Assessment of the gap and (non-)Internet users evolution based on population biology dynamics

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ABSTRACT

The evolution of Internet and non-Internet users as well as the dynamics of their divide is studied using population biology concepts. Users’ evolution and future trends are estimated by applying Lotka–Volterra model. The parameters of the proposed model are determined by both an analytical and a simulation method. The presented model is applied to two cases; Greece and Lithuania. The accuracy of the obtained results is confirmed through actual data. Internet users are constantly increasing while they outperform non-users in the last years. It is confirmed that the maximum growth rate of Internet users in both countries coincides to periods with effective regulation, broadband promotion, provision of bundle products and alternative operators’ investments. Model’s estimation and forecasting ability can be used as a valuable tool for decision and policy makers. Several policy guidelines are provided helping to achieve higher penetration levels.

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1. Introduction

In the last decades, ICT technologies and especially Internet4 have experienced an unrivaled evolution. Advanced applications such as e-commerce, teleconference, e-learning, telemedicine, video on demand and online gaming were implemented facilitating and improving human lives. The significant growth of Internet in terms of traffic volumes and users can be mainly attributed to the progress occurred in the field of computers and networks. Faster systems with increased memory have been designed and implemented allowing the production and processing of high amounts of information. At the same time broadband networks with high capacity have been adopted as a means to accommodate the increased traffic.

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4 A global computer network providing a variety of information and communication facilities, consisting of interconnected networks using standardized communication protocols.

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The extreme evolution of Internet can be described by several statistical data such as penetration rate and Internet users. It should be highlighted that the global growth rate of Internet users for 1/7/2000–1/7/2013 is 556%. Furthermore, the new figures show that, by the end of 2014, there will be almost 3 billion Internet users. However, there are also billions of people who have never used Internet because they do not want to or they have no access to it. The low average Internet penetration rate of only 38% for the first half of 2013 is highly indicative. Furthermore, a huge divide of Internet penetration rates is noted in 2013 between developing (29.9% of the population use Internet) and developed (75.7% of the population use Internet) regions.

It is a common belief that Internet and generally ICT is a major factor leading to socio-economic development (OECD, 2004). Furthermore, ICT can also play a significant role in economic growth (Jung, Na, & Yoon, 2013; Koutroumpis, 2009; Sassi & Goaied, 2013) and competition (WEF, 2009) for countries, enterprises and individuals. In detail, the use of ICT has a great impact on several fields such as trade, health and education and a power in creating new job possibilities. The creation of around one million jobs in Europe due to ICT and a broadband-related growth of economic activity of 850 billion Euros are expected between 2006 and 2015 (European Commission, 2009). On the other hand, ICT can help individuals and enterprises to remain competitive by doing things in a more efficient and effective way (European Commission, 2009).

However, inequalities of ICT investments, usage, skills and availability of infrastructures are detected affecting human development, economic growth and the creation of wealth (ITU, 2006). These inequalities are observed among both developing (Gulati & Yates, 2011) and developed (Cruz-Jesus, Oliveira, & Bacao, 2012) countries. Although diffusion keeps increasing, there are still inequalities in the rates of adoption as well as in the level of the provided digital opportunities (Mariscal, 2005).

It then become evident that more effort should be made towards this direction. This endeavor can be exemplified by the numerous programs implemented all over the world in order to boost the use of new technologies bridging the digital divide. Taking into account the observed gap, the European Union incorporated both digital convergence and the role of ICT in socio-economical structure in its strategic plans (European Commission, 2010a). The European structural funds spent almost 5.5 billion Euros on information society programs in the period 2000–2006 (Vicente & Gil-de-Bernabé, 2010) while 1 billion Euros of extra spending for broadband investments (especially for high-speed connections in rural regions) has already been budgeted (European Commission, 2009).

In the last years, there has been an increased attention for qualitative and quantitative investigation of Internet diffusion and usage (Chinn & Fairlie, 2006). Statistical information regarding individuals who regularly use Internet (at least once a week i.e. every day or almost every day or at least once a week but not every day, on average within the last 3 months before the survey) either at home, at work or at any other place (via fixed or mobile access), hereafter called Internet users, and those who have never used Internet before (hereafter called non-Internet users) is of great importance and high interest for the government, private and public sector to academics. These data are extremely useful in determining the current status and monitoring future evolution.

Special interest has also been paid for the investigation of digital divide. As described in the following section, several studies have been conducted regarding ICT inequalities (Cruz-Jesus et al., 2012) and digital convergence (Doong & Ho, 2012). It has been shown that several factors affect digital divide while a digital gap does exist even within developed countries (e.g. among countries of the European Union).

However, the majority of previous studies are limited to (a) individually investigate Internet or non-Internet users (Modis, 2005); (b) model and forecast Internet diffusion and/or digital divide as a whole (Mariscal, 2005; Michalakelis, Christodoulos, Varoutas, & Sphicopoulos, 2012) and (c) study the factors influencing both users and divide evolution (Li & Shiu, 2012), using statistical methods while ignoring the interaction between Internet and non-Internet users. Motivated by this literature gap, the evolution of Internet and non-Internet users is studied in the present work. Using historical data, this work aims to study the competition of these populations and answer the following research questions:

- How Internet and non-Internet users evolve?
- Which are the dynamics of their interaction?
- Is there an equilibrium?
- Will non-Internet users survive competition at the equilibrium?
- Is this the “end” of Internet diffusion?

Based on the evolutionary theory of population biology and dynamics, the evolution and competitive dynamics of Internet and non-Internet users are modeled, investigated and forecasted. In detail, the proposed model is based on Lotka–Volterra model describing the competition between species (Begon, Townsend, & Harper, 2006; Murray, 2007). The results obtained by the model can be supportive to other already used techniques providing a comparison reference confirming their results.

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5 According to Internet Word Stats, “An Internet User is anyone currently in capacity to use the Internet that is having: (a) available access to an Internet connection point and (b) the basic knowledge required to use web technology”.


Lotka Volterra model is a widely used model especially in biology. However, it has also been applied in several other areas, besides biology, providing precise estimates of the dynamics under consideration (Forys, 2009; Lee, Lee, & Oh, 2005; Ying & Shi, 2008). For example, in Lee et al. (2005), the dynamic relationship between two competing markets KSE and KOSDAQ was analyzed by estimating annual trajectory curves using daily data and Lotka–Volterra equations. The obtained results (coefficients) showed that the relationship between the two species – markets were changed in time from predator–prey, to symbiotic and finally to the pure competition. This alternation has been explained by several socio-economic factors such as government’s active support toward venture industry and investors’ interest in new market. Using equilibrium analysis, authors estimated an unstable point the Korean stock market.

Typical examples of the application of L-V model in telecommunications market (Kim, Lee, & Ahn, 2006; Lopez & Sanjuan, 2001; Michalakelis et al., 2012) where providers’ competitive behavior – market share are examined. An interesting study on the evolution of communication technologies using population dynamics was recently published (Baláž & Williams, 2012). An extensive analysis of the socio-economic state of Czech and Slovak Republics was performed. The evolution of both fixed lines and mobile phone subscriptions was studied and correlated to factors such as GDP. The obtained results were used to evaluate the coefficients of the proposed population dynamics model in several time periods influenced by different socio-economic changes. A comparison between the results of the two Republics was made. The resulted model has been finally solved using Runge–Kutta enabling the estimation of both fixed and mobile subscriptions. A good agreement was found between real and predicted values revealing the validity of the proposed model.

The proposed model is applied in the case of two European countries; Greece and Lithuania that are below European average values (Cruz-Jesus et al., 2012). The choice of these countries was not made arbitrary but was based on the grouping into similar digital profiles made by Cruz-Jesus et al. (2012). According to this procedure, all countries were grouped in terms of their ICT infrastructure and adoption by population (availability and use of ICT infrastructures by the people) and e-business and internet access costs (commercial use of ICTs and internet access cost) by examining variables compatible with recommendations from the OECD and the European Commission. Five different digital profiles in the EU are created resulting into grouping countries with similar digital imbalances. Greece and Lithuania fall in the same cluster named “firm-side and low access cost focused” presenting low availability and use of ICT infrastructure by their population and high commercial use of ICTs and internet access cost in comparison with “digital leaders” (Cruz-Jesus et al., 2012).

In addition, it is interesting to note that there are some differences regarding policy aspects as well as market characteristics. First of all it should be mentioned that there is a time lag between the two countries regarding the transformation and stabilization of their legal and regulatory framework. Lithuania started the transposition of the EU regulatory framework into the law of 2002 which came into force on 1 May 2004 leading to the complete liberalization of country’s telecommunications market. In the following years, there were numerous pieces of secondary legislation tackling the uncertainty of telecommunications market. Furthermore, Lithuania market is characterized by infrastructure-based competition notably in the form of fixed to mobile substitution. LLU has not taken-up and did not contribute to the development of the market due to several problems as well as to incumbent’s competitors own investments. The majority of broadband lines were not provided by DSL, and there was an increasing range of innovative service offerings to users of telecommunications services, including mobile broadband. Several projects dealing with the deployment of FTtx networks have already been completed.

On the other hand, in Greece, the transposition of the EU framework was delayed while several secondary legislation actions are still pending. This resulted in an uncertain environment with several complaints regarding the transparency of EETT decisions and discrimination in favor of the incumbent. The telecom market is also characterized by infrastructure-based competition. However, the take-up of the market can be attributed mainly to the LLU due to physical collocations and secondly to private and public funded projects dealing with the construction of optical networks. In the mobile market, it is characterized very competitive in both countries with Lithuania proceeding faster to investments and to the adoption of new technologies. Their similar digital profiles as well as the policy and market differences will facilitate the conduction of conclusions.

