Technoeconomic Analysis of Telecommunications Networks – Models for Estimating and Forecasting Diffusion and Competition in the Telecommunications Market

Christos Michalakelis*

Department of Informatics and Telecommunications, University of Athens
Panepistimiopolis, Ilissia, Athens, Greece, GR-15784
michalak@di.uoa.gr

***Abstract*—** This doctoral thesis presents a methodological framework which integrates, in a uniform way, a number of important parameters which influence the progress of penetration of telecommunications products and services. Given the above methodological framework, significant problems of high-end market regarding diffusion and competition, are faced. The approaches developed are based on an appropriate mathematical and statistical background and they are applied to specific case studies, providing highly accurate results. The objective of this thesis corresponds to an important part of the techno economic design aiming to the prediction of demand and competition in the telecommunications market. The design of networks together with their expected use in the future, are important elements related to the development of the corresponding infrastructure. Due to the rapidly developing technologies and the growing demand for access, design of these networks should provide and support innovative network services and technologies. Determination of a number of factors, such as the number of users, the expected utilization of services, the volume of mobile data and the shaped market shares due to competition, should be the drivers for the development of the infrastructure to support the network operation.

***Index Terms* —**Techno- economic Analysis, Diffusion Theory, Demand Forecasting, Hedonic Price Indices, Population Biology

I. INTRODUCTION

Diffusion of innovative products is a research field facing a high level of interest, as it is almost always connected to heavy investments and critical business plans, targeting to meet the market's demand and competition. As industrial plans rolled out in an attempt to attract and to retain customers, they must be precisely forecasted, in terms of the expected level adoption and market shares, together with the revenue consequences for both new and established products. Since forecasts predict demand, failure to produce reasonably accurate forecasts will much probably lead to dramatic sequences to corresponding supply, e.g. oversupply and unneeded over-investments, or under-utilization of a firm's capacities.

The most characteristic case to be considered as an example is the sector of telecommunications, which corresponds to one of the most significant contemporary investment, regarding new technologies and services, subject to competition. Privatization and deregulation of the telecom market, together with the effects of increasing competition and the introduction of new services, resulted in the emergence of new problems regarding technology diffusion forecasting, under a high level of uncertainty and a need for risk management.

Although literature regarding forecasting methods for established products and services is well

* Dissertation Advisor: Thomas Sphicopoulos, Professor

developed, new opportunities have emerged due to the nature of high technology products' market. Therefore, further methodological work should be done, by identifying the gaps that have opened up, due to the change of the markets' scope and structure.

II. Models for Innovation Diffusion

One of the main central themes of the innovation field is the mathematical modeling of innovation diffusion, for different types of innovations and under different assumptions. The main finding can be summarized to a bell-shaped curve depicting the frequency of adoption against time and an S-shaped curve, when the cumulative number of adopters is plotted. During the first stages of the innovation life cycle (the introduction stage), the adoption rate is relatively low, followed by the next stage (the take off), described by a high rate of adoption, until the peak of the bell curve is reached, which corresponds to the inflection point of the cumulative adoption. After that time adoption rate decreases again, until the market saturation level is asymptotically met and the maximum number of adopters is reached. This corresponds to the end of the life cycle of the innovation, which in the cases of high technology products, is usually replaced by its descendant generation. The diffusion life cycle of an innovation, together with the distribution of the adopters are depicted in Figure 1.

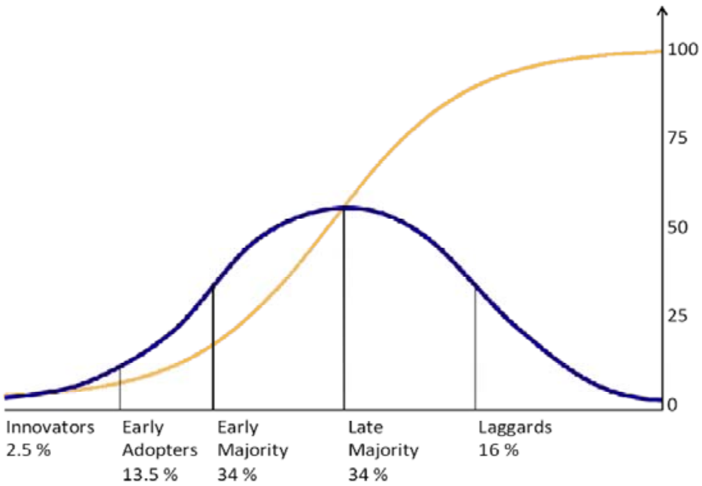

Figure 1 Idealized diffusion shape of an innovation

Apart from the early work of Gompertz [1, 2], the work of Bass [3] represents the early contemporary efforts to capture the diffusion dynamics. These ones together with the logistic family models [4], such as the linear logistic or Fisher-Pry [5] are the most widely used diffusion models employed for estimating and forecasting the market demand.

