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Study of a scheduling policy for diverse deadline-based quality of service☆

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Abstract

One of the most challenging problems in ATM network design is providing diversified Quality of Service (QoS) to applications with distinct characteristics. Buffer management schemes can play a significant role in providing the necessary diversification through the employed cell admission and service policy. A flexible priority service policy for two applications (classes) with strict — and in general distinct — deadlines and different deadline violation rates is studied in this paper. The proposed policy is a generalization of the shortest time to extinction (STE) policy (or the Earliest Due Date policy which discards expired cells), which is more flexible in providing diversified QoS. The relationship of this policy to other standard ones is also discussed. A flexible numerical analysis is presented for the derivation of performance measures such as cell loss, mean cell-delay and the tail of the cell-delay probability distribution for each class. Numerical results illustrate the effectiveness of the studied priority scheme. Finally, a low-complexity implementation scheme is proposed, which does not require time-stamp-based sorting. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Asynchronous transfer mode (ATM) has been selected as the CCITT standard for the switching and multiplexing technique of the future Broadband-Integrated Service Digital Networks (B-ISDN). These networks are expected to support services with diversified traffic characteristics and Quality of Service (QoS) requirements, such as data transfer, telephone/videophone, HDTV, multimedia conference, medical diagnosis and real-time control. To efficiently utilize the network resources through statistical multiplexing while providing the required QoS to the supported applications, it is necessary that the service to a specific

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application be dependent on its QoS requirement. As a result, some type of priority service needs to be adopted to provide for the diversification in the QoS provided to different applications.

In the absence of priority service, the network load must be set at a potentially very low level in order to provide the most stringent QoS to all applications. Considering the fact that QoS requirements can differ substantially, it is clear that a non-priority service scheme would result in severe underutilization of the networking resources. For instance, cell loss probability requirements can range from 10^{-2} to 10^{-10}; end-to-end cell delay requirements for real-time applications can range from below 25 to 1000 μs.

The QoS requirement is typically described in terms of some measure of the delay or loss that information units of the associated application suffer over the transmission path from source to destination. Since the universal information unit of an ATM network is the (fixed size) cell, QoS requirements are usually described in terms of some metric of the cell delay and cell loss. The objective of a priority service scheme is to deliver service which is as close as possible to the target one as determined by the associated cell delay/loss metric.

A priority service scheme can be defined in terms of a policy determining: (a) which of the arriving cells are admitted to the buffer(s) and/or (b) which of the admitted cells is served next. The former priority service schemes are typically referred to as “space-priority” schemes and impact substantially on the delivered cell loss metric. The latter are typically referred to as “time-priority” (or priority-scheduling) schemes and impact substantially on the delivered cell delay metric. Most of the priority service schemes proposed and studied in the past can be classified as either space- or time-priority; in these cases, the QoS is typically defined in terms of a cell loss constraint or cell delay constraint, respectively. Recently, some attempts to incorporate both space- and time-priority policies to deliver diversified service defined in terms of both cell loss and cell delay constraints, have been reported. A brief survey of the past work is presented below.

Space-priority policies — for cell loss control — may assume different levels of buffer sharing among the priority classes. A Partial Buffer Sharing policy — usually implemented by considering Nested Thresholds — assigns different cell discarding thresholds for cell streams under different cell loss constraints [1–4]. In [4], a study is presented on the trade-off between cell loss performance of marked cells — controlled by properly setting the thresholds — and the resulting cell delay performance. It turns out that such a space-priority scheme is not effective in balancing cell loss and cell delay, unless the class with strict cell loss requirement is the one with the low cell delay requirement. Even in this case, the achieved QoS range is fairly limited. Under a complete buffer sharing scheme, a space-priority policy is usually implemented by a push-out scheme [5,6]. Threshold-based Push-out and Probabilistic Push-out schemes [7] have been found to provide more flexibility in tuning the cell loss probability but their implementation seems to be complex while still having the restriction that the class with strict cell loss requirement is the one with the low cell delay requirement. Comparisons between various space-priority schemes are presented in [8–10]. It turns out that such schemes are fairly effective in inducing fairly different cell loss rates for the differentiated applications (differences of several orders of magnitude are achievable).

There is a great amount of past work on time-priority policies shaping the delay performance delivered to the different multiplexed applications. The delay performance can be controlled by a policy that guarantees certain service rate to a class or, more directly, by a policy which utilizes a time-metric, such as induced delay or associated deadline.

Examples of rate-based service polices [18] include Fair Queueing [16], Hierarchical Round Robin [17] and variants of Fair Queueing such as the Self-clocked Fair-Queueing [19] and the Generalized Processor Sharing under source traffic constrained by a leaky-bucket [20]. The Virtual Clock [15] aims to emulate the Time Division Multiplexing — and provide certain rate — by utilizing a time-metric.

Intuitively, it is expected that time-priority schemes which utilize directly a time-metric in determining the next cell to be served would be more effective in delivering a target time-metric. A well-studied policy is the Early-Due-Date (EDD) [11,26,29]. In this policy, packets are assigned a deadline and they are served in order of non-decreasing deadlines. In the classical EDD policy proposed in [13,14,26], all packets are being served, independently of whether their deadlines have expired or not. It has been shown in [12–14,26] that this policy — called sometimes dynamical priority policy — minimizes the maximum lateness and tardiness. The former is equal to service completion time minus deadline; the latter is equal to $\max\{0, \text{lateness}\}$. More references and discussion on the EDD policy may be found in [26,28,29].

An interesting policy called Head-of-the-Line with Priority Jumps (HoL-PJ) is proposed in [26]. By setting properly the priority jumping parameters, this policy gives priority to the packet with the largest queueing delay in excess of its delay deadline and thus it becomes the EDD policy. As emphasized in [26], the HoL-PJ policy provides for a mechanism to implement the EDD policy without requiring the additional processing delay and complexity necessary to identify the next packet to be served under the classical EDD policy. The study in [26] derives bounds on the average packet waiting time for the supported priority classes. The priority policy considered in the present paper has some similarities to the HoL-PJ policy. As it will be indicated in Section 2, the objectives and evaluated performance measures associated with the present study are different. Furthermore, the proposed policy may be viewed as a generalization to the Shortest-Time-to-Extinction (STE) — as explained below — in a similar way that the HoL-PJ policy may be viewed as a generalization of the EDD policy.