It was shown that the model gives very good fitting of the statistical data providing at the same time an accurate forecasting (it was compared to actual data of 2012) using the parameters obtained from both the analytical and simulation methods. The results showed that Internet (non-Internet) users as a share of population will continue to increase (decrease) in the following years tending/leading to a steady state.

Good performance of the proposed methodology would result in a twofold contribution. On the one hand, it would provide an alternative analysis and interpretation method of Internet usage data as well as of the interaction of Internet and non-Internet users. On the other hand, it would act as valuable tool for policy and decision makers saving money from expensive and frequently unnecessary investments and training of skills.

This paper is organized as follows. At first a literature review introduces digital divide and Internet usage. Then the proposed model describing the relationship and the evolution of Internet and non-Internet users is presented. Subsequently the solution procedure for the set of the nonlinear differential equations is described and details are given for the analytical method for the determination of model coefficients and the simulation method based on Artificial Bee Colony optimization.
algorithm. The linearization of the nonlinear problem as well as a closed form formula for the evolution of Internet and non-Internet users is derived in the following subsection. The results obtained by the application of the described model along with the two solving methods are finally presented and discussed before the concluding remarks.

2. Literature review – definitions

2.1. Population ecology dynamics

Population ecology is a sub-field of ecology that deals with the dynamics of species populations and how these populations interact with each other and with the environment (Hutchison, 1978; Silvertown, 2002). In the literature, one can find numerous works dealing with population ecology where several mathematical and statistical models, enabling the investigation of the behavior of populations that follow simplified rules or the estimation of several quantities such as survival rates, are described (Rafikov, Balthazar, & von Bremen, 2008; Royama, 1992).

The main goal in such studies is to estimate and forecast the population size. There are various well-known models for population growth evaluation describing the relationship between births, deaths, and the current population size. The first category of such models is the continuous time models where the exponential and the logistic models are the most common and simple models. In general, there are continuous time models for structured populations, using partial differential equations (deRoos, 1997) or delay-differential equations (Nisbet, 1997). The second category is the discrete time models such as matrix population models which are very popular in describing biologically important variability (Caswell, 2001). In case of chaotic dynamics several statistical methods have been used including linear or non-linear regression models (Turchin & Taylor, 1992), graphical analysis (Schaffer, 1985), and estimating Lyapunov exponents (Nychka, Ellner, Gallant, & McCaffrey, 1992). Contrary to deterministic models, there are also stochastic models that describe variations as extra random components or random rate changes (Dennis & Costantino, 1988; Nakaoka, 1996).

Apart from the models describing single species dynamics in an isolated population, one can also find models dealing with the dynamics of a population that interacts with other coexisting species. This category includes multispecies and spatial models. The predator–prey model, which is one of the main representatives of such models, can be attributed to the pioneering work in population ecology by Lotka (1925)\textsuperscript{10} and Volterra (1926).\textsuperscript{11}

2.2. Diffusion of innovations and new technologies

In the literature, there are numerous studies dealing with the modeling of the diffusion of innovations and new technologies, aiming to provide accurate estimates and forecasts. The first efforts in modeling diffusion dynamics appeared in 1825 (Gompertz, 1825) and were followed by the work of Bass (1969) along with the logistic family models presented by Bewley and Fiebig (1988) and Fisher and Pry (1971). In such works the life cycle of an innovative product is modeled from its launch until its possible rejection. Bell-shaped curves describe the frequency of adoption implying that the diffusion of innovations and new technologies at the aggregate level follows an S-shaped pattern.

Apart from these pioneering works, there are also extensions of the classical diffusion models studying the impact of marketing variables on the diffusion process. A representative example is the effect of prices on both adoption rate and market potential. An excellent and comprehensive review on this topic can be found in Ruiz-Conde, Leeflang, and Wieringa (2006).

Another extension focuses on integrating network externalities into diffusion models especially for network industries (Katz & Shapiro, 1985; Suomi, 2006). According to Economides (1996) network externalities influence the diffusion of a network product in two ways: (i) they accelerate the adoption rate since they increase the valuation that all consumers place to the network good; and (ii) they positively affect the total market size since higher valuation of the network good positively affects total demand, and hence, the overall penetration is higher for a given retail price. The impact of network externalities on the diffusion of innovations and new technologies is an open issue in the literature (Goldenberg, Libai, & Muller, 2010; Peres, Muller, & Mahajan, 2010; Rust, 2010; Tellis, 2010).

New models dealing with specific issues of innovations and new technologies diffusion were also derived by (a) incorporating several adjustments into the existing models such as incorporation of secondary market (Cho & Koo, 2012) and seasonality (Guidolin & Guseo, 2014) or (b) using advanced techniques and theories such as real options (Kumarabolu, Madlener, & Demirel, 2008), multi-optional modeling (Laciana & Oteiza-Aguirre, 2014) and fuzzy time series (Cheng, Chen, & Wu, 2009).


2.3. Digital divide, ICT adoption and Internet diffusion

In the mid 1990s, the term “digital divide” was introduced by former Assistant Secretary for Communications and Information of the U.S. Department of Commerce, and director of the National Telecommunications and Information Administration (NTIA), Larry Irving Jr. (Dragulansc, 2002). The term was used in order to distinguish those who can afford the necessary software and hardware from those who cannot access information services due to financial limitations. It can be deduced that the first definition had a binary nature, a choice between “has” and “has not” access to ICT resulting in a somehow inaccurate description of the term. Over the years and as new forms of information technology were occurred (van Dijk, 2006), the digital divide was enhanced by socio-economical factors (Vehovar, Sicherl, Hüsing, & Dolnicar, 2006), leading to a more precise but complex definition. According to the Organization for Economic Co-operation and Development – OECD (2001), the term digital divide refers to:

“the gap between individuals, households, businesses and geographic areas at different socio-economic levels with regard both to their opportunities to access information and communication technologies and to their use of the Internet for a wide variety of activities”.

The goal of Larry Irving Jr. to attract publics regarding "digital inequalities" was greatly achieved since, till now, numerous support programs were implemented and a plethora of studies were conducted focusing on the causes of access and use discrepancies.

Leaders from all over the world agreed to work towards a society “where everyone can create, access, utilize and share information and knowledge, enabling individuals, communities and people to achieve their full potential in promoting their sustainable development and improving their quality of life” (WSIS, 2005). The high importance of Information and Communications Technologies for Europe can be viewed by its action to include Digital Agenda in Europe 2020 Strategy. In this plan European Union describes its ambitions for 2020 that is “to reach a smart, sustainable and inclusive growth for European Economy (European Commission, 2010a) and “to exit the crisis and prepare the EU economy for the challenges of the next decade” (European Commission, 2010b). Towards this direction, the European Union has already and will continue to provide funds in order to develop a knowledge- and innovation-based “digital” economy.

Several studies have been conducted, in parallel to this endeavor and mostly constructively to it, trying to determine from the one hand, the factors affecting digital divide and Internet adoption12 (Cruz-Jesus et al., 2012; Chinn & Fairlie, 2006; Billon, Marco, & Lera-Lopez, 2009; Li & Shiu, 2012) and on the other hand to measure Internet diffusion or digital divide extent and describe its evolution (Andrés, Cuberes, Diouf, & Serebrisky, 2010; Corrocher & Ordanini, 2002; Vicente & López, 2011).

Digital divide and Internet diffusion related studies deal with both developing and developed countries cross-sectional data and cross-sectional time series (Corrocher & Ordanini, 2002; Quibria, Ahmed, Tschang, & Reyes-Macasauqit, 2003; Rao, 2005). Furthermore, there also numerous work investigating both developed and developing countries (Bagchi, 2005; Beilock & Dimitrova, 2003; Kraemer, Ganley, & Dewan, 2005).

Multiple regression analysis is the most frequently used technique in the determination of factors influencing ICT adoption. In such studies, several quantities are considered as dependent variables. Internet users, hosts and mobile telephones are the most investigated dependent variables (Andrés et al., 2010; Beilock & Dimitrova, 2003; Dasgupta, Lall, & Wheeler, 2005; Oyelaran-Oyeyinka & Lal, 2005; Wong, 2002). Other factors that have been examined as ICT adoption indicators are internet diffusion, expenditure on computer hardware per capita, imports of computer equipment and secure e-commerce hosts (Caselli & Li, 2001; Gibbs & Kraemer, 2004; Li & Shiu, 2012; Wong, 2002). In the last years, broadband penetration was also introduced in the studies of ICT adoption (Dauvin & Grzybowskii, 2014; Distaso, Lupi, & Manenti, 2006). In order to simultaneously investigate multiple dependent variables, canonical analysis can also be used. In Billon et al. (2009), the differences detected between groups of countries both in terms of ICT patterns and in terms of the factors explaining each are compared.

Although there is a lack of consensus regarding the factors determining ICT adoption and differences, the most crucial one is income per capita. It has been shown that there is a close relation between digital divide and the economic status of a country (Fuchs, 2009; Hargittai, 1999; Crenshaw & Robison, 2006). This may be attributed to the fact that ICT products and services can be forwarded more efficiently and effectively in wealthy countries. It should also be noted that a positive correlation between Internet penetration and infrastructure was also observed (Arnum & Conti, 1998; Bazar & Boalch, 1997; Billon et al., 2009; James, 2007; Maherzi, 1997). However, the separate impact of income and Internet infrastructure penetration is still unclear due to the high correlation between them.