Aggregate diffusion models, that describe cumulative penetration, derive by a differential equation of the following general form [6]:

$$\frac{dN(t)}{dt} = d(N(t),\overline{p}) \cdot [f(K,N(t))] \qquad \text{Eq. 1}$$

where $N(t)$ represents the total penetration at time t, K is the saturation level, or the maximum expected cumulative penetration of the innovation considered and $d(N(t),\overline{p})$ is a function standing as a factor of proportionality. The quantity $f(K,N(t))$ represents the function of the remaining market potential, at time t, depending on the saturation level, K and the number of adopters, $N(t)$, at this time t. Finally, $\overline{p}$ is the vector of the model parameters, which are considered constants during the period of study. All of the above models provide *"S-shaped curve"* estimations for cumulative demand and are used to describe and forecast diffusion of innovations at the aggregate level, which describes total market response, in contrast to the econometric choice-based models, which focus on the estimation of the probability of individuals to adopt the innovation, by assuming that market behavior is driven by maximization of preferences. Aggregate models are generally able to provide reliable estimations of diffusion processes, regarding the adoption of innovations into a market of reference. One of their main fields of application is the sector of high technology and especially telecommunications.

There are a number of methods developed for estimating the parameters of the diffusion models. These methods can be either time independent, as the ones used here, i.e. Ordinary Least Squares (OLS), Nonlinear Least Squares (NLS), Maximum Likelihood Estimation (MLE) and Genetic Algorithms, or time dependent and they are mainly based on feedback filters, like the Kalman filters [7].

Finally, the selection of a model to describe the diffusion path of an innovation is based on either suitable statistical measures of accuracy, like Mean Squared Error – MSE, Maen Absolute Percentage Error – MAPE etc, or on theoretical criteria like the Minimum Description Length – MDL.

III. Generation Substitution Effects in Innovation Diffusion

Markets of high technology products and services, such as telecommunications, are described by fast technological changes and rapid generational substitutions. Since the conventional modelling approaches that are based on diffusion models do not usually incorporate this important aspect into their formulations, the accuracy of the provided forecasts is consequently affected. The work presented in this section is concerned with the development of a methodology for describing innovation diffusion, in the context of generation substitution. For this purpose, a dynamic diffusion model is developed and evaluated, based on the assumption that the saturation level of the market does not remain constant throughout the diffusion process but is affected by the diffusion of its descendant generation, as soon as the latter is introduced into the market. In contradiction to the conventional diffusion models, which assume static saturation levels, the proposed approach incorporates the effects of generation substitution and develops a diffusion model with a dynamic ceiling. The importance of such an approach is especially significant for markets characterized by rapid technological and generational changes.

A. Methodology

According to the concepts of diffusion theory, the diffusion of a product among a social system is proportional to the number of the existing adopters and to the number of the remaining market potential and is described by the following differential equation:

$$\frac{dN(t)}{dt} = rN(t)\left[\overline{N} - N(t)\right] \qquad \text{Eq. 2}$$

In Eq. 2, $N(t)$ refers to the cumulative number of adopters at time t, r is the rate of diffusion and $\overline{N}$ is the saturation level, which corresponds to the maximum cumulative number of adopters that diffusion is expected to reach. The initial number of adopters, at the beginning of the process is assumed to be $N(t=0) = N_0$ and the quantity $(\overline{N} - N(t))$ corresponds to the remaining market potential, while $\overline{N}$ is initially assumed constant throughout the product's life cycle.

Following the assumptions regarding the dynamic nature of first generation's saturation level, $\overline{N}$, instead of being considered as constant it is expressed as a function of the endogenous and exogenous parameters that affect it. Thus, in the general case, it can be expressed as:

$$\overline{N}(t) = f(U(t)) \qquad \text{Eq. 3}$$

where $U(t)$ represents the vector of the parameters affecting the saturation level.

