Design of photonic crystal devices

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Abstract. The subject of this PhD thesis is the study of photonic crystal waveguides that incorporate discontinuities or structural fabrication-induced variations. For the numerical study of these devices the mode matching (MM) method is proposed. The comparison of the MM method with the couple mode theory (CMT) showed that the CMT can provide only a first approximation to the perturbation-induced scattering in photonic crystal waveguides. Additionally it was investigated the propagation of optical pulses in photonic crystal waveguides near the edge of the guided band, where the group velocity of the pulse is minimized.

Keywords: Optical waveguides, Mode matching method, couple mode theory, perturbation analysis, Solitons, delay lines.

1 Introduction

In this thesis, we demonstrate the effectiveness of a method based on Plane Wave Expansion (PWE)[1] and Mode Matching (MM)[2] in the analysis of PCW discontinuities. In order to apply the MM technique, the modes corresponding to a given frequency ω must be calculated including the evanescent modes with complex propagation constants β. By applying the PWE to the wave equation[1], one may determine the various values of ω corresponding to a given β. However, in contrast to conventional, constant cross-section waveguides, where β for the evanescent modes lie on the imaginary axis, in PCWs β may lie on the entire complex plane. To avoid sweeping the entire complex plane, an alternative formulation of the PWE is used for the first time, allowing the determination of the propagation constant and the distribution of the guided and the evanescent modes at a given frequency. It is shown that the MM method can provide accurate results without requiring significant memory resources and computational time. In the framework of the MM method, whenever a discontinuity is encountered inside a waveguide, we attempt to match the field expressed in terms of the waveguide modes to the modal fields of the discontinuity. This allows the computation of the reflection and transmission coefficients of each guided waveguide mode. The method is also applied to the study of fabrication induced disorder by calculating the performance degradation of a PCW in terms of the scattering loss and it is shown that MM can handle small perturbations without excessive computational time requirements.

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The propagation of both linear and nonlinear pulses is numerically investigated in single-mode 2-D PCWs near the band edge (where the delay is increased). Both triangular and rectangular lattice waveguides are assumed. Calculations for 1-cm-long PCWs reveal that, for 1-ns delay, linear pulses exhibit large broadenings for data rates just above 10 Gb/s. On the other hand, using either bright or dark soliton pulses may lead to significant improvement provided that the optical losses are kept low. It is numerically shown that optical solitons may be used to achieve 1-ns delay in 1-cm-long PCWs, at much higher data rates (40 Gb/s and even 100 Gb/s). Higher delays of the order of 5 ns at 10 Gb/s can also be supported.

Mode matching method

In order to implement the MM method, one first needs to estimate the propagation constants β and the modal fields of the various cells of the structure under consideration[4]. Using Bloch's theorem, the modes of a periodic dielectric structure along the z-direction can be written[1]

$$\mathbf{E}(\mathbf{r}) = \mathbf{u}(\mathbf{r})e^{j\beta z} \tag{1}$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{v}(\mathbf{r})e^{j\beta z} \tag{2}$$

where β is the propagation constant of the mode and **u,v** are periodic functions along the z direction. Defining $\left|\Psi_\beta\right\rangle$ to be a four component vector comprising of the tangential parts $\mathbf{u}_t$ and $\mathbf{v}_t$ of **u** and **v** respectively, i.e.

$$|\Psi_\beta >= \left(\mathbf{u}_t, \mathbf{v}_t\right)^T = \left(u_x, u_y, v_x, v_y\right)^T \tag{3}$$

one can write Maxwell's equations in the following form (Ref. 3)

$$\left(\hat{A} + j\frac{\partial}{\partial z}\hat{B}\right)|\Psi_\beta >= \beta\hat{B}|\Psi_\beta > \tag{4}$$

where the operators $\hat{A}$ and $\hat{B}$ are defined by

$$\hat{A} = \begin{pmatrix} \omega\varepsilon - \frac{1}{\omega}\nabla_t \times \frac{1}{\mu}\nabla_t \times & 0 \\ 0 & \omega\mu - \frac{1}{\omega}\nabla_t \times \frac{1}{\varepsilon}\nabla_t \times \end{pmatrix} \tag{5}$$

and

$$\hat{B} = \begin{pmatrix} 0 & -\mathbf{z}\times \\ \mathbf{z}\times & 0 \end{pmatrix} \tag{6}$$

The eigenvalues of the eigenproblem in (4) can be used to determine the propagation constants of both evanescent and guided modes of the structures while the eigenvectors determine their modal fields.

