Estimation of future prepaid – postpaid mobile telephony equilibrium in the OECD area

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Abstract— **The objective of the present research is to build an empirical Lotka-Volterra system using real-time data and information from the postpaid and prepaid mobile telecommunications subscribers in the OECD area. Genetic algorithms are used in order to specify the appropriate coefficients of the corresponding predator-prey competition system, where the pre-paid subscribers hold the role of the predator. The results show that both the postpaid the prepaid subscribers of the area under investigation are in the process of reaching long-run stable coexistence equilibrium. The final postpaid – prepaid market share predictions are 58,8% and 41,2%, respectively.**

***Index Terms*—Competition, Lotka–Volterra, Genetic Algorithm, Prepaid mobile telephony.**

I. Introduction

The innovation diffusion literature has established that the spread of a successful innovation over time typically follows a sigmoid curve. Fourt and Woodlock [1], Mansfield [2], Floyd [3], Rogers [4], Chow [5] and Bass [6] were the first to consider the modeling of a technology's diffusion. Their work has encouraged many modifications often adopted and studied, even in recent research efforts. The common element in all these classic models is that they assume a monopolistic market. Furthermore, except for a few recent papers [7], [8], [9], [10]. [11], there remains a scarcity of the literature considering the competition mechanism in the diffusion process.

Therefore, it is necessary to adopt a model that incorporates competition explicitly. The sigmoid diffusion shape of mobile telecommunications penetration is the result of the dynamics of two competing services: Prepaid and postpaid. Their interaction defines the long-term evolution of the total technology and their rivalry is anticipated to settle down to an equilibrium point that will be more-or-less stable.

During the analysis of an ecosystem, the equilibrium analysis of dynamic predator–prey systems is one of the most important stages, which is described based on appropriate systems of Lotka–Volterra equations [12], the latter used to model the system dynamics [13].

In this paper, the Lotka–Volterra equation, which was developed to model the interaction between the two competing species based on the logistic curve, will be considered as an alternative competitive diffusion model for analysing the mobile phone market. Applications of the Lotka–Volterra equations to the analysis of technology diffusion in a competitive market can be found in relevant literature [14--16].

The purpose of this paper is the estimation of the dynamic competitive relationship in the mobile phone market, including the existence of an equilibrium point and its stability with the estimated demand function, using the Lotka–Volterra model. To the best of the authors' knowledge, this is the first application of the Lotka – Volterra model over prepaid and postpaid competition of mobile telephony.

The organization of this paper is as follows: in the next section, a brief overview of the prepaid mobile telecommunications is presented, followed by a more specific insight in the OECD area. Next, the Lotka –Volterra prey-predator system and the Genetic Algorithm are described. The fourth section presents the application of the method in the area of interest. The paper concludes with the discussion on the application's results and the proposals for further research.

II. Prepaid Mobile Telephony in the OECD area

A. Prepaid Mobile Telephony

Mobile telecommunications continue to be one of the world's largest growth industries. Yet, as mobile technology matures and standardises, mobile services become a commodity and competition intensifies especially in developed and open markets [17-18]. In addressing market competition, mobile service providers typically offer two forms of services: 1) prepaid, where customers prepay an amount that reduces on actual usage and 2) postpaid, where customers agree to a minimum fixed monthly spend and pay at the end of the month.

Mobile firms often entice customers to become postpaid via better usage rates and mobile handset subsidies [17, 19, 21]. This is because of revenue from postpaid services are more assured and predictable, a particularly important factor for mobile telecommunications where network infrastructure incurs huge fixed operating cost regardless of short-term

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utilisation. In return, customers commit to contracts with premature termination penalties. Its prevalent and continual use attests to the effectiveness of the subsidy-contract combination as a customer acquisition and retention strategy [17-19, 21]. Prepaid customers, in contrast, enjoy the flexibility of no minimum periodic spend or contractual obligation to a service provider.

Concluding, the advantages of choosing a prepaid service include the avoidance of credit check, service fee, deposit, credit check, monthly rental fee and termination fee. On the other hand, service charges are usually more expensive than using postpaid service and, finally, there exists a time limit on the credit of the user.

B. *Mobile in the OECD area*

Over the previous decade mobile has been the most important growth area for OECD telecommunication operators but many markets are nearing mobile saturation levels. There were 96,1 mobile subscribers per 100 inhabitants at the end of 2007 so operators will face an increasingly difficult time attracting new customers and will need to migrate customers to 3G services and focus more on growing mobile data markets.

Mobile revenues account for nearly half of all telecommunication revenues (41% in 2007), up from 22% ten years earlier. Most of this revenue growth is from new subscriptions as revenues per mobile subscriber have remained relatively stable since 2000.

The percentage of prepaid mobile subscriptions to total mobile subscriptions continues to grow and just under half of all mobile subscriptions are prepaid. Interestingly, the percentage of prepaid subscribers varies widely among OECD countries. In Japan and Korea, only 2% of mobile subscriptions are prepaid. By contrast, prepaid accounts for 92% of subscribers in Mexico and 89% in Italy. Finally, only nine countries had less than one mobile subscription per person. Japan, Korea and the United States all had less than 100% penetration largely due to their relatively lower percentages of prepaid accounts [22].