The EDD policy which discards packets which have violated their deadlines has been called the STE policy in [28]. As it has been shown in [28,29], this policy minimizes the number of packets which miss their deadline. Another extension to the EDD policy is the Delay-EDD policy [12]. In this case the deadline is set to be equal to the expected arrival time plus the delay bound.

In an ATM environment supporting diversified applications, it may be necessary that a priority service policy aims at delivering a specific QoS expressed in terms of both cell delay and cell loss metrics. It may be that the QoS of a single application is expressed in terms of both metrics or that supported applications utilizing the same networking resources have QoS requirements described in terms of different metrics. For this reason, recent works have investigated the performance of priority service in terms of the induced cell loss and cell delay metrics.

In [23], the optimal buffer allocation scheme under delay constraints is investigated. In [24], a queue management scheme is proposed for ATM switches under multiple cell delay and cell loss QoS requirements; the non-flexible Head-of-Line (HoL) priority is considered to deliver the cell delay requirements and a push-out scheme is considered for cell loss management. Another approach is presented in [27], where the Self-calibrating Push-out and EDD policies are assumed for cell loss and cell delay control, respectively: Self-calibrating Push-out policy aims to push out/discard cells in a fair way according to different cell loss rate requirements and arrival rates for different classes. Cell loss and cell delay performance have also been investigated in the priority service scheme considered in [25]: a Weighted-Fair-Queueing and Buffer Threshold combination. It turns out that the delay bounds computed in [25] are loose because the buffer size cannot accurately reflect the cell delay in this service scheme.

From the past work it may be concluded that cell delay requirements are more effectively met by deadline-based time-priority policies; cell losses are more effectively managed by space-priority policies. When applications with cell delay requirements share the networking resources with cell loss sensitive applications, a service policy which assigns high time-priority to the former application and high space-priority to the latter would seem to be meaningful.

In real-time applications a strict delay deadline may be associated with its QoS requirement; cells which are delayed beyond the deadline are considered to be lost. For such applications the QoS would be defined in terms of the acceptable maximum cell delay and the acceptable maximum cell loss probability, where the latter is defined — in this case — as the probability of delay-deadline violation. This cell loss probability is controllable by a time-priority policy, rather than a space-priority one controlling losses due to buffer-space limitations. Although cell losses may also occur due to buffer space limitations, it may be assumed that discarded cells due to space limitations have already violated their deadline and thus such losses have already been "registered". In fact, an effective priority service policy for different classes of delay-constrained applications would be one which minimizes the cell delay violation probability and discards (does not serve) the violating cells. As indicated earlier, the STE policy [28] is effective in the sense that it minimizes the cell delay constraint violation probability while discarding the violating cells. Despite this effectiveness, the STE policy does not have the mechanism to induce diversified cell loss (deadline violation) probabilities to two classes of traffic with different cell delay constraints. The policy proposed here generalizes the STE policy — in the sense that the STE policy corresponds to a specific setting of some parameter — and is capable of providing the above QoS diversification.

In Section 2, the proposed scheduling policy is described. A proposal for a low-complexity implementation is presented in Section 3. In Section 4, a queueing model is formulated and the derivation of cell loss (deadline-violation) probabilities and cell delay distributions is presented. Finally, some numerical results are presented and discussed in Section 5.

2. Description of the scheduling policy

Consider two different classes of traffic (or applications) sharing a transmission link. The link is slotted and capable of serving one information unit (cell) per time slot. Let H (for high priority) and L (for low priority) denote the two classes; let superscript H (L) denote a quantity associated with class H (L).

1. H-cells (i.e., cells of application H) join the HoL service class upon arrival. HoL cells are served according to the HoL priority policy; i.e., no service is provided to other cells unless no HoL cell is present. H-cells which experience a delay of more than T^H slots are discarded.
2. L-cells (i.e., cells of application L) are served according to the D-HoL priority. That is, L-cells join the HoL service class (as fresh arrivals) only after they have waited for D time slots. Since the service policy is assumed to be work-conserving, L-cells may be served before they join the HoL class provided that no HoL class cells are present. L-cells which experience a delay of more than T^L slots are discarded. In order for the policy to be nontrivial (i.e., class-switching to be possible), it is assumed that $T^L \geq D$.
3. Within each service class (HoL or waiting to join L-cells) cells are served according to the First-Come-First-Served (FCFS) policy.

Although the priority jumping in the above scheduling policy resembles this aspect of the HoL-PJ policy proposed in [26], the proposed policy as well as the objectives here are quite different from those in [26]. First the proposed policy discards cells which miss their deadlines, unlike in [26]. Related to this is the fact that the proposed policy reduces to the STE policy (for $D = T^L - T^H$) while the policy in [26] reduces to the EDD policy under proper parameter setting. Second, the induced cell loss (deadline violation probabilities) are derived in this work as opposed to bounds on the average waiting time in [26]. Finally, while the focus in [26] has been to present a policy which implements the EDD without the processing delay and complexity of time-stamp-based sorting, the objective in this paper is to develop a policy capable of delivering diversified QoS — without wasting capacity to serve expired cells — and an approach to calculate the performance measure of interest.

Since the STE policy has been shown to minimize the total cell loss (deadline violation) probability [28,29], it is expected that the proposed policy will induce higher total cell loss probability for $D \neq T^L - T^H$. Minimizing the total cell loss probability is not necessarily the most desirable quality of a policy. For instance, it is easy to be shown that unless the STE policy is non-work-conserving,* it will induce higher cell loss probability to the class with the smaller deadline. Thus, if the QoS requirement is defined in terms of a lower cell loss probability for the class of cells with the smallest deadline, then the STE policy cannot achieve the desired balance optimally and will have to operate at a lower traffic load to deliver the QoS to both classes (while providing better than specified service to one class). The proposed policy is shown here to be capable of providing for substantial QoS diversification through the control parameter D, and thus will potentially operate at higher traffic load.

An interesting and potentially useful characteristic of the priority service policy studied here is that it can represent a number of known service policies by setting properly key parameters as indicated below.