Since the modes of the structure can be calculated, one can proceed to apply the MM technique. The field at each interface between adjacent cells must satisfy the continuity equations, i.e. the tangential fields at the left of a boundary must equal the tangential fields at the right of the boundary. At the i^{th} cell the tangential magnetic field are written as:

$$\mathbf{H}_t^i = \sum_m a_m^{(i)} \mathbf{h}_{tm}^{(i)} e^{j\beta_m^{(i)}(z-z_{i-1})} + \sum_m b_m^{(i)} \mathbf{h}'^{(i)}_{tm} e^{-j\beta_m^{(i)}(z-z_i)} \tag{7}$$

where $\mathbf{h}_{tm}^{(i)}$, and $\mathbf{h}'^{(i)}_{tm}$ are the tangential magnetic Bloch functions propagation constants of the m^{th} forward mode and the m^{th} backward mode of the i^{th} cell respectively. At each interface between two cells, the tangential fields must be continuous at the boundary $z=z_i$. This implies:

$$\mathbf{E}_t^i(z_i) = \mathbf{E}_t^{i+1}(z_i) \tag{8}$$

$$\mathbf{H}_t^i(z_i) = \mathbf{H}_t^{i+1}(z_i) \tag{9}$$

one obtains a matrix equation relating the mode coefficients in cells i and i+1:

$$\begin{pmatrix} \mathbf{A}^{i+1} \\ \mathbf{B}^{i+1} \end{pmatrix} = Z_i \cdot \begin{pmatrix} \mathbf{A}^{i} \\ \mathbf{B}^{i} \end{pmatrix} \tag{10}$$

where vectors $\mathbf{A}^i=[a_1^{(i)},\ldots,a_M^{(i)}]^T$, $\mathbf{B}^i=[b_1^{(i)}\ldots,b_M^{(i)}]^T$ contain the coefficients of the M forward and M backward modes of the i^{th} cell. If the structure consists of many cells, one can relate the modal amplitudes at its input to the modal amplitudes of its output using the transfer matrix properties leading to the following equation:

$$\begin{pmatrix} \mathbf{A}^{\mathbf{N}} \\ \mathbf{B}^{\mathbf{N}} \end{pmatrix} = Z \cdot \begin{pmatrix} \mathbf{A}^{\mathbf{1}} \\ \mathbf{B}^{\mathbf{1}} \end{pmatrix} \tag{11}$$

Solving this equation one can calculate the modal amplitudes of the modes that are related to the power reflection and transmission of the device.

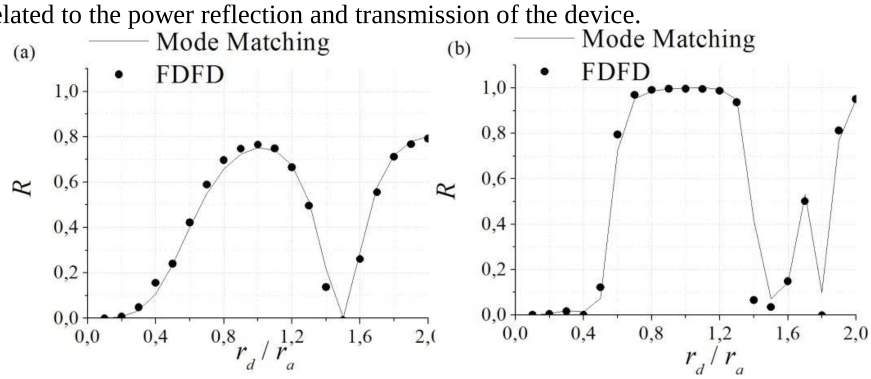

Fig 1. Comparison of power reflection coefficient of the MM and the finite difference frequency domain (FDFD) methods for a) single, and b) triple defect rods inside a PCW.

To compare the results of the MM method with the FDFD method, a sequence of 1 and 3 defect rods with radius r_d is placed inside a PC waveguide. Figures 1(a)-1(b) depict the power reflection coefficients calculated with the FDFD (dots) and the MM method (solid lines). As observed in figure 1, there is a very good agreement between the two methods in terms of the power reflection coefficients and this verifies the accuracy of the MM method.