The following figure illustrates the historical evolution of postpaid, prepaid and total Mobile Penetration in the OECD area:

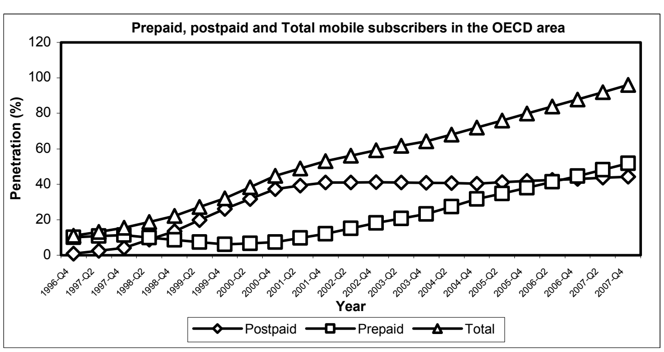

Fig. 1: Postpaid, Prepaid and Total Mobile Penetration in the OECD area

III. Prey – Predator Model and Genetic Algorithms

A. *Prey – Predator Lotka – Volterra model*

The Lotka-Volterra model is one of the earliest predator-prey models to be based on sound mathematical principles. The model is composed of a pair of differential equations that describe predator-prey dynamics in their simplest case (one predator population, one prey population). It was developed independently by Alfred Lotka and Vito Volterra in the 1920's.

The model makes several simplifying assumptions: 1) the prey population will grow logistically when the predator is absent; 2) the predator population will starve in the absence of the prey population; 3) the intensity of interaction between the prey and the predator is proportional to the product of the two populations and 4) there is no environmental complexity (in other words, both populations are moving randomly through a homogeneous environment).

According to these assumptions, the dynamics of the system can be illustrated through the following set of first order, non-linear, autonomous differential equations:

$$\frac{dx(t)}{dt} = r \cdot x(t) \cdot \left(1 - \frac{x(t)}{K}\right) - b \cdot x(t) \cdot y(t) \qquad \text{Equation 1}$$

$$\frac{dy(t)}{dt} = -c \cdot y(t) + d \cdot x(t) \cdot y(t) \qquad \text{Equation 2}$$

In the above system, $dx(t)/dt$, $dy(t)/dt$ are the transition rates of the prey and predator populations, respectively, while *r, K, b, c* and *d* are all positive parameters. The *r* parameter refers to the prey, *K* is the carrying capacity of the environment (maximum population in saturation state) and the parameter *c* refers to the reduction of the predator's population due to internal competition (intraspecies). Finally, the parameters *b* and *d* reflect the volume of interaction between the species (interspecies).

In the above mentioned system of autonomous differential equations each solution can be represented by a curve (orbit). The main purpose is the investigation of the behavior of the solutions of the system for the initial values of *x(t)* and *y(t)*. The analysis of the system results to three critical points:

$$(0,0),\ (K,0) \text{ and } \left(\frac{c}{d}, \frac{r}{b}\left(1 - \frac{c}{dK}\right)\right).$$

They come up by setting both derivatives of the system equal to zero.

B. *Genetic Algorithms*

For the solution of the Lotka – Volterra system in the specific application, Genetic Algorithms (GA) are used. GA are search algorithm based on the mechanisms of natural selection and natural genetics [23]. In other words, a genetic algorithm iterates toward a global solution through a process that in many ways is analogous to the Darwinian process of natural selection.

The mechanics of a genetic algorithm are surprisingly simple, involving nothing more complex than copying solution vectors (strings) and swapping partial solution vectors (strings). Genetic algorithm is composed of three operators: reproduction, crossover, and mutation.

1. *Reproduction* is a process in which individual strings of a generation (parent generation) are copied to the next generation (child) according to their objective function values. This operator is an artificial version of natural selection, a Darwinian survival of the fittest among string creatures.

2. After reproduction, simple *crossover* may proceed in two steps. First, members of the newly reproduced strings (or new generation) are paired at random. Second, each pair of strings undergoes crossing over.

3. *Mutation* is the process of randomly changing a cell in the string or the solution vector. Mutation is the process by which the algorithm attempts to ensure a globally optimal solution. If the algorithm is trapped in a local minimum, the mutation operator randomly shifts the solution to another point in the search space, thus removing itself out of the trap.

The above steps are repeated until the algorithm is halted. The decision to halt the program can depend either on a prefixed number of generations, the time elapsed in the evolutionary process or the difference in solutions produced between two generations.

