In this paper a system of two interconnected nodes is analyzed. Then, the behavior of the buffers is investigated under some simple routing policies adopted in the first node.

I. Introduction

In this paper, the queueing behavior of the interconnected buffers shown in Fig. 1 is investigated; \( R_{k}^{1} \) denotes the packet traffic which enters node k and is to be forwarded to node j, through possibly more than one path; \( R_{k}^{0} \) denotes the packet traffic departing from node k and forwarded directly to node j. This system of interconnected buffers may be found in the simple topology shown in Fig. 2. At first, the behavior of both buffers of the system shown in Fig. 3 is studied, under a general independent discrete time packet arrival process \( R_{12}^{1} \) and a dependent discrete time packet arrival process \( R_{23}^{1} \). Notice that no routing is incorporated in node 1 and thus \( \lambda_{12}^{1} = \lambda_{23}^{0} \), where \( \lambda_{12}^{1} \) and \( \lambda_{23}^{0} \) denote packet rates associated with the packet processes \( R_{12}^{1} \) and \( R_{23}^{0} \), respectively. Three steps are followed in the analysis of this queueing system. At first, the queueing behavior of buffer 1 is analyzed. Then, the (dependent) packet output process \( R_{12}^{0} \) is described. Finally, buffer 2 is analyzed by incorporating the (dependent) packet processes \( R_{12}^{1} \) and \( R_{23}^{1} \). In the sequel, the behavior of both buffers of the system in Fig. 3 is studied under some simple routing policies applied to node 1, by following the three steps described before. By maintaining a fixed packet output rate, \( \lambda_{23}^{0} \) (assuming that \( \lambda_{23}^{1} \) is constant), it is observed that the routing policies in node 1 result in different queueing behavior of buffer 2. This is due to the fact that, despite the equality of the intensity of the resulting packet rate, different routing policies in node 1 generate statistically different packet output processes \( R_{12}^{0} \).

II. Analysis of two buffers without routing decisions

In this section, the system shown in Fig. 3 is analyzed. The packet service rates at both nodes are constant and equal to one packet per slot; the slot is defined to be the time distance between two consecutive potential packet arrival instants. The packet input process to the first node, \( R_{12}^{1} \), is assumed to be a Generalized Bernoulli Process (GBP). That is, the number of packets arriving at node 1 at the potential arrival instants is an independent process and it follows a general distribution. This process is determined by a message (or multi packet) arrival rate, \( r \), and a general distribution, \( g(j) \), 1\(\leq j \leq N_{B} \), of the message (or multi packet) size in packets; \( N_{B} \) is the maximum message length or number of packets which may arrive during a single slot. The packet input process to node 2 is assumed to be a compound process consisting of two independent packet streams. Input \( R_{12}^{0} \) represents the packet output process from the first node. Input \( R_{12}^{1} \) is assumed to be described by a Markov Modulated Generalized Bernoulli Process (MMGBP) which is, in general, a dependent process. That is, \( R_{23}^{1} \) is described by a discrete time process \( \{a_{i}\}_{i \geq 0} \), which depends on an underlying Markov chain \( \{z_{i}\}_{i \geq 0} \) with transition probability \( \phi(z_{i}, j) \).

Furthermore, it is assumed that there is at most one state, \( x_{0} \) such that \( \phi(x_{0}, 0) > 0 \) and that the rest of the states of the underlying Markov chain result in at least one (but a finite number of) packet arrivals.

![Figure 2](image)

A 3-node element of a topology of a packet communication network.

II.1. Queueing behavior of buffer 1

The queueing behavior of the buffer in node 1 is studied for the case of finite \( K < \infty \) and infinite capacity. The outcome of this study is the derivation of the expressions for the calculation of the first two moments, \( Q_{1} \) and \( Q_{2} \), respectively, and the variance, \( V \), of the buffer occupancy and the mean packet delay, \( D \), induced by buffer 1. Let \( A \) denote any one of these quantities, \( A \in \{Q_{1}, Q_{2}, V, D\} \).

II.1-(a) Infinite buffer capacity.