Application of MM in the study of fabrication imperfections

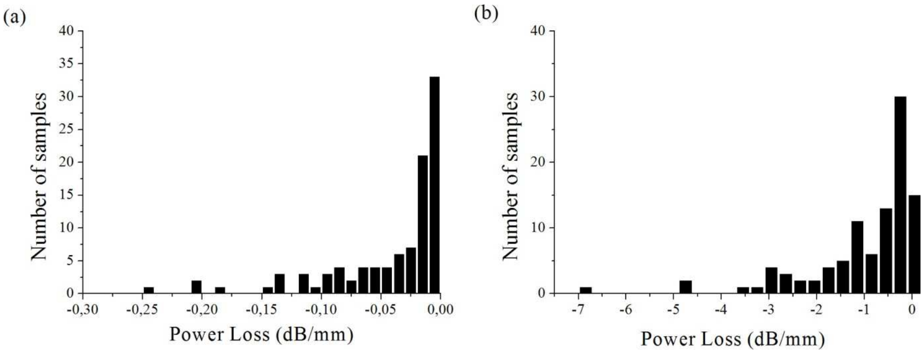

Fig 2. Power loss (expressed in dB/mm) due to scattering obtained considering 100 perturbed PCWs assuming a) Δ=1nm and b) 5nm.

In this section the MM method will be applied in the calculation of optical losses due to scattering at fabrication imperfections in a PCW. Towards this end a number of PCW cells will be considered having the centres $\left(z_i', x_i'\right) = \left(z_i + \Delta z_i, x_i + \Delta x_i\right)$ of the rods slightly displaced with respect to the centres $\left(z_i, x_i\right)$ of the rods of the ideal PCW and their radius $r_i' = r_a + \Delta r_i$ perturbed. For simplicity, the perturbations Δz_i, Δx_i and Δr_i are independently selected from the samples of a uniform distribution inside [-Δ, +Δ]. It is deduced that although small deviations of 1nm do not introduce significant losses, the losses increase significantly for Δ=5nm exceeding 1dB/mm in this case. This is illustrated in figure 2 where a bar plot of the power losses of the samples is given and it is deduced that although for the majority of the samples the loss is close to the mean value, there are some samples with significantly higher loss.

Accuracy of Coupled Mode Theory and Mode Matching Method

In this section the accuracy of the two formulations of the CMT[5] are used in the estimation of the amount of scattering due to fabrication-induced disorder. Depending on the type of orthogonality relations used, one may obtain two different formulations for the CMT: The IVG-CMT and the CCMT. To compare the accuracy of the methods a single defect rod is assumed inside a PCW. The defect rod radius r_d was altered from 0 to $2r_a$, and the power reflectivity was calculated by the IVG-CMT, the CCMT, the MM and the conventional FDFD method. In figure 3, the power reflectivity for the four methods is plotted for various values of the ratio r_d/r_a. The

MM method agrees very well with the FDFD method. However, both CMT formulations, although they roughly predict the shape of the curve, they do not provide accurate results. This is because in both formulations the field is written as a sum of the guided modes of the unperturbed waveguide and if the perturbation is not small, such an expansion is not an accurate approximation. In the region near the defect, the modes may differ significantly from the guided modes of the ideal PCW. In addition, the evanescent modes of the defect region are excited, and although they do not carry any power they alter the transmission and reflection properties of the system.

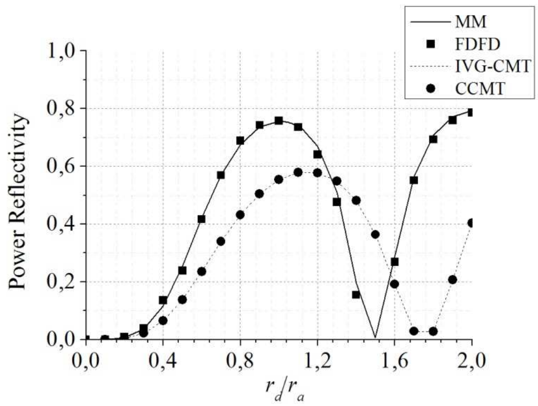

Fig 3. Comparison of power reflectivity calculated by MM, FDFD, IVG-CMT and CCMT for a single defect rod radius r_d inside a PCW.