Political, social, cultural, regulatory, security and other external conditions surrounding Internet users may also influence Internet adoption of a country (Götz, 2013; Orviska & Hudson, 2009; Pick & Nishida, in press; Wolcott, Press, McHenry, Goodman, & Foster, 2001; Zhao, Kim, Suh, & Du, 2007). Successful adoption of innovations can be affected by factors such as training and education (Nelson, 1993). It was shown that better educated populations are more advantageous regarding Internet and e-commerce adoption (Billon, Ezcurrea, & Lera-López, 2009; Vicente & Lópe, 2006). This may be explained by the fact that populations with higher levels of education have the required skills to collect, process and use information.

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12 The process of integration of the Internet by individuals to their everyday lives.
Contrary to these findings, there are also studies noted that educational attendance of the population does not seem to have a significant importance on the digital divide (Cruz-Jesus et al., 2012).

Other non-economic factors that can be critical for Internet adoption are demographic characteristics such as population age, density and urbanization (Chinn & Fairlie, 2007; Lin, 1998; Niehaves & Plattfaut, 2013). Younger generations are proved to be more prone to ICT. This can be attributed to their advanced skills and attributes as well as to the increased demand for educational purposes (Fairlie, Beltran, & Das, 2010). Furthermore, higher degrees of Internet adoption are expected in regions with high population density as well as in those with high proportion of urban population (Crenshaw & Robison, 2006; Forman, Goldfarb, & Greenstein, 2003). This is somehow expected since infrastructure cost decreases and connection speed increases as the population density increases leading to cheaper and high quality access. However, some cross-country studies find no or negative relation between urbanization and population density on ICT adoption (Chinn & Fairlie, 2006; Billon et al., 2009; Vicente & López, 2011). Internet content can also play a significant role in motivating Internet adoption (Viard, 2010).

On the other hand, there are several studies quantifying digital divide through indices. These incorporate the complexity and the various dimensions of digital divide and development. This effort can be exemplified by the synthetic index of Corrocher and Ordanini (2002) used to measure the digital divide across ten developed countries and the ICT development index (IDI) of ITU (2009) used to measure the global digital divide and examine its evolution.

### 3. Population dynamics, motivation and proposed model

According to population biology, expressing the growth or decline of the population of a given species can be achieved by the rate of its change proportional to its current size. The simplest approach implies absence of any competitors \((i=1)\) and was given in Boyce and DiPrima (2008):

\[
\frac{dy_i(t)}{dt} = r(\frac{y_i(t)}{k})y_i(t)
\]

where \(y_i(t)\) is the size of the given species population at time \(t\), constant \(r\) is called the intrinsic growth rate and presents the growth rate in absence of any limiting factors and \(k\) is the upper bound that is approached yet not exceeded by growing populations starting below this value. Constant \(k\) is usually referred to as the saturation level or the environmental carrying capacity of the given species.

When studying more than one species that coexist in the same closed environment, one would expect them to interact with each other in many ways. Their interaction would involve competition for the same resources and therefore changes in their population dynamics. The dominant species will impinge on all available resources affecting the growth rate and saturation population and of the remaining ones and vice versa. However, in the specific case where two species interact with each other, the following types of interaction can be defined (Aldrich, 1999; Kot, 2001): (i) the growth rate of one population is decreasing while that of the other one is increasing denoting a predator–prey situation; (ii) the growth rate of each population is decreasing, showing that the populations are in full competition; (iii) one population negatively affects the other, but not vice versa – partial competition; (iv) each population’s growth rate is increasing, a case known as mutualism or symbiosis; and (v) two populations are mutually indifferent, but may impacts on other populations in their community, known as neutrality.

In the present work, Internet users as a share of the population are chosen for our study group. More specifically the indicator chosen presents a percentage of the total population that is classified as Internet and non-Internet users, as described in the previous sections. Both Internet and non-Internet users are studied being regarded as two biological species.

In fact, there is no real competition between users and non-Internet users. This can also be supported by the absence of any empirical indication of switching competitive behavior. It is expected that once a non-Internet user switches to a user there is likely no switching back, unless birth of a new Internet technology disrupts access and knowledge, which is not likely in the short time period under study, or when he cannot any more afford it. On the other hand, birth processes (hided in the growth and the growth rate to carrying capacity ratio) and population growth would favor the non-Internet user sub-population, assuming newborns are non-Internet users.

The described process seems like a linear food chain (newborns → non-Internet user → Internet users) revealing an indirect competition between Internet users and non-Internet users. The move of non-Internet user to Internet users group can liken to a special type of exit–entry process. (A great work of L-V modeling with entry and exit processes between coexisting old and new technologies can be found on the IIASA study (Granstrand, 1991)). In fact, Internet users’ significantly depend on non-Internet users’ population. Thus, although there is no direct competition between the two species, the existence of the indirect competition between them forces the use of L–V competition model in order to investigate the evolution of the coexisting species. The use of L–V model in the study of other forms of indirect competition known as “apparent” competition can also be met in the literature (Krivan & Eisner, 2003).

It is expected that the growth rate of non-Internet users is decreasing while that of Internet users is increasing. The model that will be used for the species, with populations \(N_1(t)\) and \(N_2(t)\) respectively at time \(t\), is equivalent to the classical...
Lotka–Volterra model presented in (1) for \( i = 1, 2 \) and can be described by the following set of equations:

\[
\begin{align*}
\frac{dN_1}{dt} &= a_{10}N_1 + a_{11}N_1^2 + a_{12}N_1N_2 - mN_1 \\
\frac{dN_2}{dt} &= a_{20}N_2 + a_{21}N_2^2 + a_{22}N_1N_2 - mN_2
\end{align*}
\]

(2)

In (2), \( \frac{dN_i}{dt} \) and \( \frac{dN_j}{dt} \) are the rates of population change for the two species. Moreover, \( a_{10} \) and \( a_{20} \) represents the growth coefficient of populations \( N_1 \) and \( N_2 \). Coefficients \( a_{11} \) and \( a_{21} \) measure the intraspecies interaction while \( a_{12} \) and \( a_{22} \) measure interspecies interaction. Finally, \( m \) stands for the mortality rate. The inclusion of mortality rate is significant since the model describes the evolution of Internet and non-Internet users that are human beings with limited lifespans. It should be noted that mortality rate was derived from human mortality rate.

4. Methodology and solution procedure

The evolution of Internet and non-Internet users as a share of the total population is going to be modeled by solving the system of differential equations shown in (2). However, the first step will be to estimate the unknown coefficients \( a_{ij} \) using actual statistical data along with two methods: an analytical and a simulation method (Appendix A).

4.1. System linearization

After having estimated the coefficients of system (2), a linearization procedure must be adopted in order to derive a linear system of equations (Boyce & DiPrima, 2008). In order to do so, the first step is to define a stable solution for system (2) which can be rewritten in the following form:

\[
\begin{align*}
\frac{dN_1}{dt} &= N_1(a_{10} - m + a_{11}N_1 + a_{12}N_2) \\
\frac{dN_2}{dt} &= N_2(a_{20} - m + a_{21}N_2 + a_{22}N_1)
\end{align*}
\]

(3)

This system has \( 2^i \) possible equilibrium points (solutions), where \( i \) equals the number of the system’s equations. In the case of system (3), there exist 4 possible solutions only one of which contains non-zero values for parameters \( N_1 \) and \( N_2 \). In order to identify these solutions, the derivatives of the left-hand side are set to zero and the remaining system (i.e. the equations inside the parentheses) is solved in terms of \( N_1 \) and \( N_2 \):

\[
\begin{align*}
N_1 &= \frac{a_{21}(a_{10} - m) - a_{12}(a_{20} - m)}{a_{22}a_{12} - a_{21}a_{11}} \\
N_2 &= \frac{a_{11}(a_{20} - m) - a_{22}(a_{10} - m)}{a_{22}a_{12} - a_{21}a_{11}}
\end{align*}
\]

(4)

(5)

The system can be transformed to its closest linear system when the values of populations \( N_1 \) and \( N_2 \) are close to this solution. In order to do that the left-hand side of system (2) is expanded around the equilibrium point \( (N_1^{(0)}, N_2^{(0)}) \) by its tangent around that fixed point:

\[
\begin{align*}
\frac{dN_1}{dt} &= F_1(N_1, N_2) \approx F_1(N_1^{(0)}, N_2^{(0)}) + (N_1 - N_1^{(0)}) \frac{\partial F_1}{\partial N_1} (N_1^{(0)}, N_2^{(0)}) + (N_2 - N_2^{(0)}) \frac{\partial F_1}{\partial N_2} (N_1^{(0)}, N_2^{(0)}) \\
\frac{dN_2}{dt} &= F_2(N_1, N_2) \approx F_2(N_1^{(0)}, N_2^{(0)}) + (N_1 - N_1^{(0)}) \frac{\partial F_2}{\partial N_1} (N_1^{(0)}, N_2^{(0)}) + (N_2 - N_2^{(0)}) \frac{\partial F_2}{\partial N_2} (N_1^{(0)}, N_2^{(0)})
\end{align*}
\]

(6)

By definition \( F_i(N_1^{(0)}, N_2^{(0)}) = F_2(N_1^{(0)}, N_2^{(0)}) = 0 \) therefore (6) can be simplified to

\[
\begin{align*}
\frac{dN_1}{dt} &\approx (N_1 - N_1^{(0)}) \frac{\partial F_1}{\partial N_1} (N_1^{(0)}, N_2^{(0)}) + (N_2 - N_2^{(0)}) \frac{\partial F_1}{\partial N_2} (N_1^{(0)}, N_2^{(0)}) \\
\frac{dN_2}{dt} &\approx (N_1 - N_1^{(0)}) \frac{\partial F_2}{\partial N_1} (N_1^{(0)}, N_2^{(0)}) + (N_2 - N_2^{(0)}) \frac{\partial F_2}{\partial N_2} (N_1^{(0)}, N_2^{(0)})
\end{align*}
\]

