Analysis of a Statistical Multiplexer under a General Input Traffic Model

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ABSTRACT

In this paper we study the queueing behavior of a statistical multiplexer with a finite number of input lines. Each of the input lines is assumed to deliver fixed length packets of information according to a generally distributed process, whose parameters depend on the state of an underlying finite state Markov chain associated with each of the input lines. Special cases of this general model for the packet arrival processes have been studied in the past.

A method for the derivation of the moments of the buffer occupancy is developed and the first and the second moments are derived. The mean packet delay introduced by the statistical multiplexer is then derived through Little's theorem. Finally, the applicability of the analyzed multiplexer in packet communication systems is illustrated through a simple example and numerical results are provided for this case.

I. Introduction

Statistical multiplexers are common elements of communication networks used to coordinate the allocation of a single transmission media among many potential users. The statistical multiplexer consists of a buffer for the temporary storage of information packets awaiting transmission, and the server that carries out the actual transmission task over the single output channel. In this paper N users (or input lines) are assumed to be served by the statistical multiplexer. Potential packet arrival instants are synchronized and at most one packet can be served during the time interval between consecutive potential arrival instants. More than one packet arrivals per input line are possible at the same arrival instant.

Previous work on similar statistical multiplexer can be found in [1]-[5] (and the references cited there). All previous models differ significantly from the one presented here. In [1], the authors assume a single arrival line and a two state Markov Modulated Poisson arrival process. In [2], the author considers a single input line and arrivals that depend on an underlying two state Markov chain. In [3]-[5] N input lines are assumed present. In [3], it is assumed that the packet arrival process of each of the identical input lines depends on an underlying two state Markov chain (active/inactive). In [4] it is assumed that the per line packet arrival process is a first order Markov chain and at most one packet arrival is possible. A closed form solution for the mean packet delay has been derived for the latter case. In [5], a closed form expression for the mean packet delay in the case of Bernoulli per line arrivals can be found. The systems presented in [3]-[5] (and some special cases of the system in [2]), can be seen as special cases of the general system investigated here.

As it will be illustrated in the last section of this work, special cases of the general statistical multiplexer (described in the next section) result in practical systems which cannot be handled by the previous work in this area.

II. The Statistical Multiplexer

Consider a statistical multiplexer fed by N input lines. The input lines (which are mutually independent) are assumed to be slotted and packet arrivals and service completions are synchronized with the end of the slots. A slot is defined to be equal to the fixed service (transmission) time required by a packet. At most one packet can be served during one slot. The first-in first-out (FIFO) service discipline is adopted. Packets arriving at the same slot are served in a randomly chosen order. The buffer capacity is assumed to be infinite. The packet arrival process associated with line i is defined to be the discrete time process $\{a_j^i\}_{j\geq 0}$, $i=1,2,\cdots,N$, of the number of packets arriving at the end of the j^{th} slot; $a_j^i=k$, $0\leq k<\infty$, if k packets arrive at the end of the j^{th} slot through input line i.

Let $\{z_j^i\}_{j\geq 0}$, be a finite state Markov chain imbedded at the end of the slots, which describes the state of the input line i. Let $S^i=\{x_0^i,x_1^i,\cdots,x_{M^i-1}^i\}$, $M^i<\infty$, be the state space of $\{z_j^i\}_{j\geq 0}$. It is assumed that the state of the underlying Markov chain determines (probabilistically) the packet arrival process of the corresponding line. That is, if $a^i(x^i) : S^i \to Z_0$, is a probabilistic mapping from S^i into the nonnegative finite integers, Z_0, then the probability that k packets arrive at the buffer at the end of the j^{th} slot is given by $\phi(z_j^i,k)=\Pr\{a^i(z_j^i)=k\}$. Furthermore, it is assumed that there is at most one state, x_0^i such that $\phi(x_0^i,0)>0$ and that the rest of the states of the underlying Markov chain result in at least one (but a finite number of) packet arrivals, i.e. $\phi(x_k^i,0)=0$, for $1 \le k \le M^i-1$. All packet arrivals are assumed to occur at the end of the slots. To avoid instability of the buffer queue it is assumed that there is always one state x_0^i, such as described above.