1. If $\{D = 0, T^H \to \infty, T^L \to \infty\}$, the policy becomes the FCFS policy with infinite buffer capacity.
2. If $\{D = 0, T^H = T^L\}$, the policy becomes the FCFS policy with buffer capacity equal to T^H (complete buffer sharing).
3. If $\{D = 0, T^H \neq T^L\}$, the policy becomes the Nested Thresholds discarding policy, controlling cell losses due to buffer space limitations (space-priority policy); the buffer capacity in this case is equal to $\max\{T^H, T^L\}$.
4. If $\{D \to \infty, T^H \to \infty, T^L \to \infty\}$, the policy becomes the HoL priority policy with infinite buffer capacities.
5. If $\{T^L = T^H + D\}$, the policy becomes the EDD priority policy with buffer capacity equal to T^L, or the STE.

The fact that the proposed service policy contains a number of other policies does not necessarily suggest that the presented analysis should be applied for the study of those policies. Rather, it can be useful in identifying bounds on the performance that can be achieved by the proposed policy, or illustrate the increased flexibility of the proposed policy.

3. Implementation

Although the proposed scheduling scheme may be implemented in principle by considering time-stamps associated with each cell arrival, such an approach may not be realistic for a high-speed, low-management-complexity switching system. A simple implementation of the cell class-switching and discarding policy is shown in Fig. 1.

As indicated in Fig. 1, H-cells join immediately the H-buffer. If the H-buffer occupancy exceeds a threshold equal to T^H, new H-cell arrivals are discarded. Since the H-buffer is served under the HoL priority policy, the discarded H-cells are precisely the ones whose deadline T^H would be violated.

L-cells join the L-buffer which is served only when the H-buffer is empty. Cells in L-buffer which experience a delay D are moved to the end of the queue of H-buffer, provided that the H-buffer occupancy does not exceed $T^L - D$; otherwise, these L-cells are discarded. Again, since the H-buffer is served under the HoL priority policy, the discarded L-cells are precisely the ones whose deadline T^L would be violated.

* A non-work-conserving STE policy is defined to be one which does not serve any cell from those with the largest of the two (here) deadlines before they experience a delay D $(= T^L - T^H)$. Using the above scheduling policy, L-cells must join the H-cells before they are considered for service.

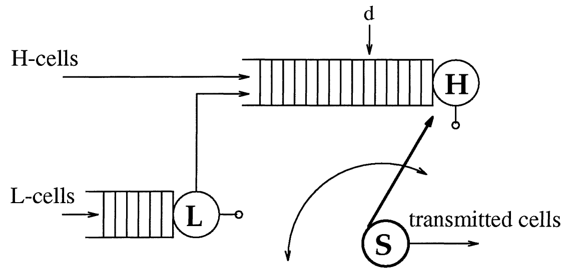

Fig. 1. Buffer management with time constraints.

Notice that the mechanism presented in Fig. 1 implements the (deadline-violating) cell discarding through a simple buffer threshold policy, avoiding a more complex time-stamping approach. The capacity of the H-buffer and L-buffer are equal to $\max\{T^H, T^L - D\}$ and $\min\{T^L, DN_{\max}\}$, respectively; $N_{\max}$ is the maximum number of L-cell arrivals per slot. The threshold d in Fig. 1 is equal to $\min\{T^H, T^L - D\}$. If $\min\{T^H, T^L - D\} = T^H$, then H-cell arrivals which find more than d cells in H-buffer are discarded; otherwise, the latter holds true for L-cells moved to H-buffer.

The identification of the time at which L-cell are moved to H-buffer can be implemented through time-stamping or by adopting the less-complex mechanism described here. Time-stamp-based sorting in every slot presents a level of complexity which may not be tolerable in a high-speed networking environment. For this reason, alternative to time-stamp-based implementation approaches have been considered in [26,29]: a list of cell arrival times and a clock are employed in [26] for each of the N queues considered; one shift register for each of the two classes is employed in [29]. It appears that when multiple arrivals per slot are possible, these implementations can become fairly complex as well. An implementation of the proposed policy utilizes a circular buffer (registers) which records the number of cell arrivals per slot and it is described next.

The L-buffer (Fig. 1) controller is equipped with a pool of D registers, a timer T and a pointer P. The timer counts from 0 to $D-1$, increasing its content by one in every slot; after count $D-1$, it is reset at the next slot and then continues as before. The current content of the timer indicates the register in which the number of L-cell arrivals during the current slot will be registered. The pool of registers may be thought of having a circular structure and the timer T may be viewed as a rotating pointer pointing to the register to be used at the current time (Fig. 2). Notice that the register visited by timer T contains the number of L-cells which have experienced a delay D in the L-buffer. These cells — which are at the head of the L-buffer — will be moved to the H-buffer and the number of new L-cell arrivals over the current slot will be registered in this register.

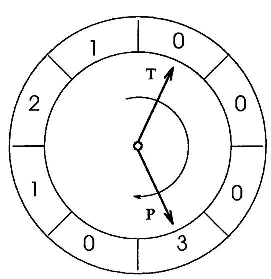

Fig. 2. The circular representation of the registers and pointers of the L-buffer controller.

The timer T identifies the time when L-cells are moved to the H-buffer. A mechanism is needed to determine changes in the content of the registers due to service provided to L-cells while still in the L-buffer. A pointer P is used for this purpose. This pointer points to the (non-zero-content) register containing the number of L-cells which are the oldest in the L-buffer. When service is provided to the L-buffer — when the H-buffer is empty — the content of the register pointed by pointer P is decreased by one. The pointer P moves from the currently pointed register to the next non-zero-content register if the content of the current register becomes zero. This occurs if the content of the currently pointed register is one and service is provided to an L-cell or if the timer T visits this register, and thus the corresponding L-cells are moved to the H-buffer. If there is currently no L-cell in the L-buffer, $P = T$.

