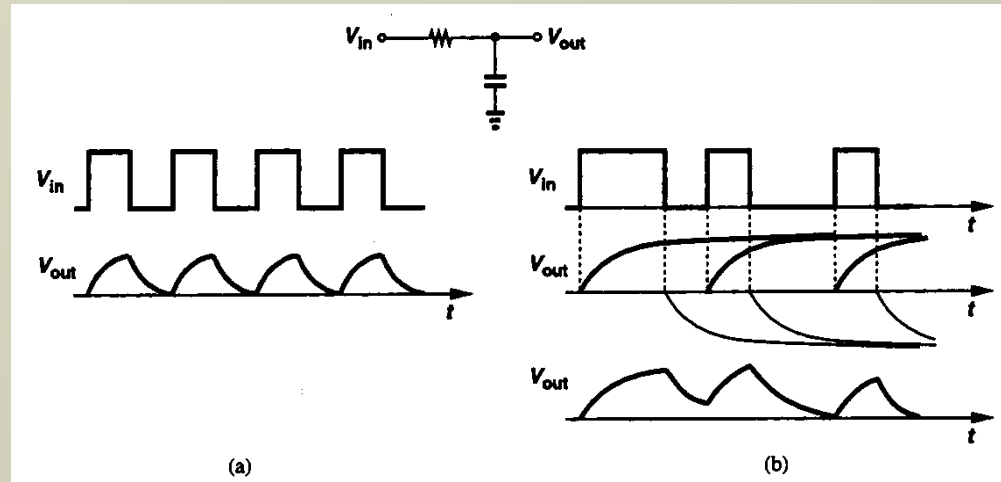


Αλληλεπίδραση Συμβόλων (Intersymbol Interference-ISI)

Μπορεί να εμφανιστεί σε γραμμικά, χρονικά αναλλοίωτα συστήματα με περιορισμένο εύρος ζώνης.

Λύση: Μορφοποίηση του παλμού στον πομπό (Nyquist signaling)
Εξίσωση του παλμού στον δέκτη (Equalization)



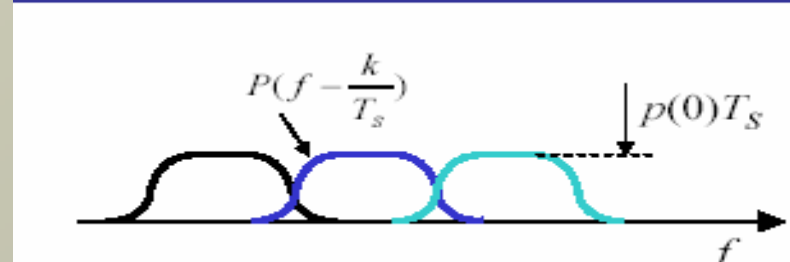
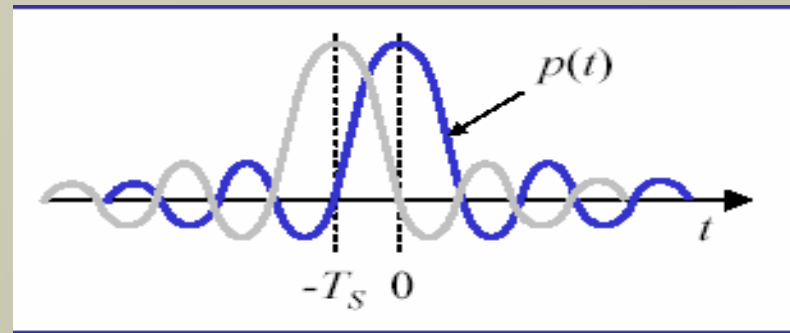
Μορφοποίηση παλμού (pulse shaping)

Για μηδενικό ISI πρέπει
$$p(kT_s) = 1 \quad \text{if } k = 0$$
$$= 0 \quad \text{if } k \neq 0$$

