Chapter 2
Fitting Planar Proximity Graphs on Real Street Networks

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Abstract Due to the rising progress of sustainable urban infrastructures, modeling realistic street networks is a fundamental challenge. This study contributes to this modeling direction, by suggesting the utilization of planar proximity graphs, and specifically the β-skeleton graphs. Their goodness of fit on producing real-like urban street networks is verified by comparison to real data. In particular, the basic topological and geometrical properties derived from synthetic β-skeleton planar graphs are compared to the properties of five urban street network datasets, all represented using the Primal approach. A good agreement with empirical patterns is found and a possible explanation is discussed.

2.1 Introduction

There are broad agreements that the street patterns shape overlay infrastructure deployment since they define a basic template which strongly constrains the further development of other webs (e.g., power grid or communication networks). Due to the rising progress of sustainable urban infrastructures, understanding and modeling the structure of street networks is an elementary challenge. Despite a large number of studies on street networks, the existing modeling methodologies are mostly long, random-based and simulation-based, which require several assumptions for generating a realistic street layout, e.g., [1].

On the other hand, the construction of planar proximity graphs can be straightforward by using analytical or simulation methods. Planar proximity graphs are planar graphs (edges intersect only in the points/nodes) where two points in Euclidean plane are connected by an edge if they are close in some sense. Each pair of points is assigned a certain neighborhood, and the points of the pair are connected by an

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S. Battiston et al. (eds.), *Proceedings of ECCS 2014*, Springer Proceedings in Complexity, DOI 10.1007/978-3-319-29228-1_2

edge if their neighborhood is empty. The Delaunay Triangulation (DT), the Relative Neighborhood Graph (RNG), the Gabriel Graph (GG) and the Minimum Spanning Tree (MST) are well known examples of proximity graphs. These are constructed by parameter-less algorithms, given the nodes positions. Specifically, the DT for a set of points in a plane is a triangulation such that no point is inside the circumcircle of any triangle; the RNG is defined by connecting two points whenever there does not exist a third point closer to both points; the GG is a graph where two points have an edge between them if no other point exists in the circumball containing the two points; last, the MST is a tree consisting of all points while having the minimum total weight (length). Though, the β-skeleton graphs [2] constitute a parameterized family of planar proximity graphs where different β values give rise to different graphs.

This study contributes to the urban street modeling, examining the fitness of planar proximity graphs, particularly the β-skeleton graphs, on real street networks with complex characteristics. Additionally, a possible explanation is discussed concerning the findings of the analysis.

The rest of this paper is structured as follows. Section 2.2, contains some preliminaries on the β-skeleton concept. The datasets and the methodology used are described in Sect. 2.3, while the results of applying the methodology are presented in Sect. 2.4. Section 2.5 discusses a possible explanation of the findings and finally Sect. 2.6 concludes the study.

2.2 The β-Skeleton Graphs

In the lune-based neighborhoods approach [2], given a spatial distribution of points S in two-dimensional space, two points u and v are connected by an edge whenever the intersection of the two disks of radius r, centered at the points $c1$ and $c2$, contains no points of S (see Fig. 2.1).

The case $\beta = 0$ corresponds to the DT, $\beta = 1$ corresponds to the GG and $\beta = 2$ corresponds to the RNG. For $1 \leq \beta < \infty$, the radius and the disk centers are defined as follows:

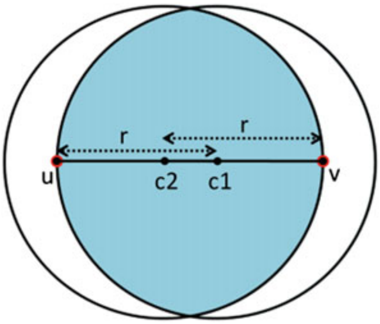

Fig. 2.1 Definition of β-skeleton in the lune-based variant for $1 \leq \beta < \infty$

$$r = \frac{\beta \cdot D(u, v)}{2} \tag{2.1}$$

$$c1 = \left(1 - \frac{\beta}{2}\right) \cdot u + \left(\frac{\beta}{2}\right) \cdot v \tag{2.2}$$

$$c2 = \left(\frac{\beta}{2}\right) \cdot u + \left(1 - \frac{\beta}{2}\right) \cdot v \tag{2.3}$$

while for $0 < \beta < 1$ the two disks pass through both u and v, with radius given by:

$$r = \frac{D(u, v)}{2 \cdot \beta} \tag{2.4}$$

The parameter β determines the size and shape of the lune-based neighbourhood. With the increase of β, the number of edges in the β-skeleton decreases (see Fig. 2.2).