4. Performance analysis

In this section, a numerical approach is developed for the evaluation of the diversified QoS provided to the two classes of traffic. The QoS is defined in terms of the induced cell loss probability; notice that cell loss and cell deadline violation are identical events under the proposed policy. The diversity in the QoS requirements for the two applications served under the proposed service policy is represented by differences in cell delay deadlines and cell loss probabilities. By controlling the delay D before the L-cells qualify for service under the HoL priority, it is expected that the induced cell loss probability will be affected substantially. The effectiveness of the D parameter is demonstrated through numerical results. As it will be shown later, the developed analysis approach can be trivially modified to yield other QoS measures such as the average cell delay and the tail of the cell delay distribution.

4.1. System model

Fig. 3 shows three axes which will be used in the performance analysis of the proposed service policy. The L-axis (H-axis) is used to conveniently describe L-cell (H-cell) arrivals at the time they occur. The system axis is used to conveniently mark the current time. Cell arrival and service completions are assumed to occur at the slot boundaries.

The H-cell (L-cell) arrival process is assumed to be an independent process and governed by geometrically distributed (per slot) batches with parameter q^H (q^L). Thus, the probability that the batch size N^H (N^L) is equal to n is given by

$$P^H(N^H = n) = (q^H)^n(1 - q^H), \qquad P^L(N^L = n) = (q^L)^n(1 - q^L), \quad n = 0, 1, 2, \ldots \tag{1}$$

and the mean H-cell (L-cell) arrival rate is given by

$$\lambda^H = \frac{q^H}{1 - q^H} \qquad \left(\lambda^L = \frac{q^L}{1 - q^L}\right). \tag{2}$$

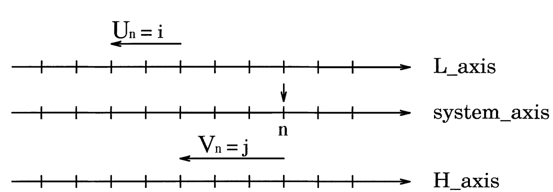

Fig. 3. Discrete axes employed in the system study.

It should be noted that the assumption on the arrival processes considered above are not to be viewed as critical, since they can be relaxed in a number of ways. For instance, two-state Markov arrival processes may be considered and the resulting system may be analyzed by applying the same approach (requiring increased computations). It is also applicable to arrival processes which are independent and identically distributed (i.i.d.) with arbitrarily distributed batch size; in this case, the maximum batch size affects the magnitude of the numerical complexity.

The analysis approach is based on renewal theory and it is similar to that followed for the study of the systems in [21,22]. Let $n, n \in N$, denote the current time, where N denotes the set of natural numbers; at this time, the server is ready to begin the service of the next cell. Consider the following definitions (see Fig. 3):

V_n: A random variable describing the current length (in slots) of the unexamined interval at H-axis. That is, all H-cell arrivals before time $n - V_n$ have been served; no H-cell arrival after time $n - V_n + 1$ has been considered for service by time n.

U_n: A random variable such that $U_n + V_n$ describes the current length of the unexamined interval on L-axis.

It is easy to observe that $\{U_n, V_n\}_{n\in N}$ is a Markov process. Let $\{s_k\}_{k\in N}$ denote a sequence of time instants (slot boundaries) at which the system is empty; $\{s_k\}_{k\in N}$ is a renewal sequence. Consider the following definitions:

Y_k: A random variable describing the length of the kth renewal cycle; Y will denote the generic random variable (associated with a renewal cycle).

$Y_k^H(Y_k^L)$: A random variable describing the number of H-cells (L-cells) transmitted over the kth renewal cycle; $Y^H(Y^L)$ will denote the generic random variable.

$L_k^H(L_k^L)$: A random variable describing the number of H-cells (L-cells) lost over the kth renewal cycle; $L^H(L^L)$ will denote the generic random variable.

It is easy to establish that

$$Y_k = Y_k^H + Y_k^L + 1, \quad Y_k \geq 1. \tag{3}$$

Fig. 4 presents a realization of a renewal cycle. Cells marked by $\times$ are the ones which are lost (due to violation of the associated deadline). Transmitted cells are shown on the system axis. In this example, $T^H = 3, T^L = 4, D = 2$. The renewal cycle begins at time t_0 at which no cell is present; the first slot of the renewal cycle is always idle (no transmission occurs). At time slot t_4, one L-cell is served and the first L-cell is discarded due to the expiration of its deadline ($t_4 - t_1 = T^L$). In fact, at t_3 the first two

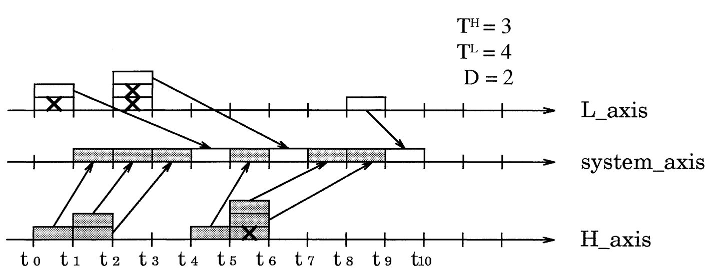

Fig. 4. An example of renewal cycle.

L-cells switch to H-buffer, but only one is served before its deadline. At t_5, the L-cells which arrived at t_3 switch to H-buffer ($t_5 - t_3 = D$); in this realization, the H-cell which arrived at t_5 happens to be served before these L-cells. At t_9, the L-cell is served since no H-cell is present. At t_{10}, no cell is present and the renewal cycle ends. In this example, $Y_k = 10$, $Y_k^H = 6$, $Y_k^L = 3$, $L_k^H = 1$, $L_k^L = 3$.