IV. Evaluation And Results

The following table illustrates the market share of prepaid – postpaid mobile subscribers' evolution in the OECD area for the years 1996-2007:

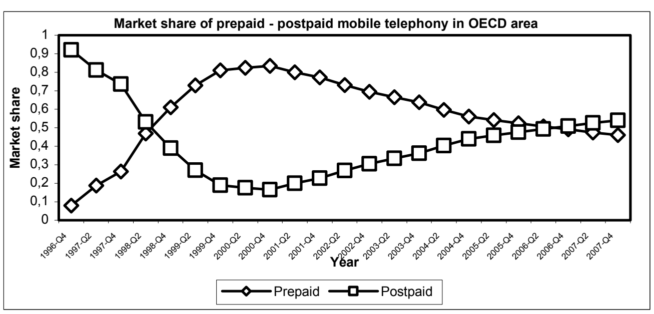

Fig. 2:Market share of prepaid – postpaid mobile telephony in the OECD area

As depicted in Fig. 2, the prepaid service started gaining market share rapidly in the period 1996 – 1997 and exceeded postpaid service's market share in 1998. Since then, the two types of service systematically approach a more-or-less stable situation, where the rivalry seems to settle down, resulting to a market equilibrium.

The following table presents the actual versus the estimated market shares for the two types of service in the specific area and for the period under investigation:

TABLE I

Actual and Estimated Market shares for Postpaid and Prepaid Subscribers in the OECD Area (Source: OECD database)

Year	Postpaid -Actual	Prepaid -Actual	Postpaid - Estimated	Prepaid -Estimated
1996-Q4	0,92	0,08	0,921	0,079
1997-Q2	0,81	0,19	*0,921*	*0,079*
1997-Q4	0,74	0,26	*0,812*	*0,188*
1998-Q2	0,53	0,47	*0,735*	*0,265*
1998-Q4	0,39	0,61	*0,532*	*0,468*
1999-Q2	0,27	0,73	*0,390*	*0,610*
1999-Q4	0,19	0,81	*0,271*	*0,729*
2000-Q2	0,18	0,82	*0,189*	*0,811*
2000-Q4	0,17	0,83	*0,175*	*0,825*
2001-Q2	0,20	0,80	*0,165*	*0,835*
2001-Q4	0,23	0,77	*0,199*	*0,801*
2002-Q2	0,27	0,73	*0,228*	*0,772*
2002-Q4	0,31	0,69	*0,269*	*0,731*
2003-Q2	0,34	0,66	*0,306*	*0,694*
2003-Q4	0,36	0,64	*0,335*	*0,665*
2004-Q2	0,40	0,60	*0,362*	*0,638*
2004-Q4	0,44	0,56	*0,403*	*0,597*
2005-Q2	0,46	0,54	*0,439*	*0,561*
2005-Q4	0,48	0,52	*0,458*	*0,542*
2006-Q2	0,49	0,51	*0,475*	*0,525*
2006-Q4	0,51	0,49	*0,493*	*0,507*
2007-Q2	0,53	0,47	*0,509*	*0,491*
2007-Q4	0,54	0,46	*0,525*	*0,475*

The last two tables, Table 2 and Table 3, depict the estimated values for the parameters of the prey – predator system after the application of genetic algorithm and the final estimated equilibrium, in terms of market shares, respectively. The forecasts indicate that the postpaid type of service will prevail, settling down to an approximation of 58,8% of market share.

TABLE II

Estimated Values for the parameters of the Lotka – Volterra System after the application of Genetic Algorithm

Paramete r	Estimated Value
r	5
K	1
b	5
c	7,81E-15
d	1,32E-14

TABLE III

Estimated Market Share Equilibrium between prepaid and postpaid mobile telephony in the OECD area

Type	Market Share
Prepaid	41,2%
Postpaid	58,8%

The application of GA for the examined system assumed for the fitness (objective) function to be the minimization of the Mean Square Error (MSE), between observed and estimated values. Moreover, the initial values of the participating parameters were set equal to the values obtained by the NLS's results, if each species operated into the market in the absence of its competitor. The confidence interval of the parameters estimates indicates the range each parameter would be expected to fell into. The stopping condition of the genetic algorithm was set to be the performance of at least 100.000 iterations, or when the reduction value would become less than 0.01% in 1000 generations. Finally, the population size was set to 500 individuals per generation, the crossover rate to 0.9 and the mutation rate to 0.01.

V. Conclusions and Future Research

This paper presented the application of the Lotka – Volterra model of prey – predator for the calculation of the mobile prepaid – postpaid equilibrium in the OECD area. Using the data published in the bi-annual publication of the Organization, the results indicate that the rivalry for greater market share acquisition will settle down to 58,8% for the postpaid and 42,2% for the prepaid subscribers. The accurate prediction of such information could be a valuable tool for telecommunications policy makers.

Nevertheless, this is just a speculation. The dynamic character diffusion of each type of service will always affect the level of equilibrium, although the changes are speculated not to affect the stable state significantly. There will always be consumers turning from prepaid to postpaid and back wise, depending on factors such as the Gross domestic Product (GDP) and the competition policy set by regulators, as well as providers. One should always outline that the Lotka – Volterra model makes some simplifying assumptions in any case of competition application.

Future research in the topic includes the inclusion of factors, such as the ones mentioned above, in the calculation of the equilibrium between prepaid and postpaid subscribers. Another major question that needs to be answered in the future is whether these two types of services are complementary or substitute goods. The use of Lotka – Volterra models and Genetic algorithm methodology can give answer to this question. The application of this method in other world areas and specific countries is also a topic of great interest.

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