III. Analysis of the statistical multiplexer

IIIa. General case : Asymmetric system.

In this section we study the statistical multiplexer described before. In particular, a method for the derivation of the moments of the buffer occupancy is developed and the first moment is computed. The mean packet delay is then derived through Little's theorem. The asymmetry of the system is due to the fact that although all packet arrival processes are described by the same general model, no two of them are identical.

Let $\pi^i(k)$ and $p^i(k, j)$, $k,j \in S^i$, denote the steady state and the transition probabilities of the ergodic underlying Markov chain, $\{z_j^i\}_{j\ge0}$, associated with the i^{th} input line, $i = 1, 2, \cdots, N$. Let also $p^n(j ; \bar{y})$ denote the joint probability that there are j packets in the system at the n^{th} time instant, or beginning of slot, (arrivals at that point are included) and the states of the Markov chains are $y^1, y^2, \cdots, y^N$, where $\bar{y} = (y^1,y^2, \cdots ,y^N)$; the arrivals which result from the state $\bar{y}$ are not included at this time instant. The vector $\bar{y}$ describes the state of a new ergodic Markov chain generated by the N independent Markov chains described before. Let $\pi(\bar{y})$ and $p(\bar{x}, \bar{y})$ be the steady state and the transition probabilities, respectively, and $\bar{S} = S^1 x S^2 x \cdots x S^N$ be its state space. The evolution of the buffer occupancy can be described by an N + 1 dimensional Markov chain imbedded at the beginning of the slots, with state space $T = (0, 1, 2, \cdots) x \bar{S}$ and state probabilities given by the following recursive equations

$$p^n(j ; \bar{y}) = \sum_{\bar{x} \in \bar{S}} \sum_{\nu=0}^{R} p^{n-1}(j+1-\nu ; \bar{x}) \, p(\bar{x}, \bar{y}) \, g_{\bar{x}}(\nu) \quad , \quad j \ge R+1 \tag{1a}$$

$$p^n(j;\bar{y}) = \sum_{\bar{x}\in\bar{S}} \sum_{k=1}^{j+1} p^{n-1}(k;\bar{x})p(\bar{x},\bar{y})g_{\bar{x}}(j+1-k) + \sum_{\bar{x}\in\bar{S}} p^{n-1}(0;\bar{x}) \, p(\bar{x}, \bar{y})g_{\bar{x}}(j) \, , \, 0 \le j \le R \tag{1b}$$

where R is the maximum number of arrivals from all input lines over a slot, $\bar{x}$ is the state of the N-dimensional Markov chain at time instant n-1 and

$$g_{\bar{x}}(\nu) = \Pr\{\sum_{i=1}^{N} a^i(x^i) = \nu\} \tag{2a}$$

$$\text{with} \quad \mu_{\bar{x}} = \sum_{\nu=1}^{R} \nu g_{\bar{x}}(\nu) \;\; , \;\; \sigma_{\bar{x}} = \sum_{\nu=1}^{R} \nu^2 g_{\bar{x}}(\nu). \tag{2b}$$

$g_{\bar{x}}(\nu)$ is the probability that the N dimensional underlying state $\bar{x}$ results in ν packet arrivals. There are totally $M^1 x M^2 x \cdots x M^N$ equations given by (1) for a fixed j and all $\bar{y} \in \bar{S}$, where M^i is the cardinality of S^i, $i = 1, 2, \cdots, N$.

Ergodicity of the Markov chains associated with the input streams implies the ergodicity of the arrival processes $\{a_j^i\}_{j\ge0}$, $i = 1, 2, \cdots, N$. The latter together with the ergodicity condition for the total average input traffic λ

$$\lambda = \sum_{\bar{x}\in\bar{S}} \mu_{\bar{x}} \pi(\bar{x}) < 1 \tag{3}$$

imply that the Markov chain described in (1) is ergodic and there exist steady state (equilibrium) probabilities. Thus, we can consider the limit of the equations in (1) as n approaches infinity and obtain similar equations for the steady state probabilities. By considering the generating function of these probabilities, manipulating the resulting equations, differentiating with respect to z and setting z=1, we obtain the following system of equations.