It is interesting to ascertain if at least the CMT formulations can be used to estimate the sensitivity $\Delta R \cong (\partial R/\partial r_d)\Delta r_d$ of R for small perturbations Δr_d in the value of r_d. Figure 4 depicts power reflectivity for the same structure as discussed before, assuming small variations of the order of 1% of the defect rod radius r_d around $1.2r_a$. Due to the small variations the AV/FDFD is applicable in this case and has been used instead of the conventional FDFD. The fact that the AV/FDFD and the MM method produce the same results for $\partial R / \partial r_d$ is a strong indication of their validity in the sensitivity analysis of fabrication induced geometric perturbations.

On the other hand, the results of both CMTs again coincide, but not only do they underestimate the power reflection R but also the derivative $\partial R / \partial r_d$.

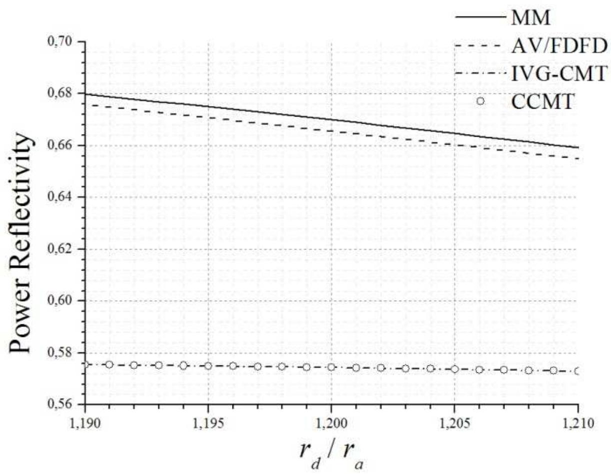

Fig 4. Normalized electric/magnetic fields along the interface z=z_1 for the device of figure 2a and r_d=r_a.

Linear and Nonlinear Pulse Propagation In PCWs

In this section, the propagation of both linear and nonlinear pulses is numerically investigated in single mode 2D PCWs near the band edge (where the delay is increased)[6]. Both triangular and rectangular lattice waveguides are assumed such as the ones depicted in figure 5.

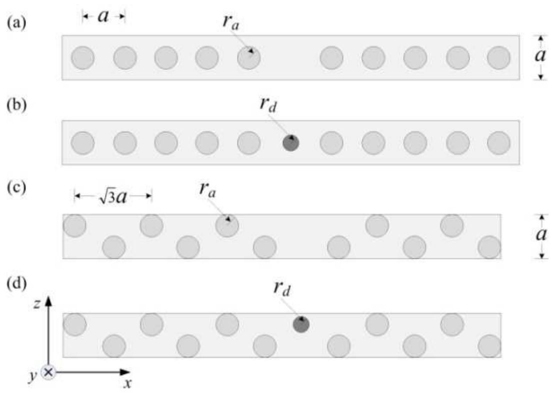

Fig 5. Examples of photonic crystal waveguides unit cells formed by a) removing a single rod from a square photonic lattice, b) inserting defect rods in a square lattice, c) removing a rod from a triangular lattice, d) inserting defect rods in a triangular lattice.

The propagation of optical pulses inside a PCW can be studied using the propagation equation[7]:

$$j\left(\frac{\partial A}{\partial z}+\frac{\Gamma}{2}A\right)+\sum_{l\geq 2} j^{m(l)}\frac{k_l}{l!}\frac{\partial^l A}{\partial T^l}+\gamma|A|^2 A=0 \qquad (12)$$

Equation (12) describes the pulse evolution $A(z,T)$ as it propagates along the PCW assuming a frame of reference $T=t-z/v_e$ moving with the group velocity v_e of the signal. The coefficient Γ is related to the optical losses which are caused either by disorder-induced scattering or by out-of-plane propagation losses. The function $m(l)$ is defined by $m(l)=\text{mod}(l,2)$ while the coefficient k_l is the group velocity dispersion coefficient (for $l=2$) or a higher order dispersion coefficient (for $l>2$). These coefficients are calculated from the derivatives of the mode propagation constant k with respect to the frequency ω, i.e.