(7)

System (7) is now transformed into the following linear system:

\[
\begin{bmatrix}
\frac{d}{dt} [nIU] \\
\frac{d}{dt} [IU]
\end{bmatrix} = \mathbf{J} \times \begin{bmatrix}
[nIU] \\
[IU]
\end{bmatrix}
\]

(8)

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where \(nIU = N_1 - N_0^0\) and \(lIU = N_2 - N_0^0\) and the matrix \(J\) is the Jacobian matrix of the system at the equilibrium point \((N_0^0, N_0^0)\):

\[
J = \begin{bmatrix}
\frac{dF_1}{dN_1} (N_0^0, N_0^0) & \frac{dF_2}{dN_1} (N_0^0, N_0^0) \\
\frac{dF_1}{dN_2} (N_0^0, N_0^0) & \frac{dF_2}{dN_2} (N_0^0, N_0^0)
\end{bmatrix}
\] (9)

According to Poincare–Lyapunov theorem (Boyce & DiPrima, 2008) if the eigenvalues of matrix \(J\) are not equal to zero or are not pure imaginary numbers, then the trajectories of the system around this point behave the same way as the trajectories of the associated linear system. More specifically if the eigenvalues are negative or complex with negative real part, then the fixed point is a sink, or if the eigenvalues are positive or complex with positive real part, then the fixed point is a source point and if the eigenvalues are real numbers with different sign then the equilibrium point is a saddle point.

The general solution of system (8) is

\[
\begin{bmatrix}
nIU \\
lIU
\end{bmatrix} = c_1 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} e^{\lambda_2 t}
\] (10)

where \(c_1\), \(c_2\) are arbitrary constants and \(v_{ij}\) are the elements of the eigenvectors derived from the Jacobian matrix \(J\). In addition, since the system under investigation is an initial value problem, substitution of these initial values to Eq. (10) allows the calculation of constants \(c_1\) and \(c_2\) resulting in the final solution.

### 5. Results and discussion

In this section the proposed model is applied in order to describe the evolution and gap of Internet and non-Internet users in the case of two European countries; Greece and Lithuania. Calculations were performed on annual data describing Internet and non-Internet users as a share of the total population of each country. Two species will be used, i.e. Internet and non-Internet users, in all cases. The selected data set was found in Digital Agenda for Europe from 2005 until 2013. In fact, data from years between 2005 and 2011 was used for the estimation of the model coefficients and the data of the last two years (2012 and 2013) was utilized in order to verify the model’s ability to forecast future trends. The authors are willing to reexamine the model’s forecasting ability when more/future data will be available. Although the used data set may seem to contain few data points in order to produce accurate estimates, both the ABC algorithm and the analytical method remain unaffected and reveal high performance.

The evaluation procedure is going to be described in detail for the case of Greece while for Lithuania’s case only the final results are going to be presented since they were derived in exactly the same way.

Internet and non-Internet users as a share of the total population for the chosen cases are shown in Table 1. The first column corresponds to the year under consideration while the second and the third, of each case, to the Internet and non-Internet users shares respectively. It should be noticed that the shares of Internet users and non-Internet users do not add to one which can be attributed to the definition of Internet users given in the Introduction. In fact, Internet users are individuals who regularly use the Internet excluding those who rarely or occasionally use the Internet.

### 5.1. Internet and non-Internet users modeling

The first step towards the Internet and non-Internet users modeling is the identification of equation’s (2) coefficients \(a_{ij}\), \(i = 1, 2\) and \(j = 0, 1, 2\). This will be achieved with the application of the ABC algorithm as well as with the analytical method, described in previous sections, for the available dataset. It should be noted that the average mortality rates (2005–2013) equal to 0.98 and 1.2 for Greece and Lithuania respectively. The coefficients derived using the aforementioned methods are illustrated in Table 2.

<table>
<thead>
<tr>
<th>Year</th>
<th>Greece Internet users</th>
<th>Greece Non-Internet users</th>
<th>Lithuania Internet users</th>
<th>Lithuania Non-Internet users</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0.183</td>
<td>0.727</td>
<td>0.298</td>
<td>0.614</td>
</tr>
<tr>
<td>2006</td>
<td>0.226</td>
<td>0.65</td>
<td>0.377</td>
<td>0.541</td>
</tr>
<tr>
<td>2007</td>
<td>0.277</td>
<td>0.616</td>
<td>0.451</td>
<td>0.487</td>
</tr>
<tr>
<td>2008</td>
<td>0.329</td>
<td>0.557</td>
<td>0.498</td>
<td>0.427</td>
</tr>
<tr>
<td>2009</td>
<td>0.379</td>
<td>0.534</td>
<td>0.552</td>
<td>0.382</td>
</tr>
<tr>
<td>2010</td>
<td>0.408</td>
<td>0.524</td>
<td>0.575</td>
<td>0.355</td>
</tr>
<tr>
<td>2011</td>
<td>0.473</td>
<td>0.446</td>
<td>0.611</td>
<td>0.331</td>
</tr>
<tr>
<td>2012</td>
<td>0.5</td>
<td>0.42</td>
<td>0.64</td>
<td>0.31</td>
</tr>
<tr>
<td>2013</td>
<td>0.56</td>
<td>0.36</td>
<td>0.65</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Table 1
Actual data of Internet and non-Internet users as a share of total population.
Source: Digital Agenda for Europe.

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As shown in Table 2, the results of both methods coincide very well. For simplicity, the ABC’s coefficients are assumed to be the model’s coefficients that will be used in the rest of the paper. It should be noted that important information can be derived regarding the process dynamics from coefficients of Table 2. For example, in both cases, coefficients $a_{12}$ are much larger than $a_{21}$ showing the rapid transformation of non-Internet users to Internet users.

Substituting the corresponding coefficients in Eq. (A4) for the case of non-Internet users ($N_1$) and in a similar equation for Internet users, one can easily evaluate the estimates of the Internet and non-Internet users’ shares. In Figs. 1 and 2, non-Internet and Internet users as a share of the total population are shown respectively. Actual data is illustrated as solid line while estimated data is depicted in dots. The $R^2$ coefficient of the fitting between estimated and actual values equals to 97.5% and 99.5% for non-Internet and Internet users respectively, demonstrating the high accuracy of the whole process.

It must be highlighted once again that the model’s coefficients were evaluated over the training period 2005–2011. As shown in Figs. 1 and 2, the estimated values (0.5057, 0.5390 and 0.4340, 0.4130) for years 2012 and 2013 almost coincide with the actual values (0.5, 0.56 and 0.42, 0.36 – Table 1) for non-Internet and Internet users respectively giving a first insight of the forecasting ability of the proposed model.

As mentioned in the previous section, the system of (3) has 4 critical points, when the derivatives of the left-hand side equal to zero. However, three out of four equilibrium solutions contain at least one $N_i$ equal to zero and must be ignored. The remaining non-zero solution in the case of Greece was found to be $(N_1, N_2) = (0.2561, 0.6615)$. The eigenvalues of Jacobian
matrix (9) are evaluated at the calculated critical point in order to investigate its stability. This analysis showed that the critical point is stable since the eigenvalues are real and negative (critical point is a sink). The calculated eigenvalues for this case are \( \lambda_1 = -0.1374, \lambda_2 = -0.8649 \). The fact that the calculated critical point is a sink bears significant value for the forecasting analysis since populations \( N_I, N_U \) of non-Internet and Internet users as a share of total population will eventually settle to the values of that non-zero solution. This is of great importance for the policy and decision makers in order to properly design their strategy according to future conditions and trends of Internet usage.

According to Poincare–Lyapunov theorem and since the eigenvalues of the Jacobian matrix (9) are negative real numbers, the trajectories of system (2) around this point behave the same way as the trajectories of the associated linear system. Therefore the system is almost linear close to the critical point.

After simple mathematical manipulations (Eqs. (6)–(9)), it is straightforward to derive the general solution (10) for the case of Greece:

\[
\begin{bmatrix}
    nIU \\
    IU
\end{bmatrix}
= c_1 \begin{bmatrix}
    0.7828 \\
    -0.6223
\end{bmatrix} e^{-0.1374t} + c_2 \begin{bmatrix}
    0.5767 \\
    0.8170
\end{bmatrix} e^{-0.8649t}
\]

where \( nIU = N_I - 0.2561 \) and \( IU = N_U - 0.6615 \) and \( c_1 \) and \( c_2 \) are arbitrary constants. These constants can be calculated by substituting the initial values of \( nIU \) and \( IU \) (at \( t = 0 \)) in (11). Therefore, the resulting final solution for the case of Greece is

\[
\begin{bmatrix}
    nIU \\
    IU
\end{bmatrix}
= \begin{bmatrix}
    0.518 \\
    -0.4118
\end{bmatrix} e^{-0.1374t} + \begin{bmatrix}
    -0.0471 \\
    -0.0667
\end{bmatrix} e^{-0.8649t}
\]

(12)

Next, illustrated in Fig. 3 are the non-Internet and Internet users’ shares evolution over time, based on the system described by (12). In order to be more specific the transformation of \( nIU = N_I - 0.2561 \) and \( IU = N_U - 0.6615 \) in (12) is reversed and the constructed model estimates that after \( t \approx 30 \) years the non-Internet users share reaches the equilibrium. As shown in Fig. 3, it is obvious that the share of Internet users will constantly increase in the next years until its predicted stabilization. On the other hand the share of non-Internet users exhibits an asymptotical decay towards the forecasted equilibrium point.