$$P'(1;\bar{y}) = \sum_{\bar{x}\in\bar{S}} P'(1;\bar{x}) \, p(\bar{x},\bar{y}) + \sum_{\bar{x}\in\bar{S}} (\mu_{\bar{x}}-1)p(\bar{x},\bar{y})\pi(\bar{x}) + \sum_{\bar{x}\in\bar{S}} p(0;\bar{x})p(\bar{x},\bar{y}) \, , \, \bar{y}\in\bar{S} \tag{4}$$

where $P'(1;\bar{y})$ denotes the derivative of the generating function of the probability distribution $p(j;\bar{y})$ evaluated at z=1,

$$\pi(\bar{x}) = \prod_{i=1}^{N} \pi^i(x^i) \;\; , \;\; p(\bar{x},\bar{y}) = \prod_{i=1}^{N} p^i(x^i,y^i)$$

and $p(0;\bar{x}) = p_0 p(\bar{x}_0,\bar{x})$; $p_0 = 1-\lambda$ is the probability that the buffer of the multiplexer is empty and $\bar{x}_0 = (x_0^1,x_0^2, \cdots ,x_0^N)$ is the only state that results in no packet arrival. The above equations are linearly dependent and thus an additional equation is required. By adding all the above equations and using L'Hospital's rule we obtain an additional linear equation which is linearly independent of those in (4) and it is given by

$$\sum_{\bar{x}\in\bar{S}} \Big[2(\mu_{\bar{x}}-1)P'(1;\bar{x}) + 2(\mu_{\bar{x}}-1)p(0;\bar{x}) + [2+\sigma_{\bar{x}} - 3\mu_{\bar{x}}]\pi(\bar{x}) \Big] = 0 \tag{5}$$

By solving the $M^1x \cdots xM^N$ dimensional linear system of equations which consists of (5) and any $M^1x \cdots xM^N - 1$ equations taken from (5), we compute $P'(1;\bar{x})$, $\bar{x} \in \bar{S}$. Then, the average number of packets in the system, $Q=P'(z)$, can be computed by adding all the solutions. The average time, D, that a packet spends in the system can be obtained by using Little's formula as the ratio $D=Q/\lambda$.

The k^{th} derivative of $P(z)$, evaluated at $z=1$, gives the k^{th} factorial moment of the number of packets in the system, [6], it turns out that the k^{th} moment of the buffer occupancy can be evaluated by differentiating (4) k-1 times, setting $z=1$ and solving the resulting system equations. By differentiating (4) and setting $z=1$, we get the following system of linear equations with respect to $P''(1;\bar{y})$, $\bar{y} \in \bar{S}$,

$$P''(1;\bar{y})=\sum_{\bar{x}\in\bar{S}} p(\bar{x},\bar{y})P''(1;\bar{x})+\sum_{\bar{x}\in\bar{S}}\Big[\mu_{\bar{x}}^{2f}\pi(\bar{x})+ +2\mu_{\bar{x}}^{1f}[P'(1;\bar{x})+p(0;\bar{x})]\Big]p(\bar{x},\bar{y}) \quad (6)$$

where

$$\mu_{\bar{x}}^{1f}=\sum_{\nu=0}^{R}(\nu-1)g_{\bar{x}}(\nu) \quad , \quad \mu_{\bar{x}}^{2f}=\sum_{\nu=0}^{R}(\nu-1)(\nu-2)g_{\bar{x}}(\nu)$$

The required linearly independent equation is obtained as in the previous case and it is given by

$$\sum_{\bar{x}\in\bar{S}} 3\Big[\mu_{\bar{x}}-1\Big]P''(1;\bar{x})=-\sum_{\bar{x}\in\bar{S}}\Big\{\mu_{\bar{x}}^{3f}+3\mu_{\bar{x}}^{2f}\Big[P'(1;\bar{x})+ + p(0;\bar{x})\Big]\Big\} \quad (7)$$

where

$$\mu_{\bar{x}}^{3f}=\sum_{\nu=0}^{R}(\nu-1)(\nu-2)(\nu-3)g_{\bar{x}}(\nu)$$

By solving the linear equations given by (6) and (7) we compute $P''(1;\bar{x})$, $\bar{x} \in \bar{S}$. The second factorial moment of the number of packets in the system, $P''(1)$, is obtained by adding all the solutions. Finally, the variance V can be obtained from, [6],

$$V=P''(1)+P'(1)-\Big[P'(1)\Big]^2 \quad (8)$$

Notice that the solution of the same number of equations is required for the computation of any moment. Furthermore, the coefficients of the unknown quanitities are the same in all systems of equations except from those of the last linearly independent equation.