$$k_m=\left.\frac{d^m k}{d\omega^m}\right|_{\omega=\omega_0} \qquad (13)$$

The group velocity v_e is simply

$$v_e=(d\omega/dk)_{\omega=\omega_0}=1/k_1 \qquad (14)$$

The coefficient γ is the self phase modulation (SPM) coefficient and can be calculated using[0]:

$$\gamma=\frac{2\omega_0}{a}\int_V dS\,\varepsilon_{NL}\left|\mathbf{e}_0\right|^4 \qquad (15)$$

The slow down factor $S=c/v_e$, where c is the speed of light in vacuum, obtained for the waveguide structures of figure 5 is plotted in figure 6. There is a large increase in S, at wavelengths near the two band-edges of the guided mode. It is interesting to note that defect type waveguides, with r_d=0.175a achieve higher slow down factors than hollow type waveguides (r_d=0) of the same lattice type.

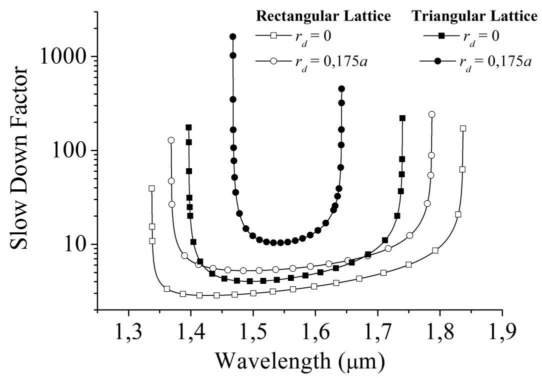

Fig 6. Slow down factors for the guided mode of PCWs depicted in figure 5.

As in the case of linear optical fibers, in a linear PCW (γ=0), it can be shown that an optical pulse having a Gaussian incident profile, $A(0,T)=\exp(-T^2/2T_0^2)$, remains Gaussian in shape and is broadened by a factor of:

$$BF_L(z) = \left(1 + \left(z / L_D\right)^2\right)^{1/2} \tag{16}$$

where L_D is the dispersion length

$$L_D = T_0^2 / \left|k_2\right| \tag{17}$$

Ignoring higher order dispersion terms (k_l=0 for l>2, the broadening factor can be estimated using (16). In figure 7 the broadening factor is plotted for the four PCWs in question, assuming T_d=1ns and 5ns respectively, and 1cm long waveguides. The launch point is taken near the left band edge where the values of k_2 are smaller. It is deduced that for $R_b\cong$10Gb/s and T_d=1ns, BF_L is lower than 1.33 (corresponding to the limit for dispersion-induced broadening) only in the case of the defect-type triangular lattice PCW. This PCW can support 12Gb/s signal at this limit. For T_d=5ns, the broadening factors are prohibitive, even for data rates slightly above 1Gb/s.

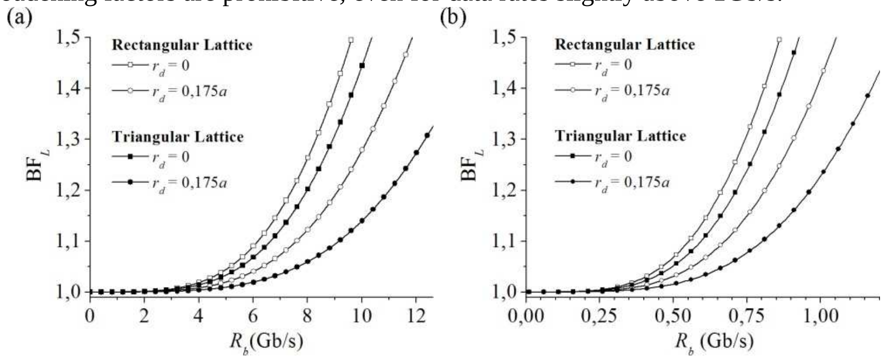

Fig 7. Linear Broadening factor for hollow and defect-type PCW waveguide formed in either rectangular or triangular PC lattice at the left band edge when a) T_d=1ns and b) T_d=5ns.