Closed form expressions for the queueing quantities of interest, \( A \), are computed by using the results of the analysis of the multiplexer in [3]; a single state underlying Markov chain describes the GBP. The following closed form expressions are obtained in this case.

\[
Q_{1}^{1} = \lambda + \frac{\sigma - \lambda}{2(1 - \lambda)}
\]

where \( \sigma \) is the second moment of the packet arrival process. By applying Little's theorem to the previous the following expression for the mean packet delay \( D_{1} \), is obtained.
The variance of the buffer occupancy is given by

\[ V^1 = Q^1_2 - (Q^1_1)^2 \]

where \( Q^1_2 \) is the second moment of the buffer occupancy given by

\[ Q^1_2 = \frac{1}{3(1-\lambda)} \left[ \mu_2^d + 3 \mu_2^d (Q^1_1 - 1 + \lambda) \right] + Q^1_1 \]

where

\[ \mu_2^d = \sum_{v=0}^{\infty} (v-1)(v-2)g(v) \quad \mu_3^d = \sum_{v=0}^{\infty} (v-1)(v-2)(v-3)g(v) \]

### 11.1-(b) Finite (moderate size) buffer capacity

When the capacity \( K \) of the buffer in node 1 is finite and of small or moderate size, then the queuing quantities \( A \) induced by node 1 can be calculated from the following equations

\[ Q^1_1 = \sum_{i=0}^{K} \pi(i) i \quad V^1 = Q^1_2 - (Q^1_1)^2 \]  

(1a)

\[ Q^1_2 = \sum_{i=0}^{K} \pi(i) i^2 \quad D^1 = Q^1_1 / \lambda \]  

(1b)

where \( \pi(i) \), \( 0 \leq i \leq K \), are the steady state probabilities of the Markov chain \( \{d_j\}_{j=0} \), where \( d_j \) denotes the number of packets in the buffer of node 1 at the \( j \)th slot. Since

\[ d_{j+1} = (d_j - 1) + a_j \]

where \( (x)^+ = x \) if \( x \geq 0 \) and 0 otherwise and \( a_j \) denotes the packet arrivals during the \( j \)th slot, it is clear that \( \{d_j\}_{j=0} \) is a Markov chain; its transition probabilities can be easily obtained. The steady state probabilities \( \pi(i) \), \( 0 \leq i \leq K \), can be obtained from the equations

\[ \sum_{i=1}^{K} \pi(i) = 1 \quad \Pi P = \Pi \]

(2)

where \( \Pi \) is the vector of the steady states probabilities and \( P \) is the matrix of the transition probabilities.

### Figure 3

A system of two interconnected buffers without packet routing.

### 11.1-(c) Finite (arbitrarily large) buffer capacity

When the buffer capacity \( K \) is finite but large, the accurate values of the quantities of interest can be obtained, in principle, from equations (1). The steady state probabilities \( \pi(i) \), \( 0 \leq i \leq K \), are obtained from the solution of a large number of equations given by (2). When the system operates outside its instability region (i.e., \( \lambda > 1 - \epsilon, \epsilon > 0 \)), the expected solutions \( \pi(i) \), \( 0 \leq i \leq K \), given by (2) become vanishingly small as \( i \) increases. These solutions may also be inaccurate particularly beyond some \( k_0, k_0 < K \). To overcome this computational difficulty, bounds on the queuing quantities of interest are derived, by introducing the concept of the dominant systems.

Consider the two nodes \( C^\alpha \) and \( C^\beta \) which are identical to node 1, except from the capacity of the corresponding buffers. Node \( C^\alpha \) has an infinite capacity buffer; node \( C^\beta \) has a buffer capacity of size \( L < K \), where \( K \) is the capacity of node 1. Let \( A_j \), denote the queuing quantity \( A \) associated with node \( C^j, j=L,\infty \). It is easy to justify that

\[ Q^1_1 \leq Q_1 \leq Q^\alpha_1 \quad Q^1_2 \leq Q_2 \leq Q^\beta_2 \]

(3a)

\[ D^1 \leq D \leq D^\alpha \]

(3b)

\[ Q^1_1 - (Q^1_1)^2 \leq V = Q_2 - (Q_2)^2 \leq Q^\beta_1 - (Q^\beta_1)^2 \]

(3c)

where \( Q_1, Q_2, V, \) and \( D \) are the queuing quantities, \( A \), associated with node 1. The previous equations establish upper and lower bounds on \( Q_1, Q_2, V, \) and \( D \). In most practical applications, where buffer overflow is not desired, \( K \) should be sufficiently large so that \( \pi(K) \) be extremely small. Under such condition it is expected that \( A=A^\alpha, A \in [Q_1,Q_2,D,\bar{V}] \).

To analyze the queuing behavior of buffer 2 an accurate description of the packet (output) process generated by node 1 is required.

### 11.2. The packet output process of node 1, \( R^2_j \)

Although the packet input process to node 1, described by a GBP, is an independent one, the process of the packets departing from this node is not, due to the dependencies introduced by the operation of the node. This process is shown to be accurately described by a MMGBP.