In the context of the developed methodology, $\overline{N}(t)$ is assumed to be affected only by the penetration level of the descendant generation, considering the rest parameters as constants, therefore it can be expressed as:

$$\overline{N}(t) = f(M(t)) \qquad \text{Eq. 4}$$

By incorporating Eq. 4 into Eq. 2 and solving the equation that derives gives the number of adopters of the product, at each point of time, *t,* as expressed by the following equation:

$$N(t) = \frac{\overline{N}(t)}{1 + (\frac{\overline{N}_0}{N_0} - 1)\exp[-rP(t)]} \qquad \text{Eq. 5}$$

In the above equation $\overline{N}_0$ is the initially estimated saturation level and in the absence of the next generation, N_0 represents the number of adopters at time t_o and *P(t)* is given by: $P(t) = \int_{t_0}^{t} \overline{N}(t)dt$.

In correspondence with the assumption of Eq. 2 and if the time of the new generation's introduction is considered to be the initial time, then N_0 corresponds to the initial number of adopters at this time, thus $N_0=N(t=t_0)$. Obviously, the formulation of Eq. 5 reduces to that of Eq. 2 if $\overline{N}(t) = \overline{N}$.

Diffusion of the next generation, introduced into the market at time t_o, can be described by the logistic model as:

$$\frac{dM(t)}{dt} = sM(t)\left[\overline{M} - M(t)\right] \qquad \text{Eq. 6}$$

In Eq. 6, *M(t)* reflects penetration of the new generation, at time *t*, $\overline{M}$ is the corresponding estimated saturation level and *s* is the diffusion rate.

The analytical solution of the above differential equation, which is presented in Appendix A, results to:

$$M(t) = \frac{\overline{M}}{1 + (\frac{\overline{M}}{Mo} - 1)\exp[-s\overline{M}(t - to)]} \qquad \text{Eq. 7}$$

In the above Eq. 7, *Mo* is the initial penetration of the new generation, at the time of introduction, t_o. In the simple case, it can be assumed that $\overline{N}(t)$ is linearly dependent on the diffusion of the successive generation, *M(t)*, and this assumption is described by the following differential equation:

$$\frac{d\overline{N}(t)}{dM(t)} = a \qquad \text{Eq. 8}$$

Integration of both parts of Eq. 8 yields:

$$\int_{t_0}^{t} d\overline{N}(t) = a\int_{t_0}^{t} dM(t) \Rightarrow \overline{N}(t) - \overline{N}_0 = a[M(t) - M_0] \qquad \text{Eq. 9}$$

and finally:

$$\overline{N}(t) = \overline{N}_0 + aM(t) - aM_0 \qquad \text{Eq. 10}$$

The final formulation of the model is described by:

$$N(t) = \frac{\overline{N}_0 - aM_0 + a\left(\frac{\overline{M}}{1 + (\frac{\overline{M}}{M_0} - 1)\exp[-s\overline{M}(t - t_0)]}\right)}{1 + (\frac{\overline{N}_0}{N_0} - 1)\exp\left[-r\left((\overline{N}_0 - aM_0)(t - t_0) + \frac{a}{s}\left[\frac{x(t)}{x(t_0)}\right]\right)\right]} \qquad \text{Eq. 11}$$

B. Evaluation

Evaluation of the proposed methodology was performed over a number of five European countries (Austria, Finland, Netherlands, Italy, and Germany), based on historical data for 2G and 3G mobile telephony diffusion, as extracted from ITU's (International Telecommunications Union) database. Each dataset consists of two parts, corresponding to 2G and 3G diffusion data, respectively and the diffusion of mobile telephones is evaluated, in total. Figure 2 illustrates Austria as a representative result of the model's performance.