The evolution of the process $\{U_n, V_n\}$ can be easily derived for the realization shown in Fig. 4. Notice that when $U_n > D$, U_n points to the oldest unexamined slot which is to be served under the HoL priority; the associated L-cells have switched priority and are the oldest cells in the system. If $U_n < D$, V_n points to the oldest unexamined slot which is to be served under the HoL priority. If $U_n = D$, the oldest unexamined intervals in both axes have the same priority and the selection is made according to a probabilistic rule introduced later; any selection rule can be adopted. One possible evolution of $\{U_n, V_n\}$ corresponding to the realization shown in Fig. 4 is as follows:

t_1: $(0, 1)$
t_2: $(0, 2) \to (1, 1)$
t_3: $(1, 2)$
t_4: $(1, 3) \to (2, 2)$
t_5: $(1, 3) \to (2, 2) \to (1, 2) \to (2, 1)$
t_6: $(2, 2)$
t_7: $(1, 3) \to (2, 2) \to (1, 2)$
t_8: $(1, 3)$
t_9: $(1, 3) \to (2, 2) \to (1, 2) \to (2, 1) \to (1, 1) \to (2, 0) \to (1, 0)$
t_{10}: $(1, 1) \to (2, 0) \to (1, 0) \to (0, 0)$

4.2. Derivation of system equations

The following definitions will be used in the analysis:

$Y^H(i, j)$: A random variable describing the number of H-cells transmitted over the interval between a time slot at which $\{U_n, V_n\}$ is in state (i, j) and the end of the renewal cycle which contains this slot.

$Y^L(i, j)$: A random variable defined as $Y^H(i, j)$ for L-cells.

$I^H(I^L)$: An indicator function assuming the value 1 if an H-cell (L-cell) is present at the slot of H-axis (L-axis) under current examination (as pointed to by V_n or $U_n + V_n$); it assumes the value 0 otherwise.

$\hat{I}^H(\hat{I}^L)$: An indicator function defined as $\hat{I}^H = 1 - I^H$ ($\hat{I}^L = 1 - I^L$).

$[i, j]^*$: It is a function which determines the next state of $\{U_n, V_n\}$ taking into consideration possible violation of T^L:

$$[i, j]^* = \begin{cases} (i, j), & i + j \le T^L, \\ (T^L - j, j) & \text{otherwise.} \end{cases}$$

m: It denotes the lowest possible value of random variable V_n; $m = 0$ if $D \neq 0$ and $m = -1$ if $D = 0$.

$I_{\{H\}}(I_{\{L\}})$: An indicator function associated with decisions regarding the slot to be examined next when $U_n = D$. $I_{\{H\}} = 1(I_{\{L\}} = 1)$ if the oldest unexamined slot in H-axis (L-axis) is considered next, and $I_{\{H\}} = 0(I_{\{L\}} = 0)$ otherwise. This is a design parameter which can impact on the induced cell losses. In Appendix A, the expected values of these functions ($\mu^H = P\{I_{\{H\}} = 1\}$, $\mu^L = P\{I_{\{L\}} = 1\}$) are derived under the assumption that all cells (from both classes) arrived over the slots to be examined when $U_n = D$ are equally likely to be selected. $\bar{X}$: Denotes $E\{X\}$, where $E\{\cdot\}$ is the expectation operator.

In the sequel, recursive equations are derived for the calculation of $\bar{Y}^H$ and $\bar{Y}^L$. Then, similar equations are derived for the calculation of $\bar{L}^H$ and $\bar{L}^L$. As it will be shown later, these quantities will yield the cell loss probabilities. Finally, the similar approach for the calculation of the average cell delay and cell delay tail probabilities is outlined at the end of this section.

It is easy to observe that no cell is transmitted in the first slot and thus, process $\{U_n, V_n\}$ actually starts from state $(0, 1)$. Thus,

$$Y^H = Y^H(0,1), \qquad Y^L = Y^L(0,1). \tag{4}$$

A careful consideration of the evolution of the recursions presented below shows that when process $\{U_n, V_n\}$ reaches the state $(0, 0)$, the renewal cycle ends. For this reason,

$$Y^H(0,0) = 0, \qquad Y^L(0,0) = 0 \tag{5}$$

to terminate the current cycle. Notice also that U_n can exceed $D+1$ only if $V_n = T^H$.

Case A. $T^L \geq T^H$.

Case A.1. $T^H > j > 0, \min(T^L - j, D+1) \geq i \geq m$.

1. $i < D$:

$$\begin{aligned} Y^H(i,j) &= I^H + Y^H([i + \hat{I}^H, j - 2\hat{I}^H + 1]^*), \\ Y^L(i,j) &= Y^L([i + \hat{I}^H, j - 2\hat{I}^H + 1]^*). \end{aligned} \tag{6}$$

2. $i = D + 1$:

$$\begin{aligned} Y^H(D+1,j) &= Y^H([i - \hat{I}^L, j - \hat{I}^L + 1]^*), \\ Y^L(D+1,j) &= I^L + Y^L([i - \hat{I}^L, j - \hat{I}^L + 1]^*). \end{aligned} \tag{7}$$

3. $i = D$:

$$\begin{aligned} Y^H(D,j) &= \{I^H + Y^H([i + \hat{I}^H, j - 2\hat{I}^H + 1]^*)\}I_{\{H\}} + Y^H([i - \hat{I}^L, j - \hat{I}^L + 1]^*)I_{\{L\}}, \\ Y^L(D,j) &= Y^L([i + \hat{I}^H, j - 2\hat{I}^H + 1]^*)I_{\{H\}} + \{I^L + Y^L([i - \hat{I}^L, j - \hat{I}^L + 1]^*)\}I_{\{L\}}. \end{aligned} \tag{8}$$

Case A.2. $j = T^H, T^L - T^H \geq i \geq m$.

1. $i < D$:

$$Y^H(i,T^H) = I^H + Y^H([i+1, T^H - \hat{I}^H]^*), \quad Y^L(i,T^H) = Y^L([i+1, T^H - \hat{I}^H]^*). \tag{9}$$

2. $i \geq D + 1$:

$$Y^H(i,T^H) = Y^H([i - 2\hat{I}^L + 1, T^H]^*), \quad Y^L(i,T^H) = I^L + Y^L([i - 2\hat{I}^L + 1, T^H]^*). \tag{10}$$

3. $i = D$:

$$\begin{aligned} Y^H(D,T^H) &= \{I^H + Y^H([i+1, T^H - \hat{I}^H]^*)\}I_{\{H\}} + Y^H([i - 2\hat{I}^L + 1, T^H]^*)I_{\{L\}}, \\ Y^L(D,T^H) &= Y^L([i+1, T^H - \hat{I}_H]^*)I_{\{H\}} + \{I^L + Y^L([i - 2\hat{I}^L + 1, T^H]^*)\}I_{\{L\}}. \end{aligned} \tag{11}$$

Case A.3. $j = 0, \min(T^L, D+1) \geq i \geq 1$.

$$Y^H(i,0) = Y^H([i-\hat{I}^L, 1-\hat{I}^L]^*), \qquad Y^L(i,0) = I^L + Y^L([i-\hat{I}^L, 1-\hat{I}^L]^*). \tag{12}$$

Case B . $T^L < T^H$.