Let \( \{d_j\}_{j=0} \) be the Markov chain defined in section 11.1-(b) which describes the number of packets in node 1 at the end of the \( j \)th slot. Let \( S = \{0,1,2, \ldots, K\} \) be its state space, where \( K, K=\infty \), is the buffer capacity of node 1. If \( K \) is finite and of small or moderate value, then \( \{d_j\}_{j=0} \) can serve as the underlying Markov chain of the MMGBP to be used for the description of \( R^2_j \). If \( K \) is very large or infinite, then a new Markov chain, \( \{d'_j\}_{j=0} \), with a state space of reduced cardinality \( L, L < K \), will be incorporated in the description of \( R^2_j \), to lead to tractable computations of the queuing quantities in node 2. Although the description of \( R^2_j \) based on \( \{d'_j\}_{j=0} \) is an approximate one, it turns out that it results in a very accurate calculation of the queuing quantities of interest at node 2. The MMGBP which describes the packet output process \( R^2_j \) is determined by the probability distribution \( \phi(d_j,k) \), where

\[ \phi(0,0)=1 \quad \phi(d_j,1) = 1 \text{ for } d_j > 0 \quad \phi(d_j,d_j^+) \]

(4)

since node 1 outputs one packet when \( d_j > 0 \) and zero packets otherwise.

Let \( \{a_j\}_{j=0} \) and \( \{a'_j\}_{j=0} \) be the packet output processes determined by the probability distribution given by (4) and the underlying Markov chains \( \{d_j\}_{j=0} \) and \( \{d'_j\}_{j=0} \), respectively. As it is mentioned before, the queuing quantities \( A \) associated with node 2 are fairly accurately calculated under the approximation of the true packet output process \( \{d_j\}_{j=0} \) by the process \( \{d'_j\}_{j=0} \), as long as node 1 operates outside its instability region. Under the latter conditions, states \( i, i > k_0 \), for some \( k_0 < L \), are almost never visited by the true Markov chain \( \{d_j\}_{j=0} \). Thus, the Markov chain \( \{d'_j\}_{j=0} \), for some \( L > k_0 \), is expected to be a good approximation of \( \{d_j\}_{j=0} \). In addition, the number of packets, \( a_j \), generated by node 1 in the \( j \)th slot, is the same under both Markov chains and independent of the their state, as long as it is a nonzero state (see (4)). Due to the latter observation, an additional refinement in the approximation of \( \{d_j\}_{j=0} \) by \( \{d'_j\}_{j=0} \), as seen from the output process \( \{a'_j\}_{j=0} \), is introduced, as long as \( L > 0 \). Finally, the queuing process in node 2 is a complex one and it may also introduce some smoothing on the differences between the
processes \(a_{j_1}^{1} \geq 0\) and \(a_{j_1}^{2} \geq 0\), and consequently deemphasize the difference between \(d_{j_1}^{1} \geq 0\) and \(d_{j_1}^{2} \geq 0\), as inferred from the values of the queueing quantities \(A\) evaluated at node 2. The previous arguments offer some non-rigorous explanations of the expected (and observed) accuracy of the approximation of \(d_{j_1}^{1} \geq 0\) by \(d_{j_1}^{2} \geq 0\) (and, consequently, of the approximation of \(a_{j_1}^{2} \geq 0\) by \(a_{j_1}^{1} \geq 0\)), as measured by the accuracy of the calculation of the queueing quantities \(A\) at node 2.

11.3 Queueing behavior of buffer 2

At this point, the queueing behavior of node 2 is studied under the packet arrival processes \(R_{23}^0\) and \(R_{23}^1\), each of which is modeled as a MMGBP; \(R_{23}^0\) is the packet output process from node 1 and it is described in section 11.2; \(R_{23}^1\) is an arbitrary MMDBP as defined at the beginning of section II. The queueing quantities \(A\) associated with node 2 are calculated when the cardinalities of the Markov chains associated with \(R_{23}^0\) and \(R_{23}^1\) are of small or moderate value.

When the capacity of buffer 2 is infinite, then the analysis presented in [3] can be applied and the queueing quantities of interest \(A\), \(A(\Omega_1,\Omega_2,V,D)\) be computed. The obtained results are exact if the underlying Markov chain associated with \(R_{23}^1\) is the true one (i.e. if the capacity of buffer 1 is of small or of moderate size) and they are approximate otherwise.