A β-skeleton of a random planar set usually becomes a disconnected graph for $\beta > 2$ and continues losing its edges with further increase of β [3]. On the other hand, as β approaches zero, more and more edges are added to the β-skeleton until it eventually forms the complete geometric graph. For $1 \leq \beta \leq 2$, the following relationships among the different proximity graphs hold for any finite set of points S in the plane:

$$DT(S) \supseteq GG(S) \supseteq \beta - skeleton(S, \beta) \supseteq RNG(S) \supseteq MST(S) \tag{2.5}$$

Since urban street networks are usually connected networks neither DT-like, nor MST-like [4], it is thus of interest to answer to the following questions; (a) is there sufficient accuracy when using β-skeletons with $1 \leq \beta \leq 2$ to reproduce urban street networks? (b) is there a particular β value or subrange of values for which the accuracy is better? (c) what is the possible mechanism that leads real street networks to be associated with particular β values?

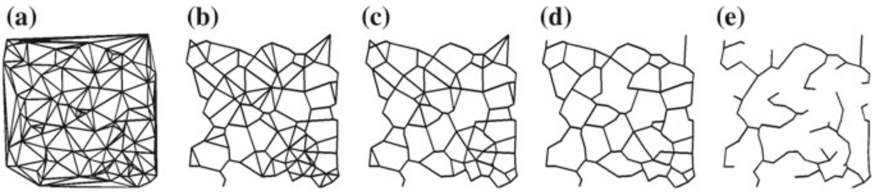

Fig. 2.2 Graph visualizations for the same set of 100 points: **a** delaunay triangulation ($\beta = 0$), **b** Gabriel graph ($\beta = 1$), **c** β-skeleton (here $\beta = 1.4$), **d** relative neighborhood graph ($\beta = 2$), **e** minimum spanning tree

Table 2.1 The datasets used for the goodness of fit verification

Dataset	Number of samples	Area
Cardillo et al. [5]	20	World
Peponis et al. [6]	118	USA
Strano et al. [7]	10	Europe
Chan et al. [8]	21	Germany
Maniadakis et al. [9]	100	Greece

2.3 Dataset and Methodology

2.3.1 Data

Five literature datasets of street networks [5–9] are used in order to compare their properties to those derived from the β-skeleton graphs. The properties of the 269 dataset samples in total (Table 2.1) are here normalized to correspond to $1\,\text{km}^2$ surface.

2.3.2 Methodology

The Primal approach [10] is used in studies [5–9] in order to turn Geographic Information System (GIS) data into spatial, weighted, undirected graphs $G(V, E, L)$ by associating nodes, V, to street intersections and edges, E, to streets (see Fig. 2.3), with length, L, as a weight.

For every sample,[1] obtained using the Primal approach, beyond the number of nodes $|V|$ (graph order), the basic statistical metrics are calculated (see Fig. 2.4); the number of edges $|E|$ (graph size), the density, the average node degree, the diameter, the average shortest path length and the cost (total wiring length).

Then, β-skeleton graphs are produced by simulations, tuning only the number of nodes $(10 \leq |V| \leq 1000)$ and the β parameter $(1 \leq \beta \leq 2)$. The same set of properties is computed for the β-skeleton graphs as well, and the resulting goodness of fit, with respect to the properties of the samples, is evaluated. In particular, the Mean Absolute Percentage Error (MAPE) measure is used for evaluating the comparison of the derived β-skeleton properties with the actual properties.

[1] In the samples where the entire set of these properties is not available, only the available properties are kept.

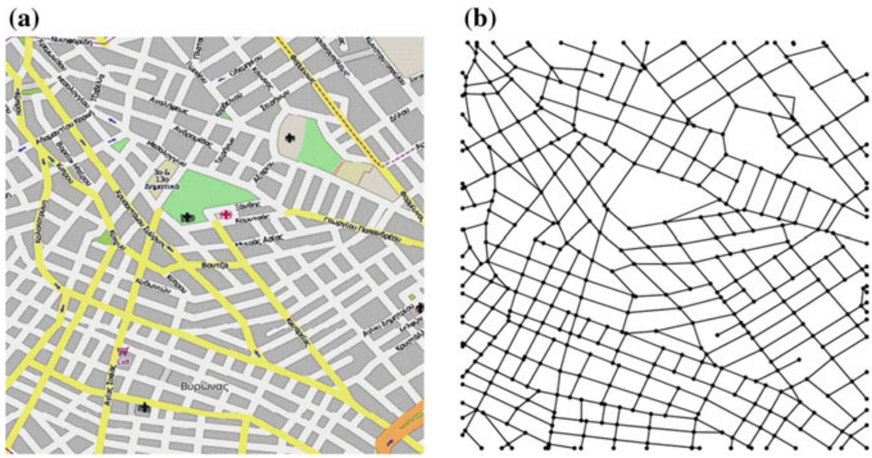

Fig. 2.3 A sample from the dataset of [9]. **a** The conventional street map, **b** the corresponding Primal graph