According to the preceding analysis for the case of Greece one can easily derive the final solution for the case of Lithuania:

\[
\begin{bmatrix}
    nIU \\
    IU
\end{bmatrix}
= \begin{bmatrix}
    0.4187 \\
    -0.3358
\end{bmatrix} e^{-0.1669t} + \begin{bmatrix}
    -0.0215 \\
    -0.0652
\end{bmatrix} e^{-1.9939t}
\]

(13)

From (13) it is deduced that the eigenvalues are real negative leading to a sink solution \( (N_I, N_U) = (0.2174, 0.699) \). This can also be confirmed by Fig. 4 where non-Internet (Internet) users share is constantly decreasing (increasing) towards its equilibrium. This is reached again after \( t \approx 30 \) years.

Considering the case of Lithuania, the model findings seem to relate closely to national socioeconomic facts, known to influence the national internet diffusion. Lithuania presented the highest economic growth rate amongst all candidate and member countries just before joining the European Union (2003), something that lasted until 2009 when a dramatic decline in GDP was marked and signaled the begging of Lithuania’s reaction to the worldwide economic crisis. Even though the country was struck by the economic crisis, there has been a gradual but consistent structural shift towards a knowledge-based economy with an IT perspective (twice as many people with higher education than the EU-15 average). It must also be noted that Lithuania has Europe’s most available fiber network and the highest FTTH penetration. However, emigration levels are constantly increasing with mostly young people seeking higher earning employment and studies abroad.

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14 According to statistical data obtained from [http://www.ftthcouncil.eu/](http://www.ftthcouncil.eu/).
Furthermore, since 2000 Lithuania presents a gradual increase of its population mean age, leading to a well-aged population by 2050, if one takes into account the country's fertility decline and mortality increase.

Similarly, in the case of Greece, the significant increase of Internet users and the future reaching of a stable point are very well reflected at several socioeconomic facts of its recent history. In 2000, after the adoption of the common European currency (€), Greece aligned with EU’s ICT policy and actually took advantage of the corresponding ICT funding for the public and private sector making a lot of progress even though it remained below the EU average R&D spending according to Eurostat. In addition, according to data from the Observatory for Digital Greece, broadband internet availability is widespread and according to 2012 data, approximately half of the Greek households used the internet regularly using a broadband connection. On the other hand Greece has been severely struck by the worldwide economic crisis, the results of which became particularly evident by the end of 2009 when Greece, in order to avert an eminent default, was required to adopt harsh austerity measures to bring its deficit under control.

5.2. Further investigation on logistic growth assumption and interaction of populations

Taking a closer look at the data of Table 1, one can note that the sum of Internet and non-Internet users remains almost constant. This may indicate a kind of collinearity between the state variables. In order to examine this feature, a fitting of the relationship between Internet and non-Internet users was performed using a linear curve of the type $IU = a nIU + b$. The results of the data fitting are shown in Fig. 5 for the case of (a) Greece and (b) Lithuania while the values found for the fitting parameters are listed in Table 3.

A shown in both Table 3 and Fig. 5, there is a strong evidence of collinearity between non-Internet and Internet users. In this case, the two equations can be reduced to a linear and a logistic equation of the type:

$$\frac{dIU}{dt} = rIU \left(1 - \frac{IU}{K}\right)$$

where $K = (a_{20} - m)\vert a_{21}\vert$ is the carrying capacity of Internet users. Using the results of Table 2, one can easily find that $K = 0.83$ and 0.85 of Internet users in Greece and Lithuania respectively.

In order to examine the consistency of the obtained results, the evolution of Internet users is investigated assuming purely logistic growth that is in the absence of any competition. By solving the differential equation of (14), it is straightforward to show that

$$IU = K \frac{e^{(t - t_i)/\tau}}{1 + e^{(t - t_i)/\tau}}$$

where $t_i$ is the inflection point corresponding to the peak of its growth rate while $\tau$ is the characteristic time constant of its growth rate.

The parameters estimated from fitting are shown in Table 4. Real data along with the estimated logistic curve are illustrated in Fig. 6 in the case of (a) Greece and (b) Lithuania. For comparison reasons, the Gompertz model ($IU = S \exp (− A \exp (− b t))$) was evaluated and plotted in the same figures.

By comparing the saturation levels obtained by both the L–V and the logistic model, it can be deduced that there is a deviation of about 9.2% and 3.3% for the case of Greece and Lithuania respectively. Although the deviation in the case of

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16 Available at http://www.stat.gov.lt.
17 Available at http://www.observatory.gr.
18 According to statistical data obtained from ELSTAT – Available at www.statistics.gr.

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Greece is larger, it is still in low levels and thus the use of the logistic growth assumption in the L-V model can be proved correct. Furthermore, as shown in Table 4, the growth rate of Internet users reach its maximum value at the end of 2008 and the middle of 2005. From then on, it slows down its rate of growth until the saturation.

In order to further investigate the appropriateness of the logistic curve in modeling the evolution of Internet users, the percent growth rate (PGR) of Internet users’ evolution is calculated as follows assuming a logistic growth:

\[
PGR(IU) = \frac{1}{r} \left( 1 - \frac{IU}{K} \right)
\]

(16)

The percentage growth rate of Internet users as a function of IU along with the estimated parameters are illustrated in Fig. 7 and Table 5 respectively in the case of (a) Greece and (b) Lithuania.

Fig. 5. Non-Internet users as a function of Internet users along with their linear fitting for the case of (a) Greece and (b) Lithuania.

| Table 3 |
|-----------------|-----------------|
| Greece | Lithuania |
| a | -0.905 | -0.9135 |
| b | 0.8725 | 0.8874 |
| R² | 0.9857 | 0.9971 |

Table 4

Results of the logistic data fitting.

<table>
<thead>
<tr>
<th>Greece</th>
<th>Lithuania</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>0.727</td>
</tr>
<tr>
<td>τc</td>
<td>2008.84</td>
</tr>
<tr>
<td>Ta</td>
<td>3.641</td>
</tr>
<tr>
<td>R²</td>
<td>0.9957</td>
</tr>
</tbody>
</table>

Fig. 6. Internet users share evolution modeling using a logistic curve in the case of (a) Greece and (b) Lithuania.
Looking at Tables 4 and 5, it can be deduced that there is a quite good agreement in the compared values with a deviation in the case of Greece. This can be attributed to the fact that in the case of Greece, PGR data is noisier.

In order to discover significant hidden information about the timing of the driving factors of the given evolutionary process, the residuals of the logistic model will be analyzed. Towards this direction, the residuals will be fitted in a series of harmonics of the following form:

$$R_s = \sum_{n=1}^{N} a_n \sin(2\pi f_0 t + \phi_n)$$

(17)

where $a_n$ and $\phi_n$ is the amplitude and the phase of the $n$th harmonic, $N$ is the total number of the harmonic modes participating in the series while $f_0$ is the frequency of the fundamental mode. The derived coefficients of Eq. (17) are listed in Table 6 while the truncated sine series curve is illustrated in Fig. 8 for the case of Greece and Fig. 9 for Lithuania.

As shown in Fig. 8, in the case of Greece, a slight increase in the number of Internet users is expected in 2016 and a larger one by 2019. It should be noted that the time distance between major-to-major peaks (M-to-M) and minor-to-minor (m-to-m) peaks is about 5 years while m-to-M and M-to-m distances are roughly 2 and 3 years respectively. Furthermore, a time distance of 56 years is observed between maximum peaks (M$^0$-to-M$^0$).

On the other hand, in the case of Lithuania as shown in Fig. 9, the next increase in the number of Internet users is expected by 2019 while increases will be observed every 5 years (M-to-M peaks). In addition, the envelope of the waveform has a period of 63 years (M$^0$-to-M$^0$ peaks).

These oscillations may be attributed to technologies substitution and investment in new products, services and infrastructures. Longer cycles (~60 year cycles) are related to critical global macroeconomic transformations due to the start of...
or the end of radical innovations. Examples of short term oscillations are the substitution between ISDN, ADSL, ADSL2 and VDSL or 2G, 3G and LTE along with the respective network investments.

In order to investigate the interaction between Internet (IU) and non-Internet (nIU) users, the rate of interaction, described by the product $\text{IU} \times \text{nIU}$, between the two populations is modeled with a log-normal function of the form:

$$f(x) = \frac{S}{\sqrt{2\pi \sigma^2}} e^{-\frac{(\ln(x/c))}{2\sigma^2}}$$ (18)

where $S$ is the surface under the curve, $c$ corresponds to the year in which the interaction rate is maximized, and $\sigma$ is the standard deviation. The coefficients of (18) derived from the data fitting are as follows: $S=3.22 (4.27)$, $\sigma=0.003 (0.004)$ and $c=2011.05 (2008.8)$ with $R^2=0.99 (0.74)$ for Greece and (Lithuania) respectively.

The obtained results (Figs. 10a and 11a) show that the interaction between Internet and non-Internet users is maximum at 2011.05 and 2008.8 for Greece and Lithuania respectively which differ from the results derived from the single logistic model. This is an evidence of the impact of the “indirect” competition on the evolution of Internet and non-Internet users.

Moving one step further, the time derivative of (18) is calculated revealing the effective interaction process time corresponding to the time between the acceleration and deceleration peaks of the process. As shown in Fig. 10b (Fig. 11b), the critical period of transformation from non-Internet to Internet users is located around 2011 (2009) and will last circa 12 (16) years for Greece and Lithuania respectively. It can also be noticed that there is still enough time (approximately 4 years) in order to influence the transformation process.