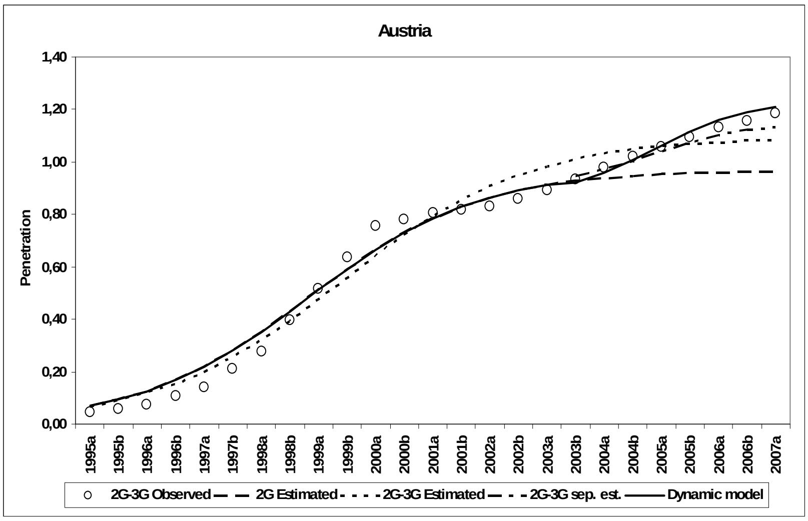

Figure 2 Estimation of 2G and 3G diffusion in Austria, vs observed values

IV. THE EFFECT OF THE POPULATION SIZE

Most of the conventional diffusion models, although they employ a number of parameters and marketing variables to describe diffusion, they do not explicitly take under consideration the influence of the population size, incorporating it into corresponding mathematical formulations. This may turn out to a major deficiency in the context of diffusion analysis, especially in the cases of rapid take offs, since the population size imposes a constraint to the further acceleration of the innovation diffusion. Such a case is the market of high technology durable products, where the probability for adopting another unit of product, apart from the first one, decreases although not totally eliminated. Mobile phones, broadband connections and personal computers could be considered as relevant examples.

The main objective of the work presented in this section is to study the influence of the targeted market's size over the diffusion process by developing an aggregate model, the "population-diffusion model" – PDM, which incorporates the population size as an explicit parameter that influences the diffusion rate. The second objective is to provide a measure for quantifying the level of uncertainty of the forecasts by proposing a stochastic variation of diffusion model.

Evaluation was performed over mobile phone historical data from 22 countries of the wider European area and showed that the "population" model can not only provide accurate diffusion estimations over a dataset that includes inflection point but it can also forecast future values for a dataset that does not include it. Moreover, evaluation of the stochastic realization provided a range of forecasted values that are quite accurate as well, since they indicated a lower and upper bound which future recorded values are expected to fall within.

A. Development of the model

According to the general formulation of diffusion models described by Eq. 1:

$$\frac{dN(t)}{dt} = rN(t)\,ln\left(a + b\frac{P}{N(t)}\right)ln\left(\frac{K}{N(t)}\right) \qquad \text{Eq. 12}$$

The quantities appear in Eq. 12 are the number of adopters, *N(t),* at time *t*, the saturation level, *K* and the population size the referenced market consists of, *P*. It should be noted that population does not necessarily refer to the number of individuals, but it can describe households or any otherwise defined units of adopters. The solution of the above equation provides the formulation of the population model:

$$N = Ke^{\frac{ln(a+\frac{bP}{K})\,ln(\frac{N_0}{K})\,e^{-r\,ln(a+\frac{bP}{K})t}}{ln(a+\frac{bP}{K}) + \frac{bP}{aK+bP}\,ln(\frac{N_0}{K})\,(e^{-r\,ln(a+\frac{bP}{K})t} - 1)}} \qquad \text{Eq. 13}$$

The solution presented in Eq. 13, is graphically represented by an S-shaped curve and it reduces again to the formulation of the Gompertz model if a and b become equal to *e* and *0* respectively.

B. Evaluation of the model

Evaluation of the "population" model was performed over historical data describing diffusion of mobile telephony, over 22 countries from the wider European area. Evaluation data were collected from the International Telecommunication Union (ITU, http://www.itu.int), corresponding to a period of time from year 1995 to year 2007. In addition, population data were extracted from Eurostat's and ITU's databases.

Figure 1 illustrates results of the model's evaluation for the case of Greece. Similar results were provided for the rest participating countries. The statistical measures of accuracy (MSE, MAPE) calculated for all the evaluated datasets verify the accuracy of the proposed model.