Δειγματοληψία του $p(t)$ δίνει:
$$p(t) \cdot \sum \delta(t - kT_s) = \delta(t)$$

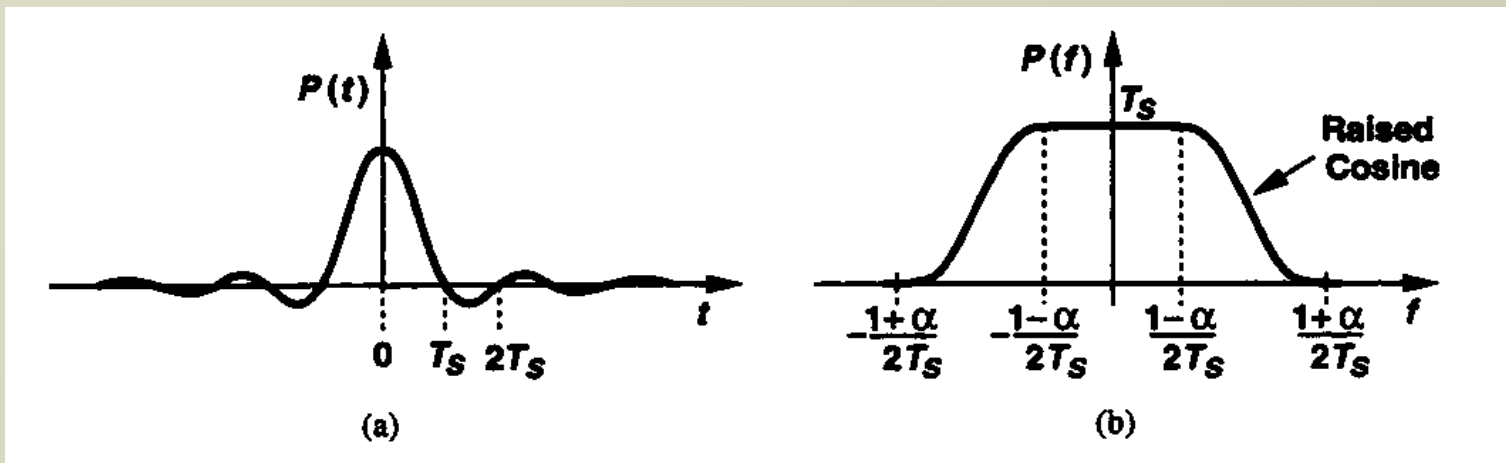
Ο μετασχηματισμός Fourier στα δύο μέλη:
$$P(f) * \frac{1}{T_s} \sum \delta\left(f - \frac{k}{T_s}\right) = 1 \Rightarrow$$

$$\frac{1}{T_s} \sum P\left(f - \frac{k}{T_s}\right) = 1$$



Ο παλμός sinc ικανοποιεί αυτές τις συνθήκες.

Προτιμάται το raised cosine.

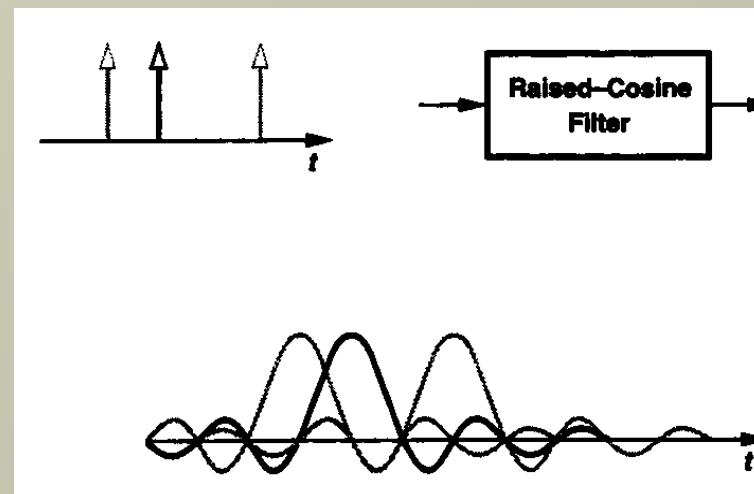


$$p(t) = \frac{\sin(\pi t/T_s) \cos(\pi \alpha t/T_s)}{\pi t/T_s \sqrt{1 - 4\alpha^2 t^2/T_s^2}}$$

and

$$P(f) = T_s \begin{cases} 0 < |f| < \frac{1-\alpha}{2T_s} \\ \frac{T_s}{2} \left[1 + \cos \frac{\pi T_s}{\alpha} \left(|f| - \frac{1-\alpha}{2T_s} \right) \right] & \frac{1-\alpha}{2T_s} < |f| < \frac{1+\alpha}{2T_s} \\ 0 & |f| > \frac{1+\alpha}{2T_s} \end{cases}$$

όπου $0 < \alpha < 1$

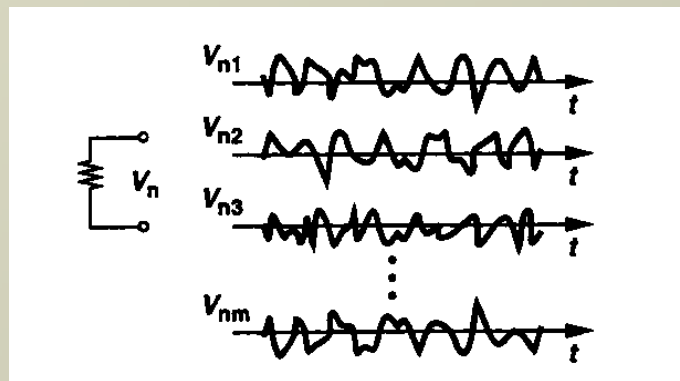
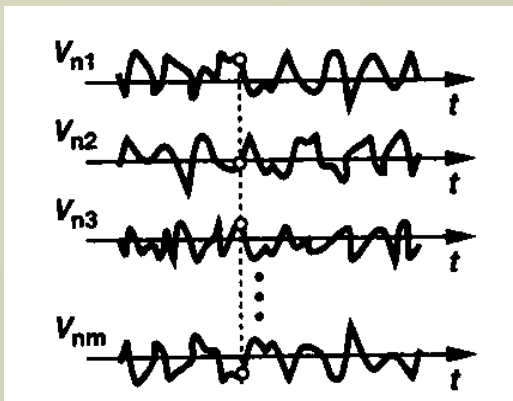