From the above analysis, it can be deduced that the maximum growth of Internet users was observed in the period 2008–2011 in the case of Greece and 2005–2008 in the case of Lithuania. It would be thus of high importance to investigate what was happened in these countries during the specific period. Studying the progress reports19 and Scoreboard reports20 published by the EC, it was deduced that these periods (and may be one year earlier) surprisingly coincided with several changes in the legal, regulatory and technological environment.

In Greece, the period before 2007 was a little bit blurred. There are complaints about the non-transparent operation and decision making of the Greek NRA (EETT) while legal uncertainty was a big issue in several areas as a result of ongoing disputes and the length of time involved in appealing decisions. In addition, the “new” regulatory framework (Telecommunications Act) has not yet been adopted. Given that the new EU regulatory framework is not yet transposed, EETT’s ability to intervene to impose ex ante measures in the market has been limited. There were also significant delays in several actions such as market analyses and the application of relevant remedies. It was thus evident that regulatory interventions were required (inter alia, rights of way, access to the local loop, CPS conditions etc.) in order to support competition and boost telecom development.

The picture totally changed in the mid of 2006 and onwards. Greece notified measures transposing the European regulatory framework into its national legislation. A great effort has been made (especially by EETT) regarding co-locations. EETT adopted a regulation in March 2008, specifying the procedures for collocation of electronic communication equipment and/or joint use of essential facilities by the network and/or service providers. The years 2007–2009 were characterized by a significant LLU growth and an increase in broadband penetration which seemed to be interdependent. Factors contributing to this incremental trend include the effective regulation and the authorities’ effort to promote broadband, the investments of alternative operators (mainly in LLU), the increase in collocated sites, and the significant increase in the take-up of bundled products. Greece ranks last among EU countries with regard to the number of broadband fixed lines using technologies other than DSL. It is also interesting to note that at the end of 2006, availability and take-up of 3G services were increased.

In this period, (co-funded) investments were intensified. At the end of 2007, the Broadband Access Development in Underserved Territories project aiming at co-financing broadband investment for local access across Greece (excluding Athens and Thessaloniki) started. This project reached its final stage in 2009, with operators concluding their investments and offering retail services contributing to the uptake of local loop unbundling. Several alternative operators also proceeded to the construction and roll-out of their own fiber-optic networks. In the same period, the deployment of metropolitan fiber optic networks was being implemented in 75 municipalities in Greece, financed by the Operation Programme ‘Information Society’. In 2008, the incumbent (OTE) also announced his plans to deploy fiber in the access network in urban and suburban areas based on FTTC architecture with VDSL at cabinet level, a project that is in progress until now. In February 2008, the Government announced a €3 billion program including investment of €2.1 billion for the installation of a fiber optic network that would give at least 2 million households and enterprises access to broadband services in at least 52 major cities. Unfortunately, this project was postponed and revisited many times in the following years.

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On the other hand, Lithuania became a member of the European Union in 2004 and characterized as the member with the faster growth especially in telecommunications. The year 2004 put the basis for a stable legal environment facilitating the development of telecommunication market. The Law on Electronic Communications transposing the 2002 EU regulatory framework on electronic communications came into force on 1 May 2004 leading to the complete liberalization of country’s telecommunications market. It is interesting to note that there were more than 50 pieces of secondary legislation while by the end of July 2004, the RRT (Lithuanian NRA) had published several draft rules that were open for public comments regarding market analysis, public consultations, 3G licensing rules, LRAIC models etc. All the necessary tools ensuring competition (e.g. sufficient powers of the NRA, number portability, carrier selection and pre-selection, local-loop unbundling, access and interconnection, possibility to impose SMP obligations on operators) were in place. In the same year, RRT had received 38 notifications from new entities about their intentions to start business in fixed market triggering positive changes such as the reduction of some prices by the fixed incumbent.

In 2005, a wide range of secondary legislative acts (new general conditions of carrying out electronic communications activities, new rights of use auction rules, new rules on installation and use of electronic communications infrastructure, new market analysis regulations) was adopted and amended by RTT. The improvement was mirrored to statement of several market players appreciated improved timing of RRT’s consultation procedures. In the same year, the operation of the first wireless broadband mobile access network started by an alternative operator. In 2006, the basis for rapid implementation of the remedies imposed have created due to the completion of virtually all market analyses, and the lower number of decisions suspended while under appeal. The belief of RTT on market forces of infrastructure competition has fully worked in the very competitive mobile market while it was proved less effective in broadband and fixed markets since the fixed incumbent has largely succeeded in escaping competition. It should also be highlighted that the majority of broadband lines were provided in 2008 by technologies other than digital subscriber lines (DSL), and there was an increasing range of innovative service offerings to users of telecommunications services, including mobile broadband. Despite the changes made in LLU, it cannot be taken-up and contributed to the development of telecommunications market. This may be attributed to the fact that many competitors of the incumbent have constructed their own networks as well as to the unattractive terms and conditions of use of LLU.

Moreover, the period 2005–2008 can also be characterized as network infrastructure construction period. The Lithuanian Government, subsidized by the European Structural Funds, was deploying a 3000 km fiber backbone, aiming to connect residential and non-residential users in rural areas. A tender procedure was announced for the choice of a private company that will operate the backbone network. The roll-out of the publicly funded rural fiber backbone network (called RAIN) was completed in 2008. Many kilometers were rented from its initial phase of operation. Two municipal broadband projects were also launched in 2008. The incumbent, Internet service providers, alternative and cable operators deployed next generation networks (NGNs) – fiber to the home (FTTH) or fiber to the building (FTTB) networks. Wireless networks (WiMAX) were also implemented in more than 20 Lithuanian cities.

### 5.3. Stability of the proposed system

In order to investigate the stability of system (11) at the specific critical point, a phase diagram is constructed and plotted in Fig. 12, based on different initial values for Internet and non-Internet users shares. As observed, all trajectories converge to the estimated critical point, shown with rectangle independently of the initial conditions (circles – starting points of the trajectories). This can be attributed to the fact that because as time increases the solutions decay exponentially and approach the critical point.

![Fig. 12. Phase portrait of dynamic system based on random initial values for Internet and non-Internet users shares. All trajectories converge to the critical point.](image-url)
5.4. Internet and non-Internet users gap – divide evaluation

It would be of great importance to model and study the divide (gap) between non-Internet and Internet users. Towards this direction non-Internet and Internet users shares should be transformed into net figures of non-Internet ($N_{nIU}$) and Internet ($N_{IU}$) users. This can be achieved by multiplying non-Internet and Internet users’ shares with the corresponding total population of the selected country respectively (source EUROSTAT). The evolution of the divide can then be investigated by introducing the following Internet Divide index (IDI):

$$IDI = \frac{N_{IU} - N_{nIU}}{N_{total}}$$

where $N_{total}$ represents the sum of the $N_{nIU}$ and $N_{IU}$.

The evolution of the divide of the two aforementioned cases is depicted in Fig. 13. In both cases, Internet users are constantly increasing against those who have never used Internet. It is interesting to note that Lithuania is in a better position (IDI is positive since 2007) compared to Greece where the shares of non-Internet and Internet are now equal. The obtained results can be further expanded beyond 2012, if one can provide a forecasting for total population of each country.

The derived results are not completely surprising since nowadays more and more people become familiar with new technologies and especially Internet. Moreover, as shown in Fig. 13, there is a lot of space regarding Internet adoption and use since IDI still remains in extremely low levels. This indicates that there are still a lot of people in both countries who have never used Internet. This may be attributed to the bad financial status of both countries and to their aged population. However, policy makers and decision makers should take into account these results in order to promote Internet adoption and increase Internet use.

6. Policy implications

The above results reveal that the policy implications which have already been undertaken had low impact on Internet diffusion and thus new approaches are needed to achieve higher penetration levels. Strategic planning and policy decisions should be mainly focused in two directions – target groups: those who want to use Internet but they do not have the skills (Billon et al., 2009) and those who do not intend to use Internet because they do not want to or simply because there is no need to (e.g. elderly people) (Vicente & López, 2006).

The first step towards this direction is to conduct a road map for the development of programs, projects and policies forming the long-term vision of Internet adoption. This can be achieved by organizing fora and workshops in which several ideas can be exchanged between experts.

Regarding the first target group, seminars and training programs should be properly designed in order to increase ICT skills and motivate Internet usage. In this endeavor, governments should play an extremely important role by funding and facilitating, through training fellowships, scholarships and exchange grants, such activities. At the same time, enterprises could also organize seminars in order to educate their employees so that to meet companies’ high-level skills needs.

As stated early, the main reason that people in the second target group do not use Internet is the lack of need and fitting to their life style. However, it seems that education is inappropriate and other activities should be performed. The adaption of Internet to the specific group of people would significantly increase their motivations. In other words, targeted contents/services should be developed addressing their needs (Viard, 2010). For example, regarding elderly people, new tele-medicine and health care services through Internet will be developed. This will lead to the increase of the perceived usefulness of Internet to these households.

It would also be of high importance, especially in the case of the second target group, to support efforts to increase the public awareness of Internet benefits. Since there is strong diffidence regarding the impact of technology on humans’ life,
privacy and security (Orviska & Hudson, 2009), the management of ethical issues seems to be the only way to calm down fears and win public support.

Finally, both governments and private enterprises should invest in developing new or up-grading and strengthening existing information and communication technology infrastructures and networks. This would increase the provided performance in terms of capacity offering in turn new opportunities and challenges to the Internet. Furthermore, they should also give incentives enhancing Internet adoption. From the one hand, governments should give subsidies to providers enabling new investments. On the other hand, Internet providers should reduce prices of new technologies or networks in order to motivate both Internet usage by those who cannot afford it and migration to new technologies and networks.