Consider the special case in which the per stream arrival process is Bernoulli. The underlying Markov chain has one state and the equations (4) and (5) become

$$p(0)=1-\mu \quad (4')$$

and

$$2(\mu-1)P'(1)+2(\mu-1)p(0)+[2+\sigma-3\mu]=0 \quad (5')$$

where

$$\mu=\sum_{i=1}^{N}\lambda^i \quad , \quad \sigma=\sum_{i=1}^{N}\lambda^i(1-\lambda^i)+\mu^2$$

and where λ^i is the packet arrival rate of the i^{th} line. From (6'), by substituting (5') and manipulating the resulting expression, we get the following equation with respect to $P'(1)$

$$Q_b=P'(1)=\frac{\sum_{i=1}^{N}\sum_{j\geq 1}^{N}\lambda_i\lambda_j+\mu(1-\mu)}{(1-\mu)} \quad (9)$$

where Q_b denotes the average number of packets in the system. The mean packet delay, D_b is given by Q_b/μ, which is a known result, [5]. The variance of the number of packets in the system, which is useful for the buffer design, has not been derived before for the case of Bernoulli per line packet arrival processes. In this case equation (9) becomes

$$P''(1)=\frac{1}{3(1-\mu)}\Big[\mu^{3f}+3\mu^{2f}Q_B+3\mu^{2f}(1-\mu)\Big] \quad (10)$$

A closed for expression for the variance of the number of packets in the system, V_b, can then be obtained in this case by using (8)-(10).

IIIb. Special case : Symmetric system.

Let us now assume that the input processes are identical, i.e. the parameters of all such processes are identical. Let M be the cardinality of any of the involved one dimensional Markov chain. As it will be shown shortly, the number of equations which need to be solved for the calculation of the mean delay in the queueing system is reduced significantly. This can be easily seen by observing that the unknown quantities in (4) and (5), $P'(1;\bar{x})$, are the same for certain values of $\bar{x}$. For instance, the quantity that corresponds to state $\bar{x}=(x_1,x_2,x_3,\cdots,x_N)$ is equal to that of state $\bar{x}=(x_2,x_1,x_3,\cdots,x_N)$.

If $\bar{v}(\bar{x})=(v_1(\bar{x}),v_2(\bar{x}),\cdots v_M(\bar{x}))$ is an M-dimensional vector with $v_i(\bar{x})$, $i=1,2,\ldots,M$, denoting the number of input processes at state S_i, then each such vector $\bar{v}(\bar{x})$ with the constraint $\sum_{i=1}^{M}v_i(\bar{x})=N$, represents a class of equivalent states $\bar{x}$. The number of equivalent states $\bar{x}$ in a class $\bar{v}(\bar{x})$ is given by (pp. 20, [9])

$$c(\bar{x})=\binom{N}{v_1(\bar{x}),v_2(\bar{x}),\cdots v_M(\bar{x})}=\frac{N!}{v_1(\bar{x})!v_1(\bar{x})!\cdots v_M(\bar{x})!}$$

Let F be the set of representative states $\bar{x}$ of the symmetric system (i.e. no two states $\bar{x}\epsilon F$ belong to the same class of equivalent states); let $v(\bar{x}_o)$ be the class of the equivalent to $\bar{x}_o$ states. For each $\bar{x}_o, \bar{y}_o \epsilon F$, equations (4) and (5) can be written as follows.