Τυχαίες Διαδικασίες και Θόρυβος

Τυχαίες διαδικασίες

• Στατιστικά Σύνολα

$$\langle n(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t) dt$$



$$\overline{n(t)} = \int_{-\infty}^{+\infty} n(t) P_n(n) dn$$

όπου $P_n(n)$ είναι η συνάρτηση πυκνότητας πιθανότητας της διαδικασίας.

$$\langle n^2(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} n^2(t) dt$$

$$\overline{n^2(t)} = \int_{-\infty}^{+\infty} n^2(t) P_n(n) dn$$

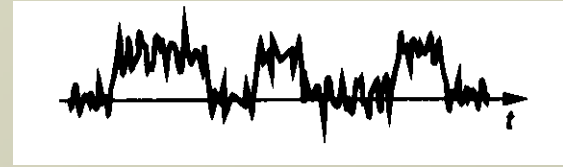
$$\langle n^2(t) \rangle = \overline{n^2(t)}$$



•Συνάρτηση Πυκνότητας Πιθανότητας (PDF)

$P_x(x)dx = \text{probability of } x < X < x + dx$

όπου X είναι η μετρούμενη τιμή του $x(t)$ σε κάποιο σημείο του χρόνου



Γκαουσιανή ή κανονική κατανομή

$$P_x(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-m)^2}{2\sigma^2}\right\}$$

όπου σ και m είναι η τυπική απόκλιση και η μέση τιμή της κατανομής, αντίστοιχα

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-m)^2}{2\sigma^2}\right\} dx$$

$$\text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp\left\{-\frac{u^2}{2}\right\} du$$

$m - \sigma$ $m + \sigma$ 68%

$m - 3\sigma$ $m + 3\sigma$ 99%

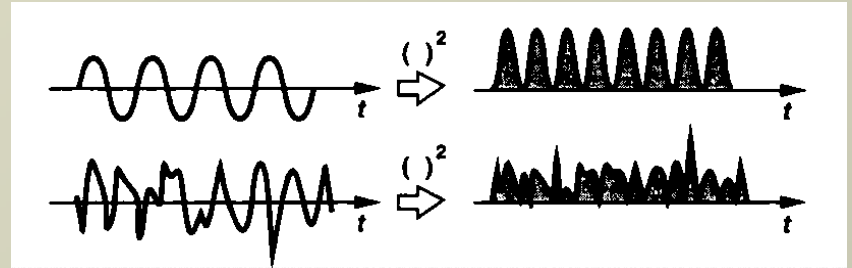


•Φασματική Πυκνότητα Ισχύος (PSD)

Ο μετασχηματισμός Fourier της $x(t)$ είναι:

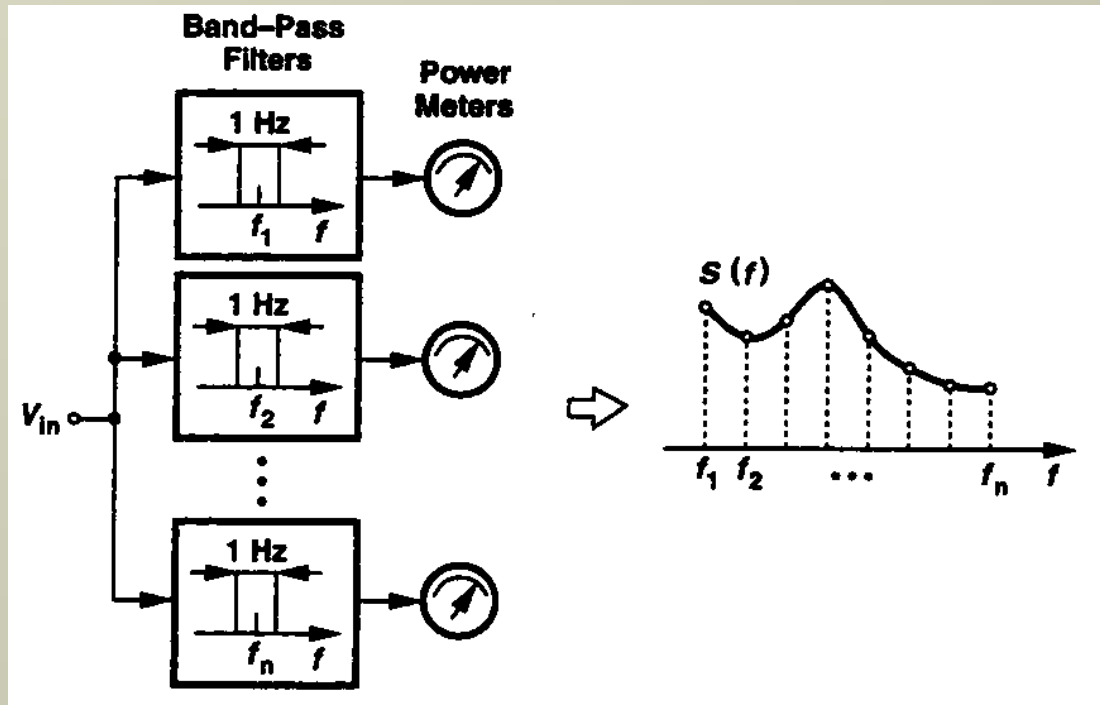
$$X(f) = \int_{-\infty}^{+\infty} x(t) \exp(-j2\pi ft) dt$$

