Structural properties of urban street networks of varying population density

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Abstract– In this study the structural properties of urban street networks of varying population density are analyzed. Samples from one hundred street networks from Greece are collected and the Primal approach is used to turn GIS data into spatial graphs. It is discovered that specific measures of street configuration are correlated with population density. Among others, it is presented that urban street networks of high population density tend to be relatively more “cost effective” compared to low population density networks. The empirical features found here may be useful for the planning of infrastructure networks that use the complex road system as a natural guide to reach customers, such as transport, telecommunications, energy or water networks.

Keywords– Infrastructure networks, spatial complex networks, street networks, urban planning.

1. Introduction
In modern society, a street network has a close relationship with human activities and city evolution. It has been shown that the urban morphology affects economic functions, since it provides a framework for interactions, and shapes the movement of populations and uses of land [1]. Also, there are broad agreements that the urban patterns affect overlay infrastructure deployment since they define a basic template that strongly constrains the further development of other webs, such as the power grid or communication networks [2]. In infrastructure networks that serve populations in urban areas, the connections usually run under streets or pavements in trenches, using the road system as a natural guide to reach other infrastructure elements or the customers. This strong dependence of infrastructure network elements (nodes) as well as their connections on the actual geography of the underlying urban street network makes it of great importance for the engineering part of the infrastructure to be able to explicitly take into account the urban street network details together with the corresponding population density.

This becomes more crucial when the underlying urban structural properties deviate from trivial structural properties that occur in planned regular topologies such as lattices. In particular, the urban morphology of an area, in terms of street network topology and geography, may depend on the history, the social processes, the economic activities, the climate, the scale of analysis or more factors that form the overall shape and properties very diverse from area to area [3]. A typology of urban layout classes has already been drawn up: grid-like or Manhattan-like, star-like, medieval, modernist, baroque, lollipop, hippocamian, organic, radio-concentric, self-organized, single-planned, etc. [2, 3]. Common traits are found either between street networks which belong to the same class, such as the levels of efficiency or cost [3], or between all street networks as universal properties, such as the average number of streets on a street intersection or the small average path length and high degree of clustering [2–4]. Nevertheless, a detailed comparison of the properties between urban areas of varying population density is still unknown and their quantitative description is lacking.

Since the population density and the topology of the network constitute two different facets of the spatial organization of an urban area, it is natural to believe that they are correlated. Obviously, the road network evolves to better serve the changing density of population. Indeed, Levinson, in a recent case study [5] about the city of London has demonstrated how the changes in population density and transportation networks deployment are strictly and positively correlated. In [6] it is studied the influence that population density distribution inside an urban area and the respective road network have on each others’ growth and evolution.
The close connection between population density and early economic development has been observed, among others, in [7]. The key idea is that when the population density in an economy is large, the per capita costs are low for the installation and maintenance of an infrastructure. Boerup [8] argued that the main advantage of a dense population is “the better possibilities to create infrastructure”. She also supported that the irrigation technology for agriculture, the building and maintenance of roads, the canalization of a river, and the laying of the railroad system were all possible only with the support of a large population, since high population density helps generate more aggregate demand for an infrastructure sector. Similarly, areas with high population density are expected to have a more “mature” road network to meet their demands. In a recent study [1], higher population densities are associated with more street length and more intersections per square kilometer.

In this contribution, the characteristic topological and geometric properties of urban street networks of varying population density are studied and compared. Street samples of 1-square-kilometer surface from one hundred urban areas from Greece are considered as spatial networks and the Primal approach [9] is used to turn Geographic Information System (GIS) data into graphs by associating nodes to street intersections and edges to streets. Then, graph-theoretic measures are used to characterize and compare the topologies. The major objectives of this study are: a) to identify patterns of urban street networks for the sample areas and b) to study the possible coupling between the topological and geometric properties of the street networks and their population density.

The rest of this paper is organized as follows. Section 2 describes the dataset of street networks used and the methodology utilized for analyzing the corresponding spatial graphs. Section 3 presents the results of the measurements and a discussion concerning the findings. The last section summarizes the contributions made and concludes the study.

2. Data and Methodology

The case of Greece is chosen since it is expected to have a large differentiation on the determinants of the street morphology, i.e., geographical restrictions (hillslope, soil properties, etc.) and historical development (different conquerors, war damages, etc.), between the individual areas along its long history. Therefore, the distinct urban areas are shaped by a variety of factors, beyond the assumed here, population density. Thus, even slight evidence of correlation between the network structural properties and the population density could be satisfactory to support our second study objective.