$$P'(1;\bar{y}_o) = \sum_{\bar{x}_o \epsilon F} \{ \sum_{\bar{x} \epsilon V(\bar{x}_o)} p(\bar{x},\bar{y}_o) \} P'(1;\bar{x}) +$$

$$+ \sum_{\bar{x}\epsilon \bar{S}} (\mu_{\bar{x}\bar{y}_o} - 1) p(\bar{x},\bar{y}_o) \pi(\bar{x}) + \sum_{\bar{x}\epsilon \bar{S}} p(0;\bar{x}) p(\bar{x},\bar{y}_o) , \quad \bar{y}_o \epsilon F \tag{4a}$$

$$\sum_{\bar{x}_o \epsilon F} c(\bar{x}_o) 2(\mu_{\bar{x}_o} - 1) P'(1;\bar{x}_o) +$$

$$+ \sum_{\bar{x}\epsilon \bar{S}} \left[2(\mu_{\bar{x}} - 1) p(0;\bar{x}) + [2 + \sigma - 3\mu_{\bar{x}}] \pi(\bar{x}) \right] = 0 \tag{5a}$$

By solving the above equations with respect to $P'(1;\bar{x}_o)$, $\bar{x}_o \epsilon F$, we obtain the first moment of the buffer occupancy from the expression

$$P'(1) = \sum_{\bar{x}_o \epsilon F} P'(1;\bar{x}_o) c(\bar{x}_o)$$

For the computation of the second moment in the case of the symmetric system we modify equations (6) and (7) in a similar way. Similar procedure can be applied for the computation of any higher moment as well. Depending on the number of input streams, the reduced number of equations, K, in (4a) and (5a) is easily computed. For the case of N=2 and 3 input streams the number of these equations is given by the next theorem. By following the procedure outlined in the proof of this theorem, similar expressions for N>3 can be easily derived.

Theorem

Let M be the cardinality of S^i (i.e. of the 1-dimensional underlying Markov chain, as defined before). The number of classes of equivalent N-dimensional states $\bar{x} = (x_1, x_2, \cdots, x_N)$, , $x_k \epsilon S^i$, $k = 1, 2, \cdots, M$, is given by K, where

$$K = M + \frac{M(M-1)}{2} \quad \text{for} \quad N=2$$

$$K = M(M + \frac{(M-1)(M-2)}{6}) \quad \text{for} \quad N=3 \quad \square.$$

Proof:

The proof is based on the enumeration of all M dimensional vectors $\bar{v}(\bar{x}) = (v_1(\bar{x}), v_2(\bar{x}), \cdots v_M(\bar{x}))$ with $\sum_{i=1}^{M} v_i(\bar{x}) = N$, and where $v_i(\bar{x})$ is the number of input processes in state x_i (pp. 20, [9]).

The above theorem indicates that significant reduction in the number of equations can be achieved in the case of the examined queueing system under symmetric inputs. In the later case the required number of linear equations is K versus M^N for the general asymmetric case. Partially symmetric inputs will also result in a significant reduction of the number of the equations.

IV. Results and conclusions

In this section we use the results of the previous analysis to evaluate the first and the second moments of the buffer occupancy and the mean packet delay induced by the statistical multiplexer, under specific packet arrival processes. Each of the input lines is assumed to carry at most one packet over a slot. The following traffic situations are considered.

(a) Bernoulli arrivals per slot (arrival / no arrival).

(b) First order Markov arrivals per slot (arrival / no arrival).

(c) Arrivals appear in blocks of length L (slots), where L follows a general distribution. The arrival of the first packet after an idle slot (occurring with probability r) is assumed to be followed by consecutive packet arrivals over the next L-1 slots.

Model (c) corresponds to an on-off line where the length of the off period is geometrically distributed and the length of the on period has an arbitrary distribution. This model can be used for the description of the traffic of a message switched line (or node), where a message may consist of more than one packets. It may also describe the output process of a multi user communication network (one successful transmission is possible over a slot). Particularly, the output traffic of a reservation multi user communication network could be described by a general distributed number of packets, transmitted over a number of consecutive slots, during a reservation period. Notice that in those message arrival models, packets are transmitted one at a time slot and the resulting packet arrival process is different from one which would assume simultaneous arrivals of all packets of a single message.