A similar behavior is observed when the launch wavelength corresponds to the right band of the guided band. Optical soliton pulses may experience less dispersion-induced broadening than linear pulses. To investigate the influence of higher order dispersion and optical loss in the stability of the soliton, one may numerically solve the propagation equation using the SSF method setting the suitable initial condition for bright and dark solitons respectively. For bright solitons one simply uses $A(0,T)=P_0^{1/2}\mathrm{sech}(T/T_0)$ as an initial conditions. Instead of using (16), the broadening factor BF_{NL} at the nonlinear regime, is numerically calculated as:

$$\mathrm{BF}_{NL}(L) = \frac{B_{3dB}(L)}{B_{3dB}(0)} \tag{18}$$

where $B_{3\mathrm{dB}}(L)$ is the numerically computed full width at half maximum of the envelope pulse $A(L,t)$.

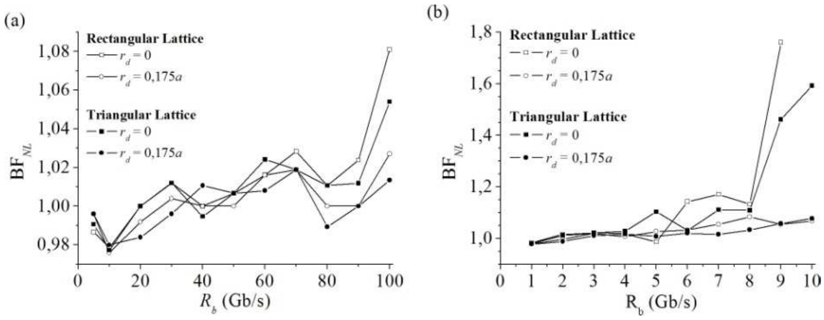

Fig 8. Broadening factors for bright solitons propagation in various geometries of PCWs obtained at a specified delay of either a) T_d=1ns for signals up to 100Gb/s and b) T_d=5ns for signals up to 10Gb/s.

The values of the broadening factor BF_{NL} of a bright soliton, with respect to the bitrate are plotted in figure 8 (a) and (b) for T_d=1ns and T_d=5ns respectively. The PCW length was set to 1cm and optical losses were neglected. It is interesting to note that the broadening factor is much smaller than 1.1, implying a broadening of less than 10% for bitrates of up to 100Gb/s at 1ns delay. The broadening factors obtained are generally much better than the linear broadening factors of figure 7, and this implies that the use of optical solitons as information carriers can significantly improve the performance of the delay line. For delay T_d=5ns, higher order dispersion has a greater degrading effect, limiting the bit rate to maximum value of 10Gb/s. However, compared to the linear case figure 7(b), this is still a significant improvement.

In the case of dark soliton propagation the maximum bitrate is similar to bright solitons. However, the optical field is better confined near the right band-edge where dark soliton are supported. This means that dark solitons maybe more suitable information carriers in nanophotonic applications where the mode field confinement is an important issue.

Optical losses can also affect the propagation of optical solitons. For a 10Gb/s bright soliton, at a specified delay of T_d=1ns, it is seen that even for losses of up to 1dB/mm, the optical pulse does not broaden significantly. However, as the data rate is increased, the influence of optical loss becomes much more critical. It is deduced that the optical loss must now be kept smaller than 0.1dB/mm in order to avoid pulse broadening beyond 30%.

Conclusion

The mode matching method has been applied in the study of PC-based waveguide discontinuities. The method is based in the expansion of the field in terms of the eigenmodes of the cells of the structure and their matching at the boundary interfaces. At a given frequency the modes are calculated by an alternative formulation of the plane wave expansion method. The MM method was verified by comparing it to

FDFD and FDTD simulations for various structures. Compared to FDFD the MM method requires much less memory while compared to the FDTD it requires less computational time.

The accuracy of two CMT formulations and the MM method for the estimation of the scattering due to geometric perturbations inside a PCW were investigated. This proves the applicability of MM in the analysis of PCW discontinuities especially for small perturbations. On the other hand, it was shown that the CMT can provide only a first approximation in the order of magnitude of the power scattering due to small perturbations.

The possibility of achieving nanosecond order delays, near the band edges of 2D photonic crystal waveguides was numerically investigated. Linear pulse propagation was shown to be severely impaired by second order dispersion. In the nonlinear regime, both bright and dark optical soliton pulses can be used to provide nanosecond delays for optical signals up to 100Gb/s. Third and higher order dispersion was shown not to significantly affect the performance of the soliton delay line. The influence of optical loss was also investigated and it was numerically shown that high bit rate soliton signals require low loss in order to limit their broadening factor.

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