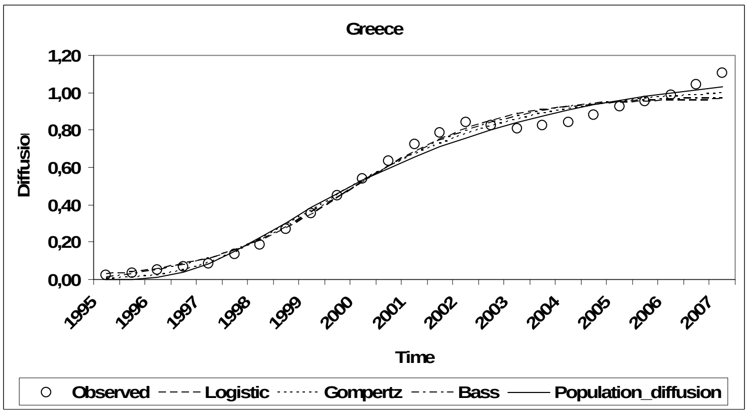

Figure 3 Diffusion estimation of the population – diffusion model, over actual data for Greece, in comparison with the performance of other diffusion

Evaluation of the proposed model's forecasting ability was performed as well, by splitting the available datasets into two parts, the "training" and the "holdback" data. The former were used to train the model and estimate its parameters, whereas the latter were used to compare the actual recorded values with the ones the models provided as forecasts. The holdback samples include the historical data from year 1995 up to one year before the inflection point (the maximum recorded penetration value, before the diffusion process slowed down), in each case considered.

V. MODELING COMPETITION IN THE TELECOMMUNICATIONS MARKET BASED ON CONCEPTS OF POPULATION BIOLOGY

A. Population Dynamics – Competing Species

Based on concepts of ecology modeling and specifically on population biology, a methodology for describing a high technology market's dynamics is developed and presented. The importance of the above methodology is its capability to estimate and forecast the degree of competition, market equilibrium and market concentration, the latter expressed by corresponding market shares, in the high technology environment. Evaluation of the presented methodology in the area of telecommunications led to accurate results, as compared to historical data, in a specific case study. Apart from a very good estimation of the market's behavior, this methodology seems to present a very good forecasting ability that can provide valuable inputs for managerial decisions, strategic planning and regulatory decisions to the players of a high technology market, described by high entry barriers.

The hypothesis concerning the variation of population is that the rate of its change is proportional to the current size of the population and the most common approach for modeling population growth of a species, in the absence of any competitors is given by Eq. 1. However, when more than one species coexist in the same environment, they are expected to compete for the same sources. Definitions and descriptions of species competition can be found in [8, 9] and they can be summarized to the following: *"Competition occurs when two or more individuals or species experience depressed fitness (reduced growth rates or saturation levels) attributable to their mutual presence in an area".* According to this approach, if two or more species are present in a closed environment, instead of only one, each of them will impinge on the available sources supply for the others. In effect, they reduce the growth rates and saturation populations of each other. A more precise definition, regarding interaction of species, is given in [10], where three types of interaction are identified: (i) If the growth rate of one population is decreased and the other increased the populations are in a predator–prey situation. (ii) If the growth rate of each population is decreased then it is competition. (iii) If each population's growth rate is enhanced then it is called mutualism or symbiosis.

Under specific conditions, in a closed established oligopolistic or competitive market, shares are reduced for each participant, due to coexistence and interaction with the others, provided that firms' decisions are based on the assumption of rationality, seeking to maximize their market shares and profit. In these cases, the second case of competition among species is considered.

The simplest expression for reducing the growth rate of each species due to the presence of the others is to incorporate suitable parameters to capture the measure of interference among species. The corresponding model is the well known Lotka–Volterra model, based on the work of Alfred J. Lotka and Vito Volterra. Analytical description together with informative examples regarding interaction and competition between two species can be widely found in literature, such as in [8-11]. In addition, theoretical analyses together with applications of interaction among three or more species can be found in [12-20].

Based on the above analysis, the dynamics of the corresponding system for a number of m competing species can be represented by the following system of first-order nonlinear differential equations:

$$\frac{dN_i}{dt} = N_i(a_i - \sum_{j=1}^{m} a_{ij} N_j), \quad i = 1,2,..,m \qquad \text{Eq. 14}$$

In Eq. 14, dN_i/dt is the rate of change of species i and a_i is the growth coefficient of the corresponding population, N_i. The coefficients a_{ij} measure the interspecies competitive effects (of one species over the others) when $i \neq j$ and to intraspecies competition when $i = j$ and they are not equal in general.