When the capacity of buffer 2 is (i) of small or moderate size or (ii) large but finite and the operation of node 2 is away from its instability region, then the queueing quantities of interest can be computed as described in sections II.1-(b) and II.1-(c), where the Markov chain \(d_{j_1}^{1} \geq 0\) is replaced by the 3-dimensional Markov chain \(a_{j_1}^{2} \geq 0\). \(d_{j_1}^{1} \geq 0\) denotes the underlying Markov chain of the packet arrival process \(R_{23}^0\); \(a_{j_1}^{2} \geq 0\) denotes the underlying Markov chain of the packet arrival process \(R_{23}^1\); \(d_{j_1}^{1} \geq 0\) denotes the buffer occupancy of node 2.

11.4 Analysis of two buffers under some routing policies

In this section, the queueing system shown in Fig. 1 is studied. The only difference between this system and the one shown in Fig. 3 (studied in section II) is that routing decisions diversify some of the packet input traffic to node 1, \(R_{13}^0\). The system shown in Fig. 2 appears in network topologies such as the one shown in Fig. 2, where the packet traffic which enters node 1 and is to be forwarded to node 3 has two alternate routes; a direct one from node 1 to node 3 and an indirect one through node 2. The routing policies at node 1 may be adopted for the regulation of the rate of the traffic which is forwarded to node 2. As a result, overloading of the links between nodes 2 and 3 and between nodes 1 and 3, may be avoided. The following routing policies will be considered; all packets to be forwarded to node 2 are stored in buffer 12.

\((P_1)\) Buffer 12 stores all packets up to a maximum number \(\Theta_{\infty}\).

\((P_2)\) Buffer 12 stores half (or a portion) of the packets arriving over a slot (or half of them plus one in case of an odd number of packet arrivals), according to a deterministic splitting, up to a maximum \(\Theta_2\); \(\Theta_2\) can be infinite.

11.1 Queueing behavior of node 1

The queueing quantities of interest \(A\), \(A(\Omega_1,\Omega_2,V,D)\), are computed by applying the analysis presented in section II. 1-(b), where the buffer capacity \(K\) is set to be equal to \(\Theta\) under policy \(P_1\). The analysis presented in sections II.1-(b), II.1-(c) or II.1-(a) is applied for the calculation of \(A\), \(A(\Omega_1,\Omega_2,V,D)\), under policy \(P_2\), depending on whether the buffer capacity \(\Theta_2\) is small, large or infinite, respectively. Notice that the splitting of the packets which arrive over the same slot modifies the message length distribution \(g(j)\), as seen from buffer 12, resulting in a better randomized packet output process under this routing policy.

11.2 The packet output process from node 1, \(R_{23}^0\)

The packet output process \(R_{23}^0\) is modeled as a MMGBP, as described in section II.2. The (exact) underlying Markov chain \(d_{j_1}^{1} \geq 0\) is incorporated in the description of the packet output process \(a_{j_1}^{1} \geq 0\) under policy \(P_1\), (if \(\Theta_2\) is small). The (approximate) underlying Markov chain \(d_{j_1}^{1} \geq 0\) is incorporated in the description of the (approximate) packet output process \(a_{j_1}^{2} \geq 0\), under policy \(P_2\), when \(\Theta_2\) is very large or infinite.

11.3 Queueing behavior of node 2

The queueing quantities of interest \(A\), \(A(\Omega_1,\Omega_2,V,D)\), are computed by applying the analysis approach presented in section 11.3.

11.4 Numerical results

The following parameters for the input process to node 1 are considered: \(N_{10}=5, g(1)=1, g(2)=3, g(3)=3, g(4)=1, g(5)=1\) and various values of the message arrival rate \(r\). Let \(r_{in}\) and \(r_{out}\) be the packet input and the packet output rates associated with node 2.

At first, the system shown in Fig. 3 is considered. For various values of \(r\) and \(K\) (the capacity of buffer 1), \(1 \leq K \leq \infty\), the queueing quantities \(A\), associated with node 1, are exactly computed. The results are shown in Table 1. From these results, the monotonicity of the quantities \(Q_1, Q_2\) and \(D\) with respect to the buffer capacity \(K\), as indicated by equations (3), is clearly observed. Notice also that the values of \(A\), computed for the case of the largest finite value of \(K\) shown in Table 1, practically coincide with those obtained under infinite buffer capacity. The latter suggests that the closed form expressions mentioned in 11.1-(a) may be used for the computation of \(A\), for any buffer capacity which is larger than a certain value.