Apart from these general policy implications, a key factor that can play a significant role in increasing Internet users is access price. However, this regulatory exercise becomes extremely complex since nowadays as well as in the near future, next generation access (NGA) networks should be deployed. The assumption that once the investment has taken place, the NGA network replaces the existing access network instantaneously is not true. In fact, the transition from the copper access networks to the NGA ones is a slow process, and hence, there would be a transition phase during which both technologies will coexist.

According to Bourreau, Cambini, and Doğan (2012) and Bourreau, Cambini, and Hoernig (2012) three conflicting effects emerge in this setting. In particular, low access prices for the copper access network increase the opportunity cost of the entrant’s investment in NGA networks making such investment less attractive, whereas high access prices for the copper access network result in reduced wholesale profits for the incumbent due to the entrant’s positive investment reaction to the incumbent’s fiber deployment. The former effect is widely known as the “replacement effect” and the latter as the “wholesale revenue effect”.

The last effect, namely the “business migration effect”, reflects the fact that low retail prices for the copper-based services discourage consumers to move from the old to the new technology unless the fiber-based services are priced sufficiently low as well. Considering regulated access to the new fiber networks in those areas without competing infrastructures, the “business migration effect” is replaced by a migration effect at the wholesale level (DotEcon, 2012). The fundamental point here is that higher access prices lead to higher retail prices. Therefore, a higher difference between fiber and copper access prices implies a higher difference between fiber and copper retail prices, which in turn, disincentivizes both entrant and consumers to move to the NGA network.

On the other hand, one can take into account that low copper or/and fiber access prices can be a great motivation for non-Internet users to be transformed to Internet users. As mentioned in the introduction (Fuchs, 2009; Vicente & López, 2006), income is one of the key determinants of ICT adoption since it may have a great impact on budget constraints. It can be thus deduced that the regulatory goal to promote effective competition and encourage efficient and timely investment in NGA networks becomes even more complex since regulators have to take into account the impact of the correlation between the access charges for both the legacy and the NGA networks on the efficiency outcomes in terms of competition and investment incentives as well as the trade-off between maintaining old Internet users and motivating new Internet users.

Furthermore, this regulatory goal should be investigated by taking into account the current situation of the two countries. In Greece, there is one incumbent operator and 3–4 main alternative operators. The incumbent operator is now starting to invest in NGA networks while the alternative operators are expected to lease incumbent’s network. On the other hand, in Lithuania there are much more operators. Fiber access is dominant while Local Loop unbundling services are not popular and operators are developing they own infrastructure. Thus, the regulatory implications should be focused on NGA networks in the form of bit-stream service in Greece and on networks infrastructures and facilities in Lithuania.

7. Conclusions and future work

In this work, a methodology based on ecological dynamics was proposed in order to investigate and forecast the evolution of Internet and non-Internet users as well as the dynamics of their gap. Towards this direction the Lotka–Volterra model was used since Internet and non-Internet users were considered as interacting species due to the existence of an “indirect” competition between them. Model’s parameters were estimated by means of both an analytical and a simulation method based on the Artificial Bee Colony optimization algorithm. An in depth analysis of the logistic growth assumption and populations interaction was also carried out. The work also introduced Internet Divide index (IDI) enabling the description and investigation of the divide (gap) between non-Internet and Internet users.

The described methodology was chosen to be applied in the case of two European countries, Greece and Lithuania, which are still below European’s averages regarding ICT adoption. In addition these countries seem to have similar digital profiles and differences in regulatory and market aspects facilitating the conduction of reliable conclusions. From the derived results, it was deduced that Internet users have recently outperformed non-Internet users, especially in Greece. Furthermore, it was illustrated that Internet users are constantly increasing against non-Internet users. The model confirmed that Internet users will shortly reach a stable point leaving a significant percentage of population away from the regular use of Internet. It was also shown that the maximum growth of Internet users was observed in the period 2008–2011 in the case of Greece and 2005–2008 in the case of Lithuania. Furthermore, it was illustrated that there is still enough time (approximately 4 years) in order to influence the transformation process of non-Internet users to Internet users by implementing several policies.
Looking closely at the history of the two model countries, the obtained results can be explained by the effective regulation and the authorities’ effort to promote broadband, the investments of alternative operators as well as by the provision of bundled products. Numerous socioeconomic milestones of the studied time period can also be assumed as contributing factors.

Although the presented research method and in general the Lotka–Volterra equations are powerful in modeling several competition processes, they are accompanied by numerous limitations that sometimes are interdependent. The most important limitation of such methods is that they cannot model “ecosystems” influenced by external interventions such as regulatory implications, governmental subsidies etc.

Furthermore, the proposed methodology assumes constant coefficients describing population growth as well as intraspecies and interspecies interactions. However in long lasting researches this is not absolutely correct since there are several factors such as demographic, technological that can be changed influencing the abovementioned coefficients. For example, the introduction of both high-speed networks and broadband services (e-commerce, video on demand, teleconference, online games) are factors that can change Internet usage as well as the interaction between Internet and non-Internet users.

In addition, whilst the set of differential equations can easily describe the competitive dynamics through the estimation of their coefficients, it cannot provide hints about the factors leading to such dynamics and maybe to the equilibrium.

Since the applied methodology is more data driven, it is highly dependent on data generation and collection. This raises limitations and difficulties to such research methods and especially in the case of socio-economic problems. Socio-economic processes in contrast to biological ecosystems cannot be analyzed through experiments limiting the available data sets, the ability for both comparisons between similar populations and repetition of the observations. Furthermore, in the case of innovative technologies there is the problem of limited data.

Last but not least, the solution of such systems is based on optimization methods. Thus, the selection of the optimization method is of high importance in order to avoid trapping to local minima or maxima. Moreover, the choice of proper parameters and stopping criteria are of equal significance for the accuracy of the obtained results.

As shown, the presented methodology possesses several limitations which however can lead to directions for future work. A possible direction for further research can be derived from the fact that the coefficients of the proposed model can be changed over time due to the rapidly changing environmental socio-economic factors. In order to investigate such cases, the data set under consideration should be divided into appropriate segments. The methodological procedure and analysis described in this paper should then be performed for each time segment. The obtained coefficients are expected to reveal the socio-economic changes between the time intervals.

Furthermore, since the proposed methodology follows a deterministic approach, an interesting research direction could be to incorporate stochastic extensions. This could be achieved by including stochastic terms and randomness into the model. A set of the possible situations of the Internet diffusion would be extracted.

As one step further, a sensitivity analysis of the possible effects on Internet diffusion can be performed. This can be induced by a rise in Internet adoption which can be attributed to policy changes such as provision of initiatives. In detail, different levels of adoption should be assumed, in order to investigate the consequences of such changes in the interaction between the two species. Towards this direction, the model illustrated in this paper should be transformed so that to describe the migration of non-Internet users to Internet users. The obtained results are expected to identify the conditions under which species interactions can dramatically be changed.

### Appendix A. Estimation of model coefficients

In this appendix, an analytical and a simulation method will be described. Both methods are based on the minimization of an objective function. More specifically, the simulation method is based on the Artificial Bee Colony, a recently proposed optimization algorithm.

#### A.1. Analytical method

The analytical method (Shatalov, Greeff, Joubert, & Fedotov, 2008) begins with time integration of the system of differential equations in (2) resulting in the following form:

\[
N_1(t) - N_1(0) = (a_{10} - m) \int_0^t N_1(\tau)d\tau + a_{11} \int_0^t N_1^2(\tau)d\tau + a_{12} \int_0^t N_1(\tau)N_2(\tau)d\tau
\]

\[
N_2(t) - N_2(0) = (a_{20} - m) \int_0^t N_2(\tau)d\tau + a_{21} \int_0^t N_2^2(\tau)d\tau + a_{22} \int_0^t N_2(\tau)N_1(\tau)d\tau
\]

where \(\tau \in [0, t]\). Using Eq. (A1), the estimation of the coefficients \(a_{11}\) can be treated independently from that of \(a_{22}\). Hence, coefficients \(a_{10}, a_{11}\) and \(a_{12}\) and coefficients \(a_{20}, a_{21}\) and \(a_{22}\) will be evaluated from the first and the second equation of (A1) respectively along with the use of actual data. The integrals in (A1) are approximated using the trapezoidal method.