Μόνο αν $E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty$



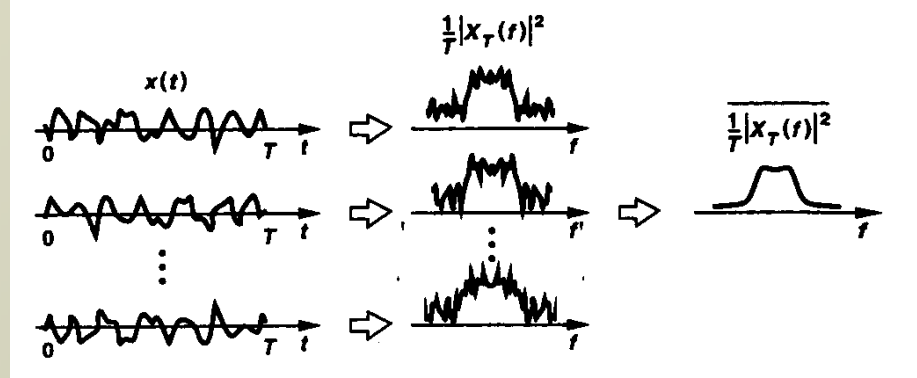
Ή έστω αν $P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} |x(t)|^2 dt < \infty$

Δεν ισχύει στα τυχαία σήματα



$$S_x(f) = \lim_{T \rightarrow \infty} \frac{\overline{|X_T(f)|^2}}{T}$$

$$X_T(f) = \int_0^T x(t) \exp(-j2\pi ft) dt$$

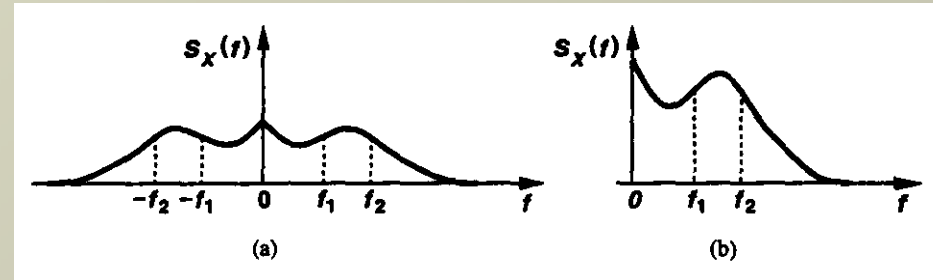


$$\int_{-f_2}^{-f_1} S_x(f) df + \int_{f_1}^{f_2} S_x(f) df = \int_{f_1}^{f_2} 2S_x(f) df$$

Παράδειγμα: τάση θερμικού θορύβου αντίστασης R

$$S_x(f) = 2kTR$$

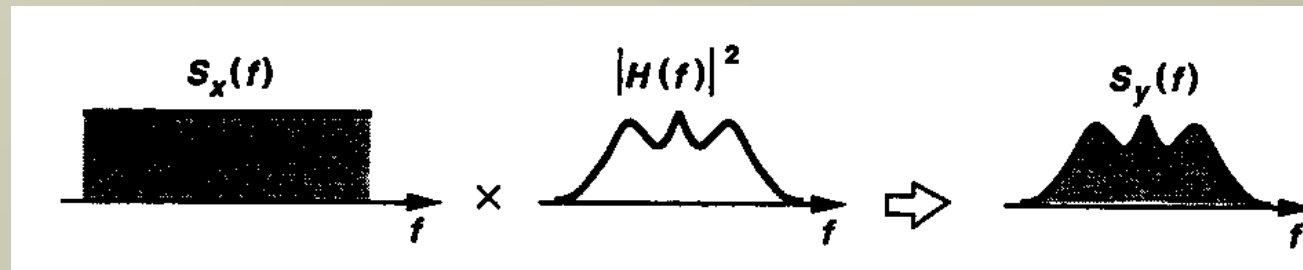
$$\overline{V_n^2} = 4kTR \cdot \Delta f$$