In Greece there are more than 6,000 municipal departments (MD) with only a small portion of them able to be considered as urban. When examining the 100 MD of the highest population density, the hundredth is about 1,200 people/km² and the first is about 27,000 people/km² [10]. Here, 1-square-kilometer samples of these hundred more densely populated MD are collected. Therefore, the street dataset represents a diverse set in terms of population density. The street data are obtained from the collaborative project OpenStreetMap [11] and the census data from the Hellenic Statistical Authority [10]. The street data are imported in a GIS environment and are turned into spatial, weighted, undirected graphs using the Primal approach [9] where streets intersections and end points are represented as nodes and street segments between successive intersections are represented as edges, as shown in Fig. 1. The samples are square delimited, in order to introduce an equivalent artificial limit for all samples following a procedure common in relevant studies [2, 3].

The derived spatial complex network whose nodes are embedded in a two dimensional Euclidean space and whose edges do not define relations in an abstract space, but are real physical connections can be represented as a graph $G(V, E)$, which consists of a finite set of nodes $V$ and a finite set of edges $E$. The graph nodes have precise position on the planar map $\{(x_i, y_i)\}_{i=1,...,n}$, while the links follow the footprints of real streets and are associated a set of real positive numbers representing the street lengths, $\{l_i\}_{i=1,...,m}$. In the following, the graph representing an area is described by the adjacency $|V| \times |V|$ matrix $A$, whose entry $a_{ij}$ is equal to 1 when there is an edge between $i$ and $j$ and 0 otherwise, and by a $|V| \times |V|$ matrix $L$, whose entry $l_{ij}$ is the weight (physical length) associated to the edge connecting $i$ and $j$. In this way both the topology and the geography metric distances are taken into account.

Beyond the number of nodes $|V|$ and edges $|E|$, the basic statistical metrics are calculated; the graph density that measures the ratio of the number of edges to the maximum number of possible edges, the average node degree which is the average number of edges connected to a node, the average shortest path length that is defined as the average number of steps along the shortest paths for all possible pairs of network nodes, the average clustering coefficient that is a measure of the fraction of triangles present in the network and the average square clustering coefficient that is a measure of the fraction of squares present in the
network. Also, the assortativity property is investigated which is a measure of the preference for network’s nodes to attach to others that are similar in some way, i.e., the node degree. Full definitions may be found in [12, 13]. Moreover, the efficiency of the topology is examined, as proposed by Latora and Marchiori [14]:

\[ E_{\text{topol}} = \frac{1}{|V||\hat{V}| - 1} \sum_{i,j,i \neq j} \frac{1}{d_{ij}} \]  

where \( d_{ij} \) corresponds to the topological shortest path length between the nodes \( i \) and \( j \). Since street networks are spatial, it is possible to work with one more type of distance, the geometric distance. The geometric path length is the sum of the lengths of the edges the path is going through. As a measure of the efficiency in the communication between the nodes of a spatial graph, the measure suggested in [3] is used, where \( d_{ij}^{\text{Euc}} \) is the Euclidian distance between the nodes \( i \) and \( j \):

\[ E_{\text{geom}} = \frac{1}{|V||\hat{V}| - 1} \sum_{i,j,i \neq j} \frac{d_{ij}^{\text{Euc}}}{l_{ij}} \]  

Of course, the counterpart of an increase in efficiency is an increase in the cost of construction, i.e., an increase in the length of the edges. The length or cost of construction can be quantified by using the measure \( W \) defined in formula [3]:

\[ W = \sum_{i,j,i \neq j} d_{ij} l_{ij} \]  

The abovementioned two measures should be normalized in order to be able to compare graphs of different size. While the common procedure in relational non-spatial complex networks is to compare the properties of the original graph with those of some randomized versions of the graph, the main problem with spatial planar graphs is that the random graph or the complete graph are not good ways to normalize the results. Thus, following ref. [15] and [16] respectively, both Minimum Spanning Trees (MST) and Greedy Triangulations (GT) are considered, induced by the real distribution of nodes in the square. MST are the planar graphs with the minimum number of edges in order to assure connectedness, while GT are graphs with the maximum number of edges compatible with the planarity. For each of the hundred MD the respective MST and GT are constructed, which serve as the two extreme cases to normalize the above two structural properties. Following ref. [3] and [17], the values of \( E_{\text{MST}} \) and \( E_{\text{GT}} \) serve to normalize the geometric efficiency results, being respectively the minimum and the maximum value of efficiency that can be obtained in a planar graph having the same number of nodes as in the original graph (Eq. 4). Similarly, the values of \( W_{\text{MST}} \) and \( W_{\text{GT}} \) serve to normalize the cost results, being respectively the minimum and the maximum value of cost (Eq. 5). Therefore, relative geometric efficiency and relative cost are calculated as follows:

\[ E_{rel} = \frac{E_{\text{geom}} - E_{\text{MST}}}{E_{\text{GT}} - E_{\text{MST}}} \]  

\[ W_{rel} = \frac{W - W_{\text{MST}}}{W_{\text{GT}} - W_{\text{MST}}} \]  

Figure 1. The sample of Vironas MD (a) the conventional street map, (b) the corresponding spatial Primal graph, (c) the corresponding auxiliary MST graph and (d) the corresponding auxiliary GT graph.
3. Results and Discussion

Figures 2-4 summarize the main characteristics of the sample networks. Their size varies greatly, ranging from 65 to 633 nodes (per square kilometer) and all define a single connected component. As observed in Fig. 2, the number of edges, the topological efficiency and the total length are all correlated to the number of nodes fitting well a power law.