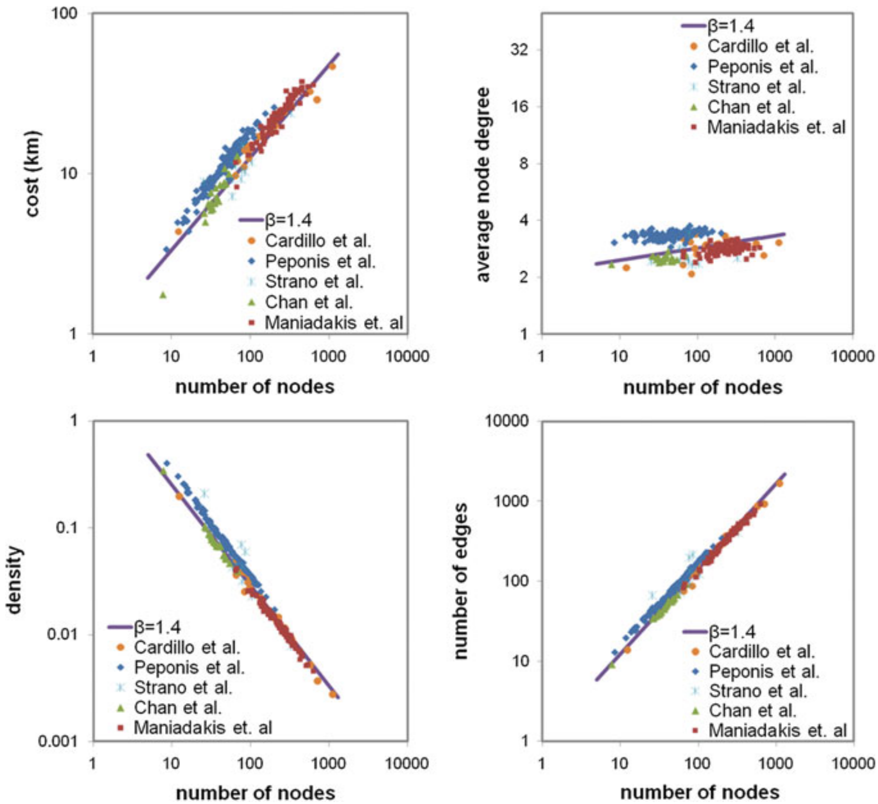

Fig. 2.4 Four of the properties as derived from real street datasets and from synthetic β-skeletons with $\beta = 1.4$ (the rest of the properties are not presented here due to lack of space)

2.4 Results

What the results of this study indicate is the sufficient ability of the β-skeleton graphs on the reproduction of the real urban street networks properties. The MAPE variance is observed in Fig. 2.5 per β value and per dataset. The β-skeletons with $1 \leq \beta \leq 2$ can lead to fitting errors of less than 10 % for certain β values. Specifically, the values of β yielding the highest averaged goodness of fit (lower MAPE), for the entire set of the examined statistical properties and for all five datasets, are found to span between 1.2 and 1.4 with $\beta = 1.4$ having the less errors (Fig. 2.6). Especially, for parameter $\beta = 1.4$ the average of MAPE of all properties and all datasets is as low as 8 %. A recent study by Osaragi and Hiraga [11] has concluded to similar β values too. In particular, they found that β lying between 1.15 and 1.45 corresponds to a maximum "agreement rate" between synthetic β-skeletons and streets of the Tokyo metropolitan region. Even though their study was geographically restricted and limited in the "agreement rate" index without investigating global topological and geometrical properties, however this matching of β values is remarkable.