• Τυχαία Σήματα στα Γραμμικά Συστήματα

$$S_y(f) = S_x(f) |H(f)|^2$$

$$H(f) = H(s = j2\pi f)$$



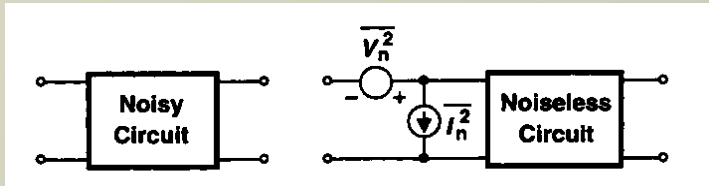
Θόρυβος

Θερμικός θόρυβος στα MOS $\overline{V_n^2} = 4kT \left(\frac{2}{3} g_m \right)$

Θόρυβος βολής $\overline{I_n^2} = 2qI$

Θόρυβος flicker $\overline{V_n^2} = \frac{K}{WLC_{OX}} \frac{1}{f}$

•Θόρυβος Ανηγμένος στην Είσοδο



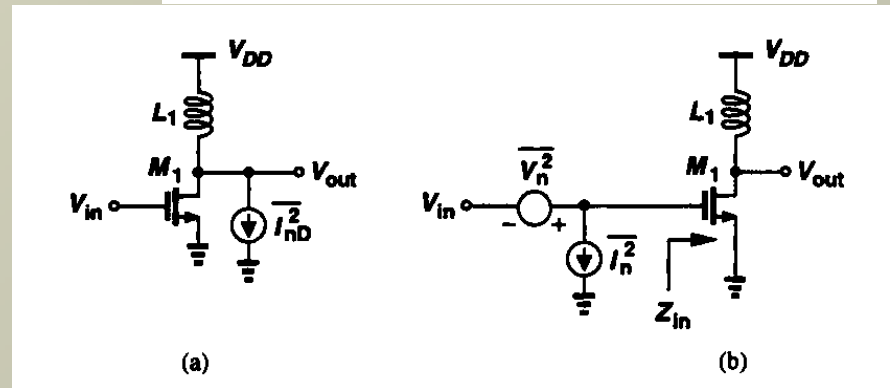
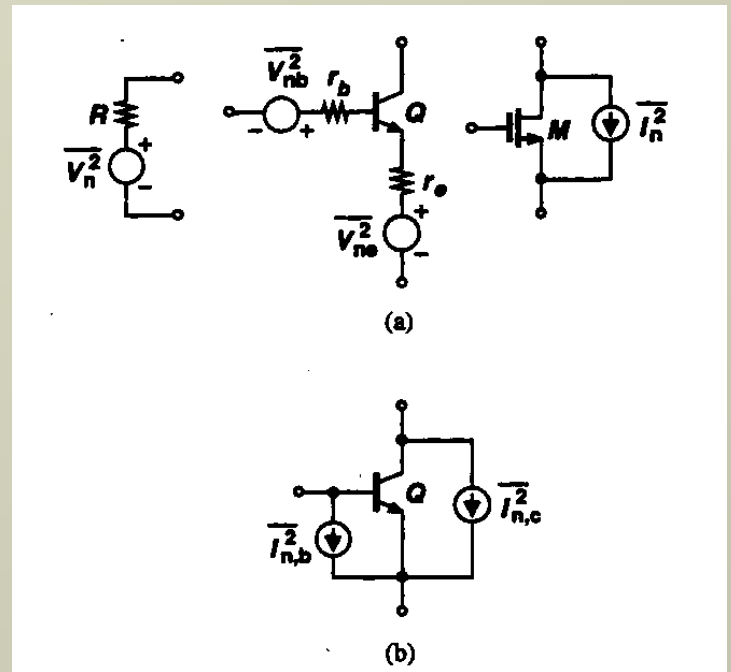
$$g_m^2 \overline{V_n^2} = \overline{I_{nD}^2}$$

$$g_m^2 \overline{I_n^2} |Z_{in}|^2 = \overline{I_{nD}^2}$$

$$\overline{I_{nD}^2} = 4kT(2g_m/3)$$

$$\overline{V_n^2} = 8kT/3g_m$$

$$\overline{I_n^2} = 8kT/(3g_m |Z_{in}|^2)$$



•Εικόνα Θορύβου

$$\text{noise figure} = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}}$$

Υπολογισμός της εικόνας θορύβου

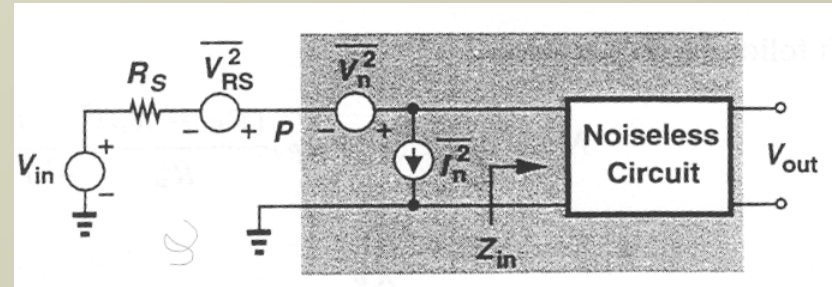
$$\text{SNR}_{\text{in}} = \frac{a^2 V_{\text{in}}^2}{a^2 V_{\text{RS}}^2}$$