When the buffer capacity of buffer 1 is finite and equal to 20 and the capacity of buffer 2 is infinite then, the queueing quantities of interest \(A\), associated with node 2 are shown in Table 2; \(\gamma\) is the clustering coefficient associated with the packet output process from node 1, \(R_{23}^0\), defined as \(\gamma = p(0,0)-p(0,0)\), where 0 is the zero packet generating state of the underlying Markov chain of the MMGBP which models the process \(R_{23}^0\) (as described in section II.2), and 0 is the union of all the packet generating states of this Markov chain; \(p(0,0)\) denotes the transition probability from 0 to 0. As it is observed below, the parameter \(\gamma\) affects the values of the queueing quantities of interest associated with node 2. The (independent from \(R_{23}^0\) packet arrival process \(R_{23}^0\) is assumed to be a 2-state MMGBP with clustering coefficient \(\gamma = 2\) and packet arrival rate equal to \(s_{out}\), where \(r_{out}\) is the packet rate of \(R_{23}^0\) and \(s_{out}\) is the total packet traffic load offered to node 2. A 2-state MMGBP is completely determined by \(r\) and \(\gamma\). Notice that for \(r=0.20\) the behavior of buffer 1 is identical to that of a buffer with infinite capacity \(r_{out}^K\). Where \(r_{out}^K\) is the packet output rate of a buffer of capacity \(K\) and that the clustering coefficient \(\gamma\) is identical to that corresponding to the packet output process of an infinite capacity buffer. The latter is true since practically no packet rejection, which would modify the clustering of the packet output process, takes place and, thus, \(\gamma\) is completely determined by the message size distribution \(g(j)\) and not by the message input rate to node 1, \(r\). For \(r=0.3\), some packet rejection takes place \(r_{out}^K < 0.778 < r_{out}^{out}\) which leads to a slightly reduced clustering coefficient \(\gamma\).

When the capacity of the buffer 1 is infinite, then the queueing results in node 2 are obtained as presented in 11.1-(a), where the MMGBP describing the packet output process \(R_{23}^0\) is approximately described based on a truncated Markov chain with state space of cardinality \(L\), as described in section II.2. The results for the queueing quantities of interest associated with node 2 are...
shown in Table 3, for various message input rates to the first node, r, and different values of L. For message input rates r less than .20, a truncation of the true underlying Markov chain associated with R_{2} as L=20 gives result which remain, in essence, unchanged for any L>20; the latter is observed at L=50 when r=.3. If processes R_{2} and R_{3} are approximated by a Bernoulli process, then the mean packet delay induced by node 2 is given in Table 3, for the various input rates r and total traffic load offered to node 2 equal to .9. By comparing the mean delay results in node 2, shown in Table 3, under the MMGDP and the Bernoulli process modeling R_{2}, it is established that the Bernoulli approximation leads to significant underestimation of the queueing problems induced by node 2.

Finally, when the packet routing policies P_{1} and P_{2} are considered in buffer 1, the queueing quantities of interest associated with node 1 and 2 are shown in Tables 4 and 5, respectively. The message size probability distribution g(j), 1≤j≤5, remains the same as before. The message arrival rate is different in each case and it is such that the output rate r_{a} be equal to .45; r_{a} = .45 and γ = .3. Various values of the buffer limit θ have been considered under policy P_{1}. Notice that the value of A, increases in both nodes as θ increases, although r_{a} remains constant, due to the increased clustering γ (affecting the value of A associated with node 2) and the increased buffer size θ (affecting the value of A associated with node 1), under the same probability distribution g(j), 1≤j≤5. Finally, the results under policy P_{2} and for various values of the buffer constraint θ (denoted by θ) are given in the same table. The new probability distribution g(j), 1≤j≤3, is given by g(1)=g(1)+g(2)=.5, g(2)=g(3)+g(4)=.4 and g(3)=g(5)=.1. Notice that as the buffer constraint θ increases the values of A, also increase for the reasons stated before. Notice that the values of A under policy P_{2} are smaller than these under policy P_{1} corresponding to the same value of θ, the reason being that the probability distribution g(j), 1≤j≤3, has been changed under policy P_{2} and less clustered packets are generated by g(j). The values of A obtained for θ=10, under policy P_{2} remain unchanged if a larger value of θ is considered. Thus, these values of A can be considered to be equal to the ones obtained for θ=∞.

### Table 1

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### Table 2

Queueing behavior of buffer 2 without routing policies in node 1 and buffer 1 capacity K=20 (γ=.615).

### Table 3

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### Table 4

Queueing behavior of buffer 2 without routing policies in node 1 and infinite capacity of buffer 1 (γ=.615).

### Table 5

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<th>D</th>
</tr>
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### Table 6

Queueing behavior of buffer 1 under some routing policies with r_{a}=.45.

### References