B. Evaluation of the model

Evaluation of the proposed methodology was performed over historical data describing diffusion and market shares of 2G and 3G mobile telephony in Greece. The data correspond to semiannual observations for the years 1995 to 2007, as collected by the corresponding operators and the Greek National Regulator Authority (NRA). In order to evaluate the effectiveness of the proposed model, the parameters of the system should first be estimated. Such estimations are usually achieved by making reasonable assumptions based on the available data, but in the present methodology heuristic methods are employed by the means of Genetic Algorithms. Genetic algorithms are applied over a particular dataset in order to "train" the system that is to estimate the model's parameters. Results are illustrated in Figure 4 to Figure 6

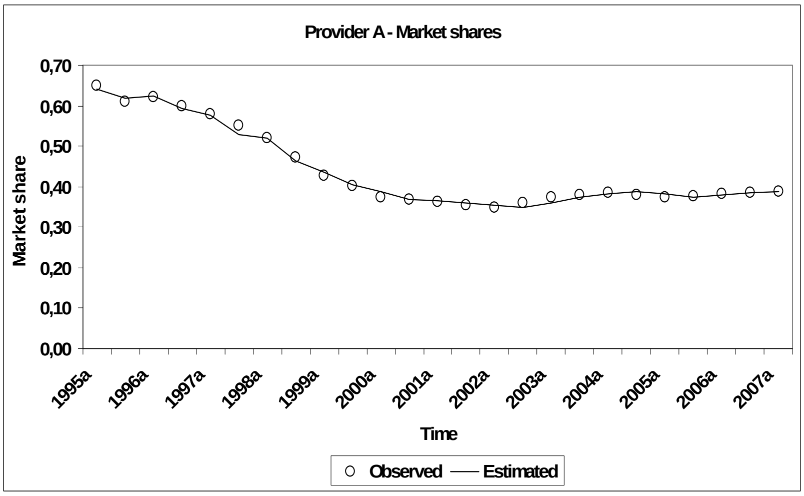

Figure 4 Estimated vs Observed market shares, for Provider A

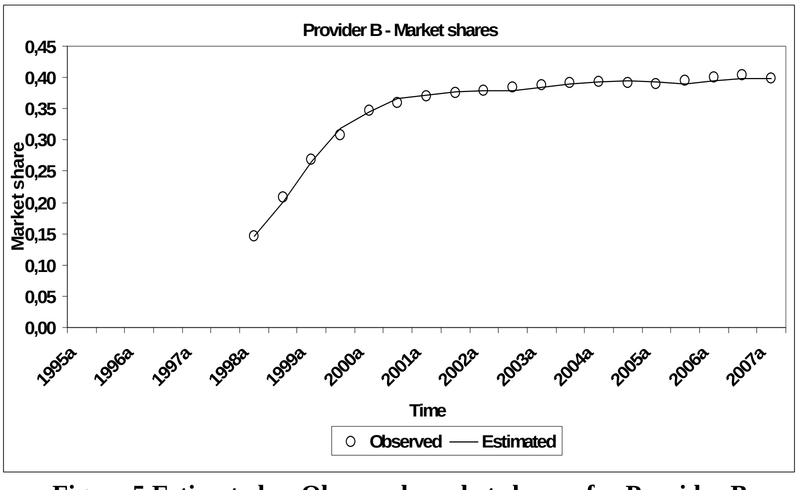

Figure 5 Estimated vs Observed market shares, for Provider B

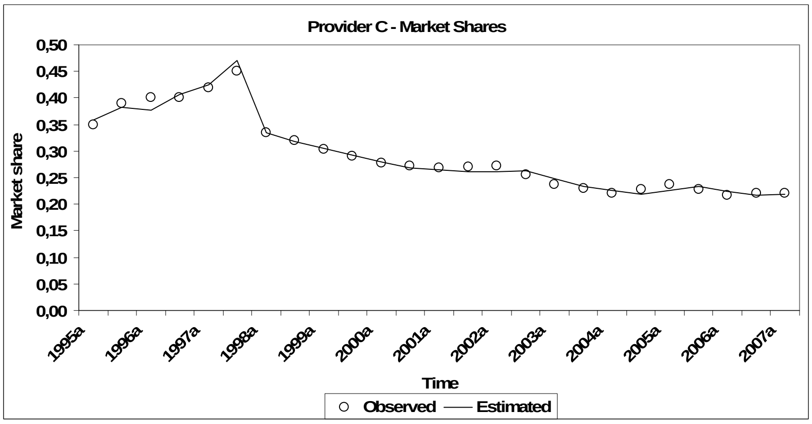

Figure 6 Estimated vs Observed market shares, for Provider C

VI. CONCLUSIONS

The main conclusions drawn by this doctoral thesis, which also constitute its contribution, lie in the following points:

- Development of a framework for estimating and forecasting diffusion, in the telecommunications market. This framework can be also applied in markets of high-technology products, in general.
- Evaluation of the existing methods for selecting appropriate demand models, in each examined case.
- Overview of the social, economic and other parameters (such as pricing and advertising) that influence the diffusion process of a product or service, in the telecommunications market.
- Design and integration into the above framework of methodologies related to the construction of price indices (price indices) and the assessment of the interaction and the elasticity of demand to price reporting service.
- Development of complex theoretical models to assess the interaction of urban centers in the region, neighboring countries, continents and general geographic areas, for estimating the demand for telecommunications services and products.
- Development of models for estimating and forecasting the level of competition and concentration in the telecommunications market, based on procedures from the natural world, such as population biology. According to these concepts, market competition is simulated by the fight of species for survival.
- Development of a model for estimating and forecasting diffusion of technological innovations by incorporating the process of generational replacement. The model provided results of greater accuracy than conventional methods.

VII. REFERENCES

[1] L. P. Rai, "Appropriate models for technology substitution," *Journal of Scientific & Industrial Research,* vol. 58, pp. 14-18, Jan 1999.

[2] B. Gompertz, "On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies," *Philosophical Transactions of the Royal Society of London,* vol. 115, pp. 513-585, 1825.

[3] F. M. Bass, "A new product growth model for consumer durables," *Management Science,* vol. 15, pp. 215-227, 1969.

[4] R. Bewley and D. G. Fiebig, "A Flexible Logistic Growth-Model with Applications in Telecommunications," *International Journal of Forecasting,* vol. 4, pp. 177-192, 1988.

[5] J. C. Fisher and R. H. Pry, "A simple Substitution Model of Technological Change," *Technological Forecasting and Social Change,* vol. 3, pp. 75-88, 1971.

[6] G. A. F. Seber and C. J. Wild, *Nonlinear regression*. Hoboken, New Jersey: Wiley, 2003.

[7] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems," *ASME TRans. - J. Basic Engineering,* vol. 82, (Series D), pp. 35-45, March 1960.

[8] D. Neal, *Introduction to population biology*. New York: Cambridge University Press, 2004.

[9] M. Begon, C. Townsend, and J. Harper, *Ecology:From individuals to ecosystems*: Blackwell Publishing, 2006, 4th ed.

[10] J. D. Murray, *Mathematical biology*, 3rd ed. New York: Springer, 2002.

[11] W. E. Boyce and R. C. DiPrima, *Elementary differential equations and boundary value problems*, 8th ed. Hoboken, NJ: Wiley, 2005.

[12] T. H. Fay and J. C. Greeff, "A three species competition model as a decision support tool," *Ecological Modelling,* vol. 211, pp. 142-152, Feb 24 2008.

[13] H. I. Freedman and P. Waltman, "Mathematical-Analysis of Some 3-Species Food-Chain Models," *Mathematical Biosciences,* vol. 33, pp. 257-276, 1977.

[14] H. I. Freedman and P. Waltman, "Persistence in Models of 3 Interacting Predator-Prey Populations," *Mathematical Biosciences,* vol. 68, pp. 213-231, 1984.

[15] H. I. Freedman and P. Waltman, "Persistence in a Model of 3 Competitive Populations," *Mathematical Biosciences,* vol. 73, pp. 89-101, 1985.

[16] P. G. L. Leach and J. Miritzis, "Analytic behaviour of competition among three species," *Journal of Nonlinear Mathematical Physics,* vol. 13, pp. 535-548, Nov 2006.

[17] J. X. Li and J. R. Yan, "Partial permanence and extinction in an N-species nonautonomous Lotka-Volterra competitive system," *Computers & Mathematics with Applications,* vol. 55, pp. 76-88, Jan 2008.

[18] R. M. May and W. J. Leonard, "Nonlinear Aspects of Competition between three Species," *Siam Journal on Applied Mathematics,* vol. 29, pp. 243-253, 1975.

[19] C. V. Pao, "Global asymptotic stability of Lotka-Volterra 3-species reaction-diffusion systems with time delays," *Journal of Mathematical Analysis and Applications,* vol. 281, pp. 186-204, May 1 2003.

[20] A. Shair and A. Tineo, "Three-dimensional population systems," *Nonliear Analysis:Real World Applications,* vol. 9, pp. 1607-1611, 2007.