$$\text{SNR}_{\text{out}} = \frac{a^2 A_u^2 V_{\text{in}}^2}{\left[V_{\text{RS}}^2 + (V_n + I_n R_s)^2 \right] a^2 A_u^2} = \frac{V_{\text{in}}^2}{\left[V_{\text{RS}}^2 + (V_n + I_n R_s)^2 \right]}$$

$$\text{NF} = \frac{\overline{V_{\text{RS}}^2} + \overline{(V_n + I_n R_s)^2}}{\overline{V_{\text{RS}}^2}} = 1 + \frac{\overline{(V_n + I_n R_s)^2}}{\overline{V_{\text{RS}}^2}} \qquad \text{NF} = 1 + \frac{\overline{(V_n + I_n R_s)^2}}{4kTR_s}$$

$$\text{NF} = \frac{4kTR_s + \overline{(V_n + I_n R_s)^2}}{4kTR_s} = \frac{A^2 \left[4kTR_s + \overline{(V_n + I_n R_s)^2} \right]}{A^2} \frac{1}{4kTR_s} = \frac{V_{n,\text{out}}^2}{A^2} \frac{1}{4kTR_s}$$

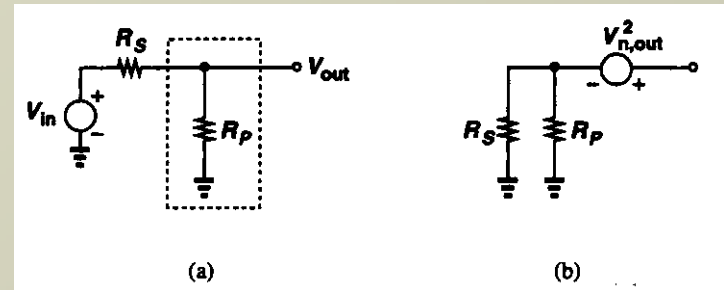
Όπου $A = aA_u$



α) Παράδειγμα υπολογισμού εικόνας θορύβου

$$V_{n,out}^2 = 4kT(R_S \parallel R_P) \quad A = \frac{R_P}{R_S + R_P}$$

$$NF = 4kT(R_S \parallel R_P) \frac{(R_S + R_P)^2}{R_P^2} \frac{1}{4kTR_S} = 1 + \frac{R_S}{R_P}$$



β) Παράδειγμα υπολογισμού εικόνας θορύβου

$$R_S = \frac{1}{g_{m2}} \frac{1}{1 + g_{m1} R_D} \quad (I_{n2} R_S / 2) g_{m1} R_D$$

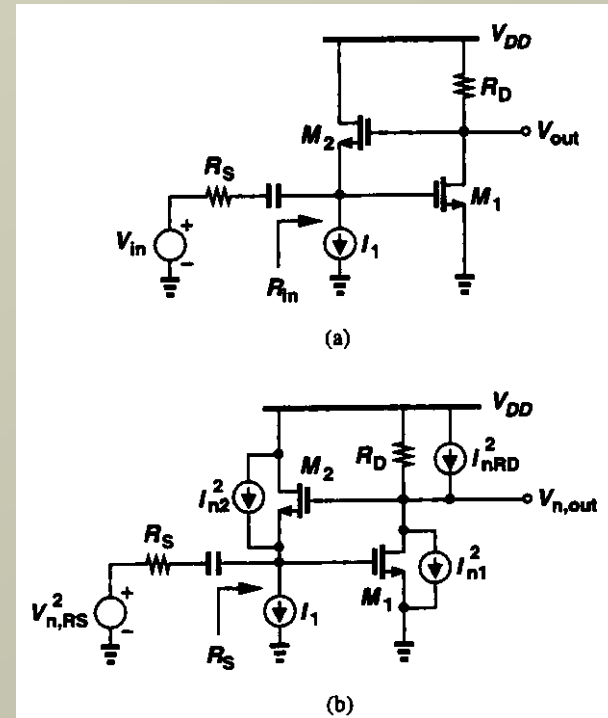
$$R_{out} = \frac{R_D}{2} (1 + g_{m2} R_S)$$

$$V_{n,out}^2 = 4kTR_S \left(\frac{1}{4} g_{m1}^2 R_D^2 \right) + \frac{1}{4} I_{n2}^2 R_S^2 g_{m1}^2 R_D^2 + (I_{RD}^2 + I_{n1}^2) \frac{R_D^2}{4} (1 + g_{m2} R_S)^2$$