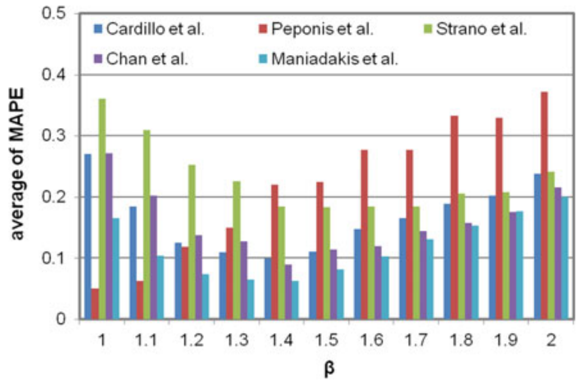

Fig. 2.5 The average of MAPE for each of the five datasets

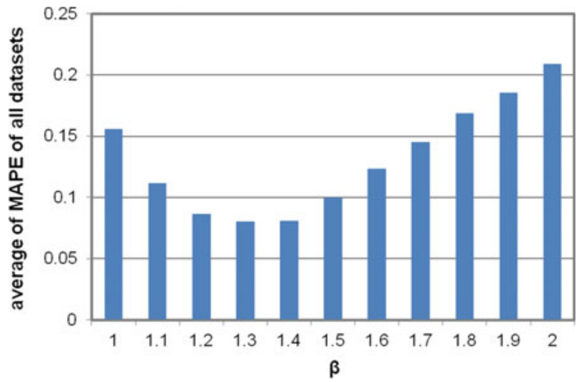

Fig. 2.6 The average of MAPE of all five datasets

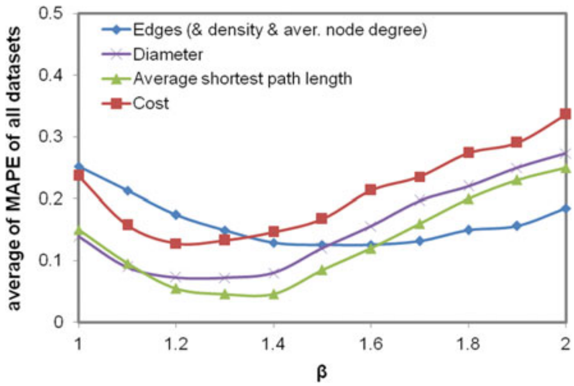

Fig. 2.7 The average of MAPE of all five datasets for the set of the basic statistical properties

The GGs ($\beta = 1$) and the RNGs ($\beta = 2$) can exhibit large errors when fitting real street properties. For instance, for the same planar set of nodes, the GGs produce higher number of edges and the RNGs generate graphs of quite less length compared to actual street networks. More specifically, as observed in Fig. 2.7, the diameter, the average shortest path length and the cost, all have the same behavior in terms of MAPE minimization with corresponding β values varying between 1.2 and 1.4. Regarding the rest of the basic properties, i.e., edges, density and average node degree, the MAPE minimization is shifted to slightly higher β values, approximately between 1.4 and 1.7. This implies that β-skeletons for $1.2 \leq \beta \leq 1.4$ may have almost identical properties with real street networks; however this arises with slightly increased number of edges compared to real street networks of the same order.

2.5 Discussion

Following the findings presented in the previous section, an intriguing question emerges. What is the mechanism that leads the majority of real street networks to have β-skeletons as equivalent graphs with β ranging between 1.2 and 1.4? Since it is natural for one to expect lower β values, as this would imply larger efficiency, is there a mechanism that restricts the network size and structure from going toward the DT characteristics ($\beta = 0$)? In this section, a possible explanation is attempted.

The hypothesis assumed here about the mechanism behind deriving the particular β values is that it may be a consequence of the percentage of land occupied by streets.

In their book [12] Meyer and Gómez-Ibáñez used data from the 1960s to examine the relationship between population density and land area in streets for 15 large cities in the United States. The majority of the investigated cities had a share of land in streets between 20 and 30 %. In addition, the results of a more recent study [13] indicate similar percentages of land for street space. It is likely to believe that this proportion is bounded at a certain level in urban areas, since the remaining land is needed for buildings, parks, plazas, parking, etc., and according to the above-mentioned empirical data this is around 30 %. Starting from this share of land used in streets, i.e., 20–30 %, and then constructing β-skeleton as street network, it is expected that particular β values will derive, as this percentage would limit the street network size and thereafter affect its structure.

More specifically, it is found that specific shares of land in streets correspond to specific values of normalized street network cost. Normalized cost ($cost_{rel}$) is a cost measure defined in [5, 9] with the introduction of two auxiliary graphs for each sample, which serve as two extreme cases; the respective MST (minimum cost) and the respective DT (maximum cost):

$$cost_{rel} = \frac{L_{graph} - L_{MST}}{L_{DT} - L_{MST}} \tag{2.6}$$

Let s be the share of land in streets, e be the surface and w be the width of the street. Then, L_{graph} is defined as:

$$L_{graph} = \frac{s \cdot e + |V| \cdot <k> \cdot w^2}{w} \tag{2.7}$$

where the term $<k>$ stands for the average node degree, therefore $|V| \cdot <k> \cdot w^2$ is added in order to take into account the multiple counting of the land at street intersections. Even though L_{graph} can vary with the number of nodes, the $cost_{rel}$ only slightly varies with the number of nodes, since it is normalized between the MST and the DT values. Thus, the figures that follow depict an average of values, whereas no large deviation is observed when testing 10–1000 nodes/km^2.