![Figure 2. Empirical findings of topological and geometric patterns, as a function of the number of nodes](image)

Concerning trends over population density, considerable variation is observed in Fig. 3. However, street networks of low population density appear to have a perceptible different behavior from those of high population density. These trends are here quantified by the linear Pearson product-moment correlation coefficient \( r \). In particular, as population density increases, the street networks tend to have: a higher number of nodes \( (r=0.53) \), a higher number of edges \( (r=0.54) \), a lower graph density \( (r=-0.48) \), a tolerably higher average node degree \( (r=0.25) \), a higher shortest path length \( (r=0.45) \), a lower topological efficiency \( (r=-0.47) \), a lower average street segment length \( (r=-0.41) \), a higher total length \( (r=0.61) \), a lower street length per person \( (r=-0.67) \), a moderately higher \( E_{rel} \) \( (r=0.29) \), a higher \( W_{rel} \) \( (r=0.39) \) and a slightly higher quantity \( E_{rel} - W_{rel} \) \( (r=0.24) \) which may characterize the “cost effectiveness” of a network. The rest of the measures examined here, do not show any recognizable trend with the population density. Though, some observations are worth mentioning. For example, low values are noticed for the average clustering coefficient, but somewhat higher values for the average square clustering coefficient indicating that squares are more present in street networks. In addition, regarding the assortativity property, it is found that most samples appear to be disassortative (values below zero), as also found in [2], thus nodes with large degree tend to be unconnected among them, implying a higher resilience [13].

The quantity \( E_{rel} - W_{rel} \) shows considerable variability from 0.18 (Agios Pavlos MD), to 0.70 (Nea Ionia Volou MD) as seen in the last plot of Fig. 3. The plots in Fig. 4 present some observations on \( E_{rel} \) and \( W_{rel} \). Particularly, \( E_{rel} \) is increasing as \( W_{rel} \) increases, while the quantity \( E_{rel} - W_{rel} \) shows an overall increasing behavior as the average node degree increases. The “cost effective” sample of Nea Ionia Volou MD is depicted in Fig. 5 and corresponds to almost grid-like structure, together with the less “cost effective” sample of Agios Pavlos MD.

The observed tendencies overall support the correlation between specific structural properties and population density, but underline that the variations of the properties are not exclusively determined by the population density. In the scarce literature measurements available [1], this dependence appears even stronger. The correlation between street length per square kilometer and population density appears higher \( (R^2=0.81) \) to what observed here \( (R^2=0.37) \). As well, correlation between street segments and population density is there reported higher \( (R^2=0.50) \) to what is found here \( (R^2=0.17) \). Finally, correlation between intersections per square kilometer and population density is also described stronger \( (R^2=0.72) \) to the correlation observed here \( (R^2=0.28) \).

4. Conclusions

This study examines the urban morphology of one hundred Greek urban areas from a topological and geometric perspective. Here, the samples are heterogeneous in terms of population density. Despite the limitations discussed above, the correlations found in this study support the assumption of coupling between the topological and geometric properties of the street networks and their population density. Among others, it is presented that urban street networks of high population density tend to be relatively more “cost effective” compared to low population density networks. Understanding the complex structure of street networks of
varying population density may bring new insights about the evolution of the urban networks and help the planning of infrastructures whose development occurs in planar constraints and is based on the street system. Future research should focus on incorporating the above findings on the set-up of street network models that will take as input parameter the population density of the area under study. Also, the effect of particular road-dependent infrastructure architectures on the infrastructure performance, e.g., underserved population, during disruptions, e.g., floods, is as well worth studying.

Figure 3. Empirical findings of topological and geometric patterns in 1-square-kilometer street network samples of varying population density
Figure 4. Observations on $E_{rel}$ and $W_{rel}$ (a) MD with high $E_{rel}$ tend to have high $W_{rel}$ and (b) MD with high value of $E_{rel} - W_{rel}$ tend to have high average node degree.

Figure 5. Two extreme street graph cases (a) Nea Ionia Volou MD has the highest value of $E_{rel} - W_{rel}$ and (b) Agios Pavlos MD has the lowest value of $E_{rel} - W_{rel}$.

References