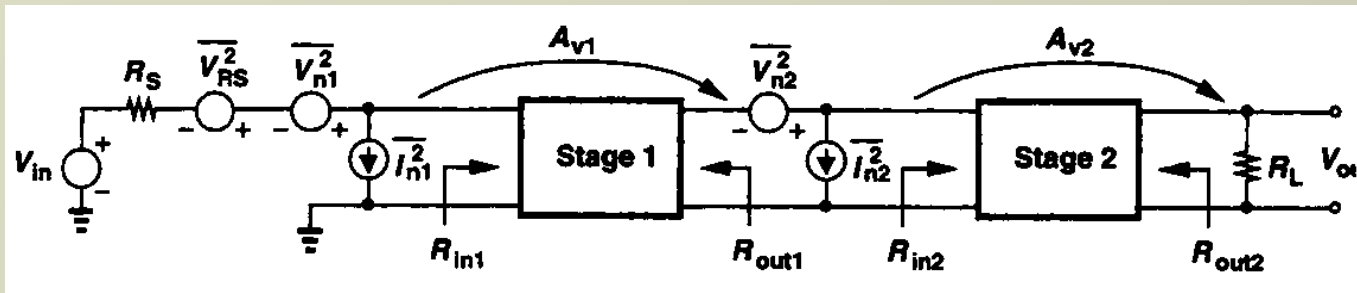
$$= kTR_S g_{m1}^2 R_D^2 + \frac{2}{3} kT g_{m2} R_S^2 g_{m1}^2 R_D^2 + \left(\frac{4kT}{R_D} + \frac{8}{3} kT g_{m1} \right) \frac{R_D^2}{4} (1 + g_{m2} R_S)^2$$

$$NF = \frac{V_{n,out}^2}{A^2} \frac{1}{4kTR_S} = 1 + \frac{2}{3} g_{m2} R_S + \left(\frac{1}{R_D} + \frac{2}{3} g_{m1} \right) \frac{(1 + g_{m2} R_S)^2}{g_{m1}^2 R_S}$$

$$g_{m2} R_S = (1 + g_{m1} R_D)^{-1}$$



•Εικόνα Θορύβου Διαδοχικών Βαθμίδων



$$V_{n,in1}^2 = \left[I_{n1}(R_S \parallel R_{in1}) + V_{n1} \frac{R_{in1}}{R_{in1} + R_S} \right]^2 + \overline{V_{RS}^2} \frac{R_{in1}^2}{(R_{in1} + R_S)^2}$$

$$V_{n,in2}^2 = V_{n,in1}^2 A_{u1}^2 \left(\frac{R_{in2}}{R_{out1} + R_{in2}} \right)^2 + \left[I_{n2}(R_{out1} \parallel R_{in2}) + V_{n2} \frac{R_{in2}}{R_{in2} + R_{out1}} \right]^2 + A_{u2}^2 V_{n,in2}^2 R_L^2 / (R_L + R_{out2})^2$$

$$A_{u,tot} = \frac{R_{in1}}{R_S + R_{in1}} A_{u1} \frac{R_{in2}}{R_{out1} + R_{in2}} A_{u2} \frac{R_L}{R_{out2} + R_L}$$

$$NF_{tot} = \frac{1}{A_{u,tot}^2} A_{u2}^2 \left(\frac{R_L}{R_L + R_{out2}} \right)^2 V_{n,in2}^2 \frac{1}{4kTR_S}$$

$$NF_{tot} = \frac{4kTR_S + (I_{n1}R_S + V_{n1})^2}{4kTR_S} + \frac{(I_{n2}R_{out1} + V_{n2})^2}{A_{u1}^2} \frac{1}{\left(\frac{R_{in1}}{R_S + R_{in1}} \right)^2} \frac{1}{4kTR_S}$$

$$NF_{tot} = NF_1 + \frac{(I_{n2}R_S + V_{n2})^2}{a^2 A_{u1}^2} \frac{1}{4kTR_S} = NF_1 + \frac{NF_2}{a^2 A_{u1}^2}$$

$$P_{out,av} = V_{in}^2 \left(\frac{R_{in1}}{R_S + R_{in1}} \right)^2 A_{u1}^2 \frac{1}{4R_{out1}}$$

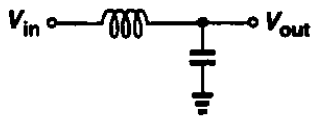
$$P_{source,av} = \frac{V_{in}^2}{4R_S}$$

$$A_p = \left(\frac{R_{in1}}{R_S + R_{in1}} \right)^2 A_{u1}^2 \frac{R_S}{R_{out1}}$$