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For example, the third integral of the first equation in (A1) at the ith time instant can be written as:

\[
I_{1}^{(i)} = \int_{0}^{t_{i}} N_{1}(\tau)N_{2}(\tau)d\tau \approx \left[\frac{N}{2} \left(\frac{N_{1}(0)N_{2}(0)}{2} + N_{1}(t_{1})N_{2}(t_{1})\right)\right. \\
+ \frac{N}{2} \left(\frac{N_{1}(t_{1})N_{2}(t_{1})}{2} + N_{1}(t_{2})N_{2}(t_{2})\right) \\
\left. + \cdots + \frac{N}{2} \left(\frac{N_{1}(t_{i-1})N_{2}(t_{i-1})}{2} + N_{1}(t_{i})N_{2}(t_{i})\right)\right]
\]

(A2)

It should be noted that at \( t_{0}=0, \) \( h_{0}=0 \) for \( i=1,2,\ldots,6. \) Using the approximations of the integrals \( I_{i} \) in each time instance \( t \in [0, t_{6}] \), where \( N \) is the number of actual data, a system of \( N \) linear equations for each equation in (A1) is derived. In the case of the first equation in (A1), the obtained system is:

\[
(a_{10} - m)f_{1}^{(1)} + a_{11}f_{2}^{(1)} + a_{12}f_{3}^{(1)} = N_{1}(t_{1}) - N_{1}(t_{0})
\]

\[
(a_{10} - m)f_{1}^{(2)} + a_{11}f_{2}^{(2)} + a_{12}f_{3}^{(2)} = N_{1}(t_{2}) - N_{1}(t_{0})
\]

\[
\vdots
\]

\[
(a_{10} - m)f_{1}^{(N)} + a_{11}f_{2}^{(N)} + a_{12}f_{3}^{(N)} = N_{1}(t_{N}) - N_{1}(t_{0})
\]

(A3)

In other words one can assume that \( N_{1}(t_{i}), i=1,2,\ldots, N \) is the actual data while

\[
\hat{N}_{1}(t_{i}) = N_{1}(t_{0}) + (a_{10} - m)f_{1}^{(i)} + a_{11}f_{2}^{(i)} + a_{12}f_{3}^{(i)}
\]

(A4)

is an estimation of \( N_{1}(t_{i}) \). Consequently, in order to determine the coefficients \( a_{ij} \), the mean squared error (objective function) between the observed – actual data and the estimations should be minimized:

\[
MSE = \frac{1}{N_{1}} \sum_{i=1}^{N} \left[ (a_{10} - m)f_{1}^{(i)} + a_{11}f_{2}^{(i)} + a_{12}f_{3}^{(i)} - \Delta N_{1}^{(i)} \right]^{2}
\]

(A5)

The coefficients \( a_{ij} \), for which the MSE is minimized, will be estimated by setting the corresponding partial derivatives equal to zero, leading to the following system of linear equations:

\[
\frac{\partial MSE}{\partial a_{10}} = a_{10} \sum_{i=1}^{N} \left( f_{1}^{(i)} \right)^{2} + a_{11} \sum_{i=1}^{N} f_{1}^{(i)}f_{2}^{(i)} + a_{12} \sum_{i=1}^{N} f_{1}^{(i)}f_{3}^{(i)} - \sum_{i=1}^{N} f_{1}^{(i)} \left( \Delta N_{1}^{(i)} + mf_{1}^{(i)} \right) = 0
\]

\[
\frac{\partial MSE}{\partial a_{11}} = a_{10} \sum_{i=1}^{N} f_{1}^{(i)}f_{2}^{(i)} + a_{11} \sum_{i=1}^{N} \left( f_{2}^{(i)} \right)^{2} + a_{12} \sum_{i=1}^{N} f_{2}^{(i)}f_{3}^{(i)} - \sum_{i=1}^{N} f_{2}^{(i)} \left( \Delta N_{1}^{(i)} + mf_{1}^{(i)} \right) = 0
\]

\[
\frac{\partial MSE}{\partial a_{12}} = a_{10} \sum_{i=1}^{N} f_{1}^{(i)}f_{3}^{(i)} + a_{11} \sum_{i=1}^{N} f_{2}^{(i)}f_{3}^{(i)} + a_{12} \sum_{i=1}^{N} \left( f_{3}^{(i)} \right)^{2} - \sum_{i=1}^{N} f_{3}^{(i)} \left( \Delta N_{1}^{(i)} + mf_{1}^{(i)} \right) = 0
\]

(A6)

Therefore, the coefficients of the first equation of (2) equal to the unique solution of (A6) are given by

\[
[a_{10}, a_{11}, a_{12}]^{T} = \begin{bmatrix}
\sum_{i=1}^{N} \left( f_{1}^{(i)} \right)^{2} & \sum_{i=1}^{N} f_{1}^{(i)}f_{2}^{(i)} & \sum_{i=1}^{N} f_{1}^{(i)}f_{3}^{(i)} \\
\sum_{i=1}^{N} f_{1}^{(i)}f_{2}^{(i)} & \sum_{i=1}^{N} \left( f_{2}^{(i)} \right)^{2} & \sum_{i=1}^{N} f_{2}^{(i)}f_{3}^{(i)} \\
\sum_{i=1}^{N} f_{1}^{(i)}f_{3}^{(i)} & \sum_{i=1}^{N} f_{2}^{(i)}f_{3}^{(i)} & \sum_{i=1}^{N} \left( f_{3}^{(i)} \right)^{2}
\end{bmatrix}
\]

\[
\times \begin{bmatrix}
\sum_{i=1}^{N} f_{1}^{(i)} \left( \Delta N_{1}^{(i)} + mf_{1}^{(i)} \right) \\
\sum_{i=1}^{N} f_{2}^{(i)} \left( \Delta N_{1}^{(i)} + mf_{1}^{(i)} \right) \\
\sum_{i=1}^{N} f_{3}^{(i)} \left( \Delta N_{1}^{(i)} + mf_{1}^{(i)} \right)
\end{bmatrix}
\]

(A7)

Following the same procedure one may estimate the coefficients of the second equation of (2).

A.2. Simulation method

In order to estimate the model’s coefficients through simulation, the Artificial Bee Colony optimization, a recently proposed optimization algorithm, is implemented in MATLAB. By using this kind of algorithm, the objective function defined as the mean squared errors (Eq. (A5)) between the observed (actual data) and the estimated values given by (A4) is going to be minimized.

The ABC optimization method was chosen since it is able to address frequent problems of the traditional methods (ordinary least squares, nonlinear least squares, and maximum likelihood estimation) such as convergence problems, values outside the allowable range, or bias and systematic variation in parameter estimates.

The Artificial Bee Colony algorithm (ABC) was proposed by Karaboga (2005) as an optimization method of multivariable continuous objective functions. ABC algorithm belongs to the wider family of swarm optimization algorithms that are based on swarm intelligence. These kinds of algorithms model the collective behavior of self-organized interacting swarms. Immune system, particle swarm, flock of birds and ant colony are some examples of swarm optimization methods (Sedighizadeh & Masehian, 2009). It has been shown that ABC performance is comparable to other population-based methods (Goldberg, 1989; Karaboga & Akay, 2009) and has been used in several problems (Pan, Fatih Tasgetiren, Suganthan, & Chua, 2011; Szeto, Wu, & Ho, 2011) due to its simple and ease of implementation as well as its reduced number of control parameters. The strongest advantage of ABC algorithm is its independency on the initial values of the examined variables.
In fact, ABC algorithm resembles the foraging operation of honeybees and their swarming around the hive. The interaction between three types of bees: employed, onlooker and scout results in the collective intelligence of the colony. This interaction is presented by the waggle dance of bees during food procuring. It should be noticed that employed bee is a bee currently procuring a food source; onlooker bee is staying in the hive in order to decide a source food while scout bees are those that randomly search for new food sources. The solutions obtained by the ABC algorithm correspond to food sources while the fitness of the solution is described by the amount of nectar of the investigated food source. The interaction of bees with food sources and the sharing of information about the direction and distance to patches of flower and nectar amount through waggle dance lead to the best solution.

ABC algorithm is an iterative process and requires three user parameters: (a) number of food sources (solutions), (b) number of iterations (MCN) and (c) number of cycles before a constant solution (with no improvement) is replaced by a new one, randomly selected by the scout bees. The number of employed and onlooker bees are set equal to the number of solutions that is an employed bee corresponds for every food source.

The initial solutions (during the first step of ABC) are randomly selected as follows:

\[ x_{ij} = LB_j + (UB_j - LB_j) \varphi_{ij}, \quad j = 1, 2, \ldots, n \text{ and } i = 1, 2, \ldots, SN \]  \hspace{1cm} (A8)

where \( LB_j \) and \( UB_j \) is the minimum and maximum value of dimension \( j \) and \( \varphi_{ij} \) is a uniformly distributed random number in the range of \([0, 1]\). The employed bees are sent to the initial sources, evaluate their fitness functions and then return to their hive in order to inform the bees waiting on the dance area about the amount of nectar of the examined sources.

At the next step, the employed bees return to the last known sources and chose a new source in this neighborhood.

\[ v_{ij} = x_{ij} + (x_{ij} - x_{kj}) \varphi_{ij}, \quad j \in [1, 2, \ldots, n] \text{ and } k \in [1, 2, \ldots, SN], \quad k \neq i \hspace{1cm} (A9) \]

where \( x_{ij} \) is the current position (source) of the employed bee and \( \varphi_{ij} \) is a uniformly distributed random number in the range of \([-1, 1]\). It should be noted that the deviation from the current position \( x_{ij} \) decreases as the difference between \( x_{ij} \) and \( x_{kj} \) decreases. Hence, the step adaptively decreases as the algorithm converges. After the selection of the new position, its

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fitness function should be evaluated and compared through a greedy selection mechanism. The current position should be replaced by the new one in the case that its fitness function is smaller than that of the new source.

An onlooker bee selects a food source $x_i$ by calculating its probability:

$$p_i = \frac{f_i}{\sum_{j=1}^{SN} f_j} \quad \text{(A10)}$$

where $f_i$ is the fitness function of source $i$ that is the nectar information gathered by the employed bees. It is evident that the source with the highest fitness function has a bigger probability to be selected. Similar to the employed bees, the onlookers generate a new source using (A9) which is finally selected if its amount of nectar is higher or equal than that of the current source.

If a source cannot be further improved in a predetermined number of cycles, it is abandoned and replaced with a new one produced by scouts through (A8). It should be noted that at most one scout searches for new food source at each cycle of the ABC algorithm.

The basic ABC algorithm is shown in Fig. 14 while its main steps are summarized in the following list:

1. Define the objective and fitness functions of the problem.
2. Initialize control parameters.
3. Allocate food sources to the employed bees.
4. Place the onlooker bees on the food sources according to the amount of their nectar.
5. New food sources are investigated by scouts.
6. Memorize the best source so far.
7. Terminate the process and show the best source if the stopping criteria are satisfied; otherwise return to step 3.

References