To describe the arrival process in terms of the general model introduced before, we define the state of line i at the end the j^{th} slot to be given by z_j^i, where $z_j^i = 0$, if no packet arrived in the j^{th} slot and $z_j^i = k$, $1 \leq k \leq L$, if there are k packets of a message to be transmitted over the next k slots, starting with the $j+1^{st}$ slot. In this environment, a message describes a block of packets arriving through the same line over consecutive slots. According to the message arrival model described above, a message is generated during a slot with probability r if the slot is empty, and with probability 0 if the slot is occupied. This scenario of the message arrival process could describe the output of a reservation multi user random access slotted communication network, where an idle slot is necessary for the release of the channel. If no such a slot is necessary, we allow a nonzero message generation process over slots in states 0 and 1; in this case the next message transmission may start right after the end of the previous one (the coming end is declared by the line state 1). This second scenario may also represent the output of a single message buffer which can receive a new message while in the last stage of the transmission of the previous one. We can also generalize, by defining the line state to be the content of the buffer at the other end of the line; L in this case denotes the buffer capacity. Different message acceptance disciplines may also be incorporated. For instance, if the length of the new message arriving at the buffer exceeds the available capacity at that time, the message can either be rejected or be accepted in part. All these cases can easily be translated into the appropriate transition probabilities of the process $\{z_j^i\}$, which can be easily shown to be a Markov chain with state space $S=\{0,1, \ldots ,L\}$.

The process $\{z_j^i\}$ in the case of the initial scenario (a message can be generated only when the line state is 0), is a Markov chain with state transition probabilities given by (we omit the superscript i for simplicity)

$$p(k,j) = \begin{cases} 1 & j=k-1 \ , \ 1\le k\le L \\ rd(j) & k=0 \ , \ 1\le j\le L \\ 1-r & k=j=0 \\ 0 & \text{otherwise} \end{cases}$$

where d(j) is the probability that the length of a message (block) is j, $1\le j\le L$. The probabilistic mapping in this case is

$$a(k) = \begin{cases} 1 & 1\le k\le L \\ 0 & k=0 \end{cases}$$

The steady state probabilities of $\{z_j^i\}$ can be easily obtained from the system of equations

$$\Pi = \Pi \mathbf{P}$$

$$\sum_{k=0}^{L} \pi(k)=1$$

where Π is the vector of steady state probabilities and $\mathbf{P}$ is the matrix of the transition probabilities.

The Bernoulli approximation for the packet arrival process described before has parameter p (packet arrivals per slot) equal to $p=1-\pi(0)$. The first order Markov approximation (arrival=1, no arrival=0) has the following parameters

$$\pi_m(0)=\pi(0) \ , \ \pi_m(1)=1-\pi_m(0)$$

$$p_m(0,0)=1-p_m(0,1) \ , \ p_m(1,0)=1-p_m(1,1)$$

where

$$p_m(0,1)=r \ , \ p_m(1,1)=1-p(0,1)\frac{\pi_m(0)}{\pi_m(1)}$$

For this model, we define the burstiness coefficient γ to be equal to

$$\gamma=p_m(1,1)-p_m(0,1)$$

To obtain some numerical results we assume that N=3 , L=5 and d(1)=.1, d(2)=.2 d(3)=.3 d(4)=.3 d(5)=.1. The exact value of the mean packet delay, D, induced by the statistical multiplexer can be obtained by solving the equations (5) and (6) and applying (7). The approximate delay results under the Bernoulli, D_b ,(see (7) and (11) or [5]) and the Markov, D_m (see [4]) models are calculated from the closed form expressions that are available for these cases and are given by

$$D_b = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda^n \lambda^m}{(1 - \sum_{n=1}^{N} \lambda^n) \sum_{n=1}^{N} \lambda^n} \right]$$

and

$$D_m = \left[1 + \frac{\sum_{n=1}^{N} \sum_{m>n}^{N} \lambda^n \lambda^m \left(1 + \frac{\gamma^n}{1-\gamma^n} + \frac{\gamma^m}{1-\gamma^m}\right)}{(1- \sum_{n=1}^{N} \lambda^n) \sum_{n=1}^{N} \lambda^n} \right]$$

where λ^i is the packet arrival rate of the i^{th} line.