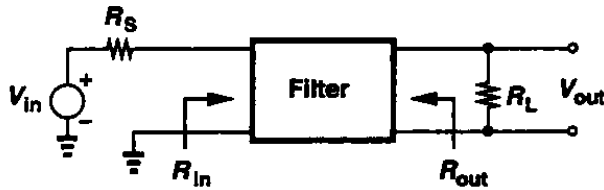
$$NF_{2,R_{out1}} = 1 + \frac{(I_{n2}R_{out1} + V_{n2})^2}{4kTR_{out1}}, \quad NF_{tot} = NF_{1,RS} + \frac{NF_{2,R_{out1}} - 1}{A_p}, \quad NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{p1}} + \dots + \frac{NF_m - 1}{A_{p1} \dots A_{p(m)}}$$



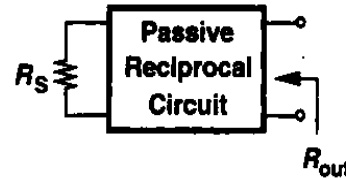
•Εικόνα Θορύβου Κυκλωμάτων με Απώλειες



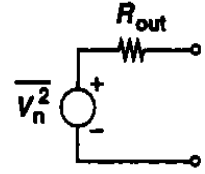
(a)



(b)



(a)



(b)

$$P_{in} = V_{in}^2 / (4R_S)$$

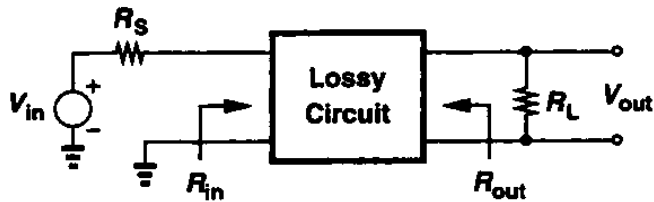
$$P_{out} = V_{TH}^2 / (4R_{out})$$

$$L = \frac{V_{in}^2}{V_{TH}^2} \frac{R_{out}}{R_S}$$

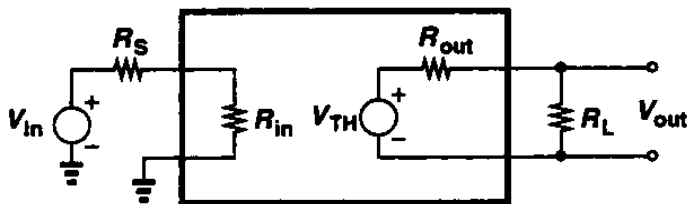
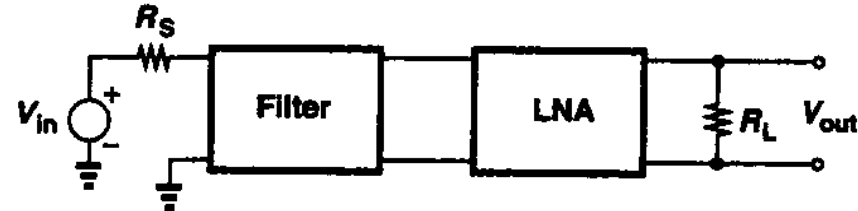
$$V_{n,out}^2 = 4kTR_{out} \frac{R_L^2}{(R_L + R_{out})^2}$$

$$A_u = \frac{V_{TH}}{V_{in}} \frac{R_L}{R_L + R_{out}}$$

$$NF = 4kTR_{out} \frac{V_{in}^2}{V_{TH}^2} \frac{1}{4kTR_S} = L$$



(a)



(b)

$$NF_{tot} = NF_{filt} + \frac{NF_{LNA} - 1}{L^{-1}} = L + (NF_{LNA} - 1)L = L \cdot NF_{LNA}$$



Ευαισθησία και Δυναμική Περιοχή

•Ευαισθησία

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig}/P_{RS}}{SNR_{out}} \quad P_{sig} = P_{RS} \cdot NF \cdot SNR_{out} \quad P_{sig,tot} = P_{RS} \cdot NF \cdot SNR_{out} \cdot B$$

$$P_{in,min} |_{dBm} = P_{RS} |_{dBm/Hz} + N |_{dB} + SNR_{min} |_{dB} + 10 \log B$$

$$P_{RS} = \frac{4kTR_S}{4} \cdot \frac{1}{R_{in}} = kT = -174dBm/Hz \quad P_{in,min} = -174dBm/Hz + NF + 10 \log B + SNR_{min}$$

•Δυναμική Περιοχή

$$P_{IIP3} = P_{in} + \frac{P_{out} - P_{IM,out}}{2} \quad P_{out} = P_{in} + G \quad P_{IM,out} = P_{IM,in} + G$$

$$P_{IIP3} = P_{in} + \frac{P_{in} - P_{IM,in}}{2} = \frac{3P_{in} - P_{IM,in}}{2}$$

$$P_{in} = \frac{2P_{IIP3} + P_{IM,in}}{3} \quad P_{in,max} = \frac{2P_{IIP3} + F}{3}$$

$$SFDR = \frac{2P_{IIP3} + F}{3} - (F + SNR_{min}) = \frac{2(P_{IIP3} - F)}{3} - SNR_{min}$$