The equations under this case are derived similarly and are presented in Appendix B. By applying the expectation operator to the above equations, the following systems of linear equations are obtained, details are presented in Appendix C:

$$\bar{Y}^H(i,j) = a^H(i,j) + \sum_{(i',j')\in R_0} b^H(i,j,i',j')\bar{Y}^H(i',j'),$$
$$\bar{Y}^L(i,j) = a^L(i,j) + \sum_{(i',j')\in R_0} b^L(i,j,i',j')\bar{Y}^H(i',j'), \tag{13}$$

where $R_0 = \{(i,j)\colon\ m \leq i \leq T^L,\ \ 0 \leq j \leq T^H\}$. It should be noted that these systems of linear equations are extremely sparse: only two to four coefficients are not zero per equation. Thus, it can be solved efficiently by using an iterative approach. The computation complexity is of the order of DT^H. For T^H and $D < 100$, it takes less than a couple of hours to solve these equations in a SUN SPARC20 workstation. From the solution of these equations, $\bar{Y}^H$ and $\bar{Y}^L$ are obtained from (see (4))

$$\bar{Y}^H = \bar{Y}^H(0,1), \qquad \bar{Y}^L = \bar{Y}^L(0,1). \tag{14}$$

The expected value of the number of H-cells (L-cells) lost over a renewal cycle, $\bar{L}^H(\bar{L}^L)$, is derived by following a similar approach. The following quantities need to be defined first.

$L^H(i,j)$ $(L^L(i,j))$: A random variable describing the number of H-cells (L-cells) discarded over the interval between a time slot at which $\{U_n, V_n\}$ is in state (i,j) and the end of the renewal cycle which contains this slot.

$S_{\{i,j\}}$: An indicator function assuming the value 1 if there is a possibility to discard an L-cell as a result of the service to be provided in the current slot (due to resulting violation of its deadline T^L):

$$S_{\{i,j\}} = \begin{cases} 1 & \text{if } i+j = T^L, \\ 0 & \text{otherwise.} \end{cases}$$

The equation for the derivation of L_n^H and L_n^L are similar to those for the derivation of Y_n^H and Y_n^L and are given below. Notice again that

$$L^H = L^H(0,1), \qquad L^L = L^L(0,1) \tag{15}$$

and

$$L^H(0,0) = 0, \qquad L^L(0,0) = 0. \tag{16}$$

Two cases need to be considered: $T^L \geq T^H$ and $T^L < T^H$.

Case A. $T^L \geq T^H$.

Case A.1. $T^H > j > 0, \min(T^L - j, D+1) \geq i \geq m$.

1. $i < D$:

$$L^H(i,j) = L^H([i+\hat{I}^H, j-2\hat{I}^H+1]^*),$$
$$L^L(i,j) = I^H S_{\{i,j\}} N^L + L^L([i+\hat{I}^H, j-2\hat{I}^H+1]^*). \tag{17}$$

2. $i = D+1$:

$$L^H(D+1, j) = L^H([i-\hat{I}^L, j-\hat{I}^L+1]^*),$$
$$L^L(D+1, j) = I^L S_{\{D+1,j\}} N^L + L^L([i-\hat{I}^L, j-\hat{I}^L+1]^*). \tag{18}$$

3. $i = D$:

$$L^H(D, j) = L^H([i+\hat{I}^H, j-2\hat{I}^H+1]^*) I_{\{H\}} + L^H([i-\hat{I}^L, j-\hat{I}^L+1]^*) I_{\{L\}},$$
$$L^L(D, j) = \{I^H S_{\{D,j\}} N^L + L^L([i+\hat{I}^H, j-2\hat{I}^H+1]^*)\} I_{\{H\}} + \{I^L S_{\{D,j\}} N^L + L^L([i-\hat{I}^L, j-\hat{I}^L+1]^*)\} I_{\{L\}}. \tag{19}$$

Case A.2. $j = T^H, T^L - T^H \geq i \geq m$.

1. $i < D$:

$$L^H(i, T^H) = I^H N^H + L^H([i+1, T^H - \hat{I}^H]^*),$$
$$L^L(i, T^H) = I^H S_{\{i,T^H\}} N^L + L^L([i+1, T^H - \hat{I}^H]^*). \tag{20}$$

2. $i \geq D+1$:

$$L^H(i, T^H) = I^L N^H + L^H([i-2\hat{I}^L+1, T^H]^*),$$
$$L^L(i, T^H) = I^L S_{\{i,T^H\}} N^L + L^L([i-2\hat{I}^L+1, T^H]^*). \tag{21}$$

3. $i = D$:

$$L^H(D, T^H) = \{I^H N^H + L^H([i+1, T^H-\hat{I}^H]^*)\} I_{\{H\}} + \{I^L N^H + L^H([i-2\hat{I}^L+1, T^H]^*)\} I_{\{L\}},$$
$$L^L(D, T^H) = \{I^H S_{\{D,T^H\}} N^L + L^L([i+1, T^H - \hat{I}_H]^*)\} I_{\{H\}} + \{I^L S_{\{D,T^H\}} N^L + L^L([i-2\hat{I}^L+1, T^H]^*)\} I_{\{L\}}. \tag{22}$$

Case A.3. $j = 0, \min(T^L, D+1) \geq i \geq 1$.

$$L^H(i, 0) = L^H([i-\hat{I}^L, 1-\hat{I}^L]^*),$$
$$L^L(i, 0) = I^L S_{\{i,0\}} N^L + L^L([i-\hat{I}^L, 1-\hat{I}^L]^*). \tag{23}$$

Case B. $T^L < T^H$.

The equations under this case are derived similarly (see also Case B in the derivation of Y^H and Y^L) and are not presented due to space consideration. Details can be found in [30].

