hardin [8] generalised the construction of Massopust to allow for more general boundary data and domains.

Some problems in the construction by [11] remained unsolved, amongst which was the lack of free contractivity factors, which are necessary in modelling complicated surfaces. A general constructive method of generating affine FISs is presented in [17]. Xie and Sun in [15] and Xie, Sun, Ju, Feng in [16] presented a construction of a compact set that contains the interpolation points defined in

* Dissertation Advisor: Sergios Theodoridis, Professor