The derived normalized cost[2] values are similar to those actually observed in real street networks, e.g., [5]. The street width obviously can vary by city and by country. Here, the indicative values $w = 10$ m, $w = 15$ m, $w = 20$ m are set, with more realistic the case of $w = 15$ m. Actually, a large width w implies less total wiring and thus lower normalized cost. It is observed that 20–30 % share of land in streets corresponds to 20–30 % normalized street network cost (see Fig. 2.8a).

Then, it is possible to associate the normalized cost values with β values, since each β-skeleton corresponds to a cost between the respective MST and the respective DT costs. By running simulations, synthetic β-skeletons are produced and the relationship between normalized cost and β values is found (see Fig. 2.8b). This relationship is used for mapping the normalized cost to specific β values. Indeed, for 20–30 % normalized cost, the corresponding β values belong to the subrange between 1.2 and 1.4 (see Fig. 2.8b). As expected, these are the values of β associated with real urban street networks. This is more clearly seen in Fig. 2.9, where the overall relationship between parameter β and the share of land in streets is drawn, depicting the sensitivity of street width as well. In short, a certain level of structural

[2] It should be noted that the normalized cost is not a measure of construction cost, but only an index of how long the wiring of the graph is, compared to the respective extreme planar graphs (MST and DT).

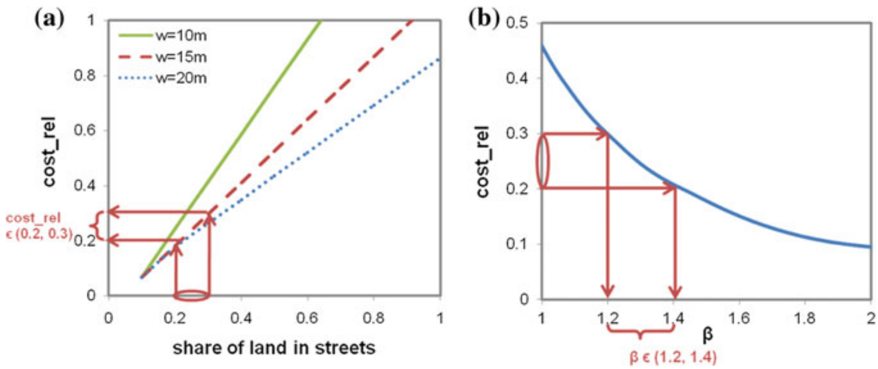

Fig. 2.8 **a** The relationship between normalized cost and share of land in streets, **b** the relationship between normalized cost and parameter β

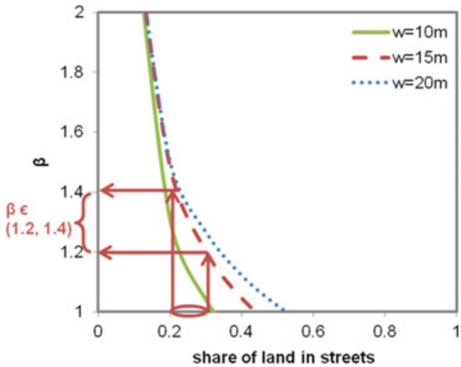

Fig. 2.9 The relationship between parameter β and the share of land in streets

similarities across urban areas as well as some differences may be well captured by the different shares of land in streets, and thus the different β values. Though, for realistic values of street width and reasonable shares of land used in streets, the urban street networks are found to be associated with a specific range of β values; the same range that was empirically observed.

2.6 Conclusion

Planar proximity graphs based on β-skeletons, which change as a response to variations in parameter β, were employed in the present study. Particularly, they were considered from the viewpoint of the topological and geometrical structure and were compared to real street networks.

The good agreement with empirical street data that was found to characterize the β-skeleton graphs induces their utilization—particularly for β values between 1.2 and 1.4—in modeling complex urban street networks and assisting various applications, more essentially those concerning street-constrained processes. Besides, the reasoning behind associating real street networks with the particular β values was discussed and a possible mechanism based on the share of land in streets was sketched.

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