The delay results for different values of per line message (block) arrival rate r, which result in a per line packet arrival rate r_{out}, and a total packet arrival rate r_{tot}, together with the corresponding burstiness coefficient γ, are shown in Table I. From these results, a number of interesting conclusions may be drawn. It can be noticed that the Bernoulli approximation results in smaller delay than the one calculated under the Markovian approximation. This is always the case; the latter can be shown directly from the corresponding equations, keeping in mind that $\gamma=0$ in the case of the Bernoulli model while $\gamma>0$ under the Markovian model. The latter fact can be explained intuitively as well. Under the Markov model, packet arrivals tend to arrive in bursts. Whenever simultaneous bursts of arrivals occur, the content of the buffer of the node will keep increasing until the end of all but one burst and cannot start decreasing before the end of all bursts. Clearly, this situation (not present under the Bernoulli model) results in the increased packet delays. We believe that the latter behavior of the Markov model (or the geometrically distributed message length) is the reason for the larger delay results obtained under this approximation, when the true arrival process has the general length distribution described before. The Markov model creates concrete blocks of packets of average length equal to the average length of the generally distributed message length. On the other hand, generally distributed message lengths result in better randomized empty slots which reduce the intensity of the queueing problems.

In Table II, the results for the mean and the variance of the number of packets in the multiplexer are shown for packet arrival processes as in Table I. Both the exact results and those obtained under the Bernoulli approximation on the true input processes are shown. It can be easily observed that the error due to the Bernoulli oversimplification of the true input processes is very large. Since both the mean and the variance of the queue length are critical quantities for the determination of the appropriate buffer size, it can be easily concluded that a buffer design based on the results of the approximate analysis may be completely inappropriate.

In Table III, similar results are presented. In this case it is assumed that two of the packet arrival processes are exactly described by the Bernoulli model and one by the message length distribution used before. The total input traffic is .90 packets per slot; r_{out} is the intensity of the non-Bernoulli line and $(.9\text{-}r_{out})/2$ is the intensity of each of the Bernoulli lines. The delay error introduced by the adoption of the Bernoulli model for the packet arrival process of the non-Bernoulli line, is also shown. Notice that the error is significant (~20%) even when the dependent input line carries less than 15% of the total load. This observation implies that even if more than 85% of the total traffic is accurately described, the error in the approximation can still be large.

The simple application presented in this section indicates that our general model for the statistical multiplexer studied in this paper can be useful for the analysis of simple but difficult to analyze systems. A great deal of previous work can also be seen as a special case of the system considered here.

r	r_{out}	r_{tot}	D	D_b	D_m	γ
.05	.134	.403	1.654	1.224	1.982	.63
.10	.236	.710	2.997	1.816	4.045	.58
.12	.271	.813	4.330	2.453	6.113	.56
.14	.303	.908	8.064	4.287	11.97	.54
.15	.317	.952	14.84	7.643	22.47	.53

Table I.
Results for the mean delay for N=3 input lines.

r	r_{out}	r_{tot}	Q	Q_b	V	V_b
.05	.134	.403	.666	.493	1.222	.455
.10	.236	.710	2.127	1.289	6.213	1.548
.12	.271	.813	3.522	1.995	14.364	3.385
.14	.303	.908	7.324	3.893	55.685	13.132
.15	.317	.952	14.126	7.278	201.313	48.325

Table II.
Results for the mean and the variance of the queue length for N=3 input lines.

r	r_{out}	D	D_b - error	D_m - error
.05	.134	4.861	3.771 -22.4%	5.693 +17.1%
.10	.237	5.229	3.967 -24.1%	6.350 +21.4%
.20	.383	4.809	3.943 -18.0%	5.953 +23.7%
.30	.482	4.105	3.724 -09.3%	5.082 +23.8%
.40	.554	3.481	3.464 -00.05%	4.282 +23.0%
.50	.608	2.984	3.210 +07.6%	3.636 +21.8%
.60	.650	2.597	2.977 +14.6%	3.128 +20.4%

Table III.
Results for the mean delay for N=3 input lines.

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