By taking expectation operation to both sides of the equations, the following systems of linear equations are obtained:

$$\bar{L}^H(i, j) = a'^H(i, j) + \sum_{(i',j') \in R_0} b^H(i, j, i', j') \bar{L}^H(i', j'),$$
$$\bar{L}^L(i, j) = a'^L(i, j) + \sum_{(i',j') \in R_0} b^L(i, j, i', j') \bar{L}^H(i', j'), \tag{24}$$

where R_0 and coefficients $b^H(i, j, i', j')$ and $b^L(i, j, i', j')$ are identical to those in system (13), and constants $a'^H(i, j)$ and $a'^L(i, j)$ are derived as the corresponding ones in the system in (13). Finally,

$$\bar{L}^H = \bar{L}^H(0, 1), \qquad \bar{L}^L = \bar{L}^L(0, 1). \tag{25}$$

The cell loss probabilities $P^H_{\rm loss}$ for H-cells and $P^L_{\rm loss}$ for L-cells are obtained from the following expressions:

$$P^H_{\rm loss} = \frac{\bar{L}^H}{\bar{Y}^H + \bar{L}^H}, \qquad P^L_{\rm loss} = \frac{\bar{L}^L}{\bar{Y}^L + \bar{L}^L}. \tag{26}$$

Notice that $\bar{Y}^H + \bar{L}^H$ is the average number of H-cells over a renewal cycle which is also given by $\lambda^H \bar{Y}$. Similarly, $\bar{Y}^L + \bar{L}^L$ is the average number of L-cells over a renewal cycle which is also given by $\lambda^L \bar{Y}$.

By invoking renewal theory, the rate of service provided to H-cell and L-cell streams — denoted by $\lambda^H_{\rm s}$ and $\lambda^L_{\rm s}$, respectively — is given by

$$\lambda^H_{\rm s} = \frac{\bar{Y}^H}{\bar{Y}^H + \bar{Y}^L + 1}, \qquad \lambda^L_{\rm s} = \frac{\bar{Y}^L}{\bar{Y}^H + \bar{Y}^L + 1}. \tag{27}$$

Alternatively, the cell loss probabilities — given by (26) — can be obtained as

$$P^H_{\rm loss} = \frac{\lambda^H - \lambda^H_{\rm s}}{\lambda^H}, \qquad P^L_{\rm loss} = \frac{\lambda^L - \lambda^L_{\rm s}}{\lambda^L}. \tag{28}$$

Notice that computation of $P^H_{\rm loss}$ and $P^L_{\rm loss}$ from (27) and (28) does not require computation of L^H and L^L. It should be noted, however, that Eq. (28) can potentially introduce significant numerical error, especially if the cell loss rates are very low. For this reason, results have been obtained by invoking Eq. (26) in this paper.

4.3. Other performance metrics

As stated earlier, other measures of the QoS can be derived by following the above approach. The calculation of the average delay of the successfully transmitted cells and the tail of the delay probability distribution are outlined below. Equations similar to those presented for $L^H(i, j)$ and $L^L(i, j)$ can be derived, where the associated quantities of interest — instead of discarded H-cells in $L^H(i, j)$ and L-cells in $L^L(i, j)$ — are properly defined below.

$C^H(i, j)$ ($C^L(i, j)$): A random variable describing the cumulative delay of successfully transmitted H-cells (L-cells) over the interval between a time slot at which $\{U_n, V_n\}$ is in state (i, j) and the end of the renewal cycle which contains this slot.

$B^H_h(i, j)$ ($B^L_l(i, j)$): A random variable describing the number of H-cells (L-cells) which have experienced a delay less than or equal to h (l) over the interval between a time slot at which $\{U_n, V_n\}$ is in state (i, j) and the end of the renewal cycle which contains this slot.

Then,

$$C^H = C^H(0,1), \qquad C^L = C^L(0,1), \qquad B^H_h = B^H_h(0,1), \qquad B^L_l = B^L_l(0,1). \tag{29}$$

The average delays for H-cells and L-cells are given by

$$\bar{D}^H = \frac{\bar{C}^H}{\bar{Y}^H}, \qquad \bar{D}^L = \frac{\bar{C}^L}{\bar{Y}^L}. \tag{30}$$

The tail of the delay probability distribution is given by

$$P^H(D^H > h) = 1 - \frac{\bar{B}^H_h}{\bar{Y}^H + \bar{L}^H} = 1 - \frac{\bar{B}^H_h}{\lambda^H \bar{Y}}, \quad P^L(D^L > l) = 1 - \frac{\bar{B}^L_l}{\bar{Y}^L + \bar{L}^L} = 1 - \frac{\bar{B}^L_l}{\lambda^L \bar{Y}}. \tag{31}$$

To derive the quantities in (31), similar equations to those associated with $L^H(i,j)$ or $L^L(i,j)$ can be derived by replacing the first of the two right-hand side terms in those equations — counting discarded cells — by functions that count the total delay of the currently transmitted cell (in determining $C^H(i,j)$ or $C^L(i,j)$) or count the number of cells transmitted over the current slot which experienced a delay of less than or equal to h or l slots (in determining $B_h^H(i,j)$ or $B_l^L(i,j)$). These functions — denoted by $F_{\{i,j\}}^H$ ($F_{\{i,j\}}^L$) and $G_h^H(i,j)$ ($G_l^L(i,j)$), respectively — are given by the following:

$$F_{\{i,j\}}^H = \begin{cases} j & \text{if an H-cell is served,} \\ 0 & \text{otherwise,} \end{cases}$$

$$F_{\{i,j\}}^L = \begin{cases} i+j & \text{if an L-cell is served,} \\ 0 & \text{otherwise,} \end{cases}$$

$$G_h^H(i,j) = \begin{cases} 1 & \text{if an H-cell is served and } j \le h, \\ 0 & \text{otherwise,} \end{cases}$$

$$G_l^L(i,j) = \begin{cases} 1 & \text{if an L-cell is served with } i+j \le l, \\ 0 & \text{otherwise.} \end{cases}$$

5. Numerical result

In this section, some numerical results are presented to illustrate the potential effectiveness/flexibility of the proposed priority service policy. Results are presented for the induced cell loss, mean delay and tail of the delay probability distribution for each of the classes (applications).

Results for the L-cell and H-cell loss probabilities (P_{loss}^L and P_{loss}^H, respectively) as functions of D are shown in Figs. 5 and 6, respectively, under the parameters: $\lambda^H = \lambda^L$ and $\lambda^H + \lambda^L = 0.9, 0.86, 0.82$; $T^H = 50$ and $T^L = 100$. The expected monotonic increase (decrease) of P_{loss}^L (P_{loss}^H) as D increases is clearly observed. P_{loss}^H is minimized for $D = 100$, which corresponds to the HoL priority service policy for the H-class. The FCFS policy with different cell admission thresholds (T^H and T^L) is determined for $D = 0$: all cells are admitted under buffer occupancy less than 50 ($= T^H$); only L-cells are admitted under buffer occupancy greater than 50; no cell is admitted under buffer occupancy greater than 100 ($= T^L$). Due to the difference in deadlines (or admission thresholds), this policy induces unequal cell loss probabilities (10^{-5} for H-cells and 10^{-23} for L-cells for $\lambda = 0.86$). Practically, the application with the largest deadline suffers almost no losses, while the induced losses for the other applications may be unacceptably high. Finally, the STE policy is determined for $D = T^L - T^H = 50$. It is clear from the results shown in Figs. 5 and 6 that the parameter D can provide for enormous diversification of the induced cell losses. For the deadlines and rates considered in Figs. 5 and 6, the range of achieved values for P_{loss}^H and P_{loss}^L is more than 15 orders of magnitude.

To illustrate the efficiency and flexibility of the proposed policy, consider two applications with deadlines as in Figs. 5 and 6. Let QoS be defined by a cell loss requirement given by $\{P_{\text{loss}}^H < 10^{-10}, P_{\text{loss}}^L < 10^{-5}\}$. Figs. 5 and 6 indicate that none of the "standard" policies — determined by $D = 50 = T^L - T^H$ (STE), $D = 0$ (FCFS) or $D = 100 = T^L$ (HoL) — can deliver this QoS under total load equal to 0.90. For $D = 63$, the proposed policy can deliver this QoS at the load equal to 0.90.

The previous example illustrates that the STE policy can be sub-optimal with respect to the achieved resource utilization. The results shown in Fig. 7 (discussed later) as well as the discussion in Section 2

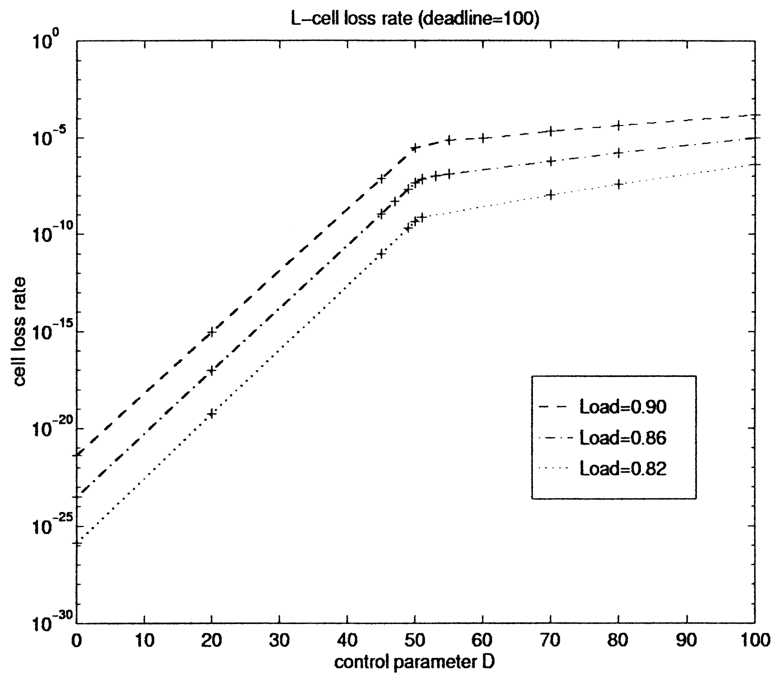

Fig. 5. L-cell loss probability vs. D under different loads: $\lambda^H = \lambda^L$; $T^H = 50$; $T^L = 100$.

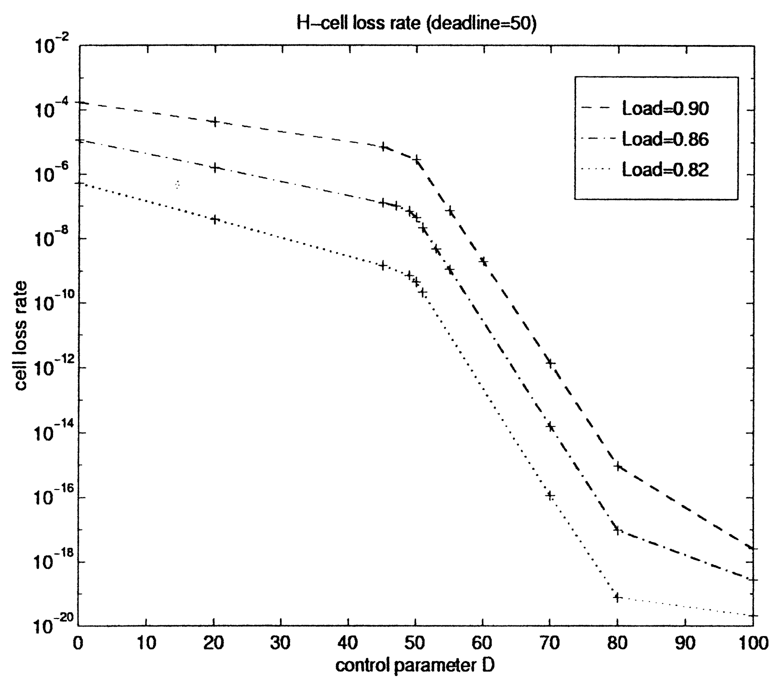

Fig. 6. H-cell loss probability vs. D under different loads: $\lambda^H = \lambda^L$; $T^H = 50$; $T^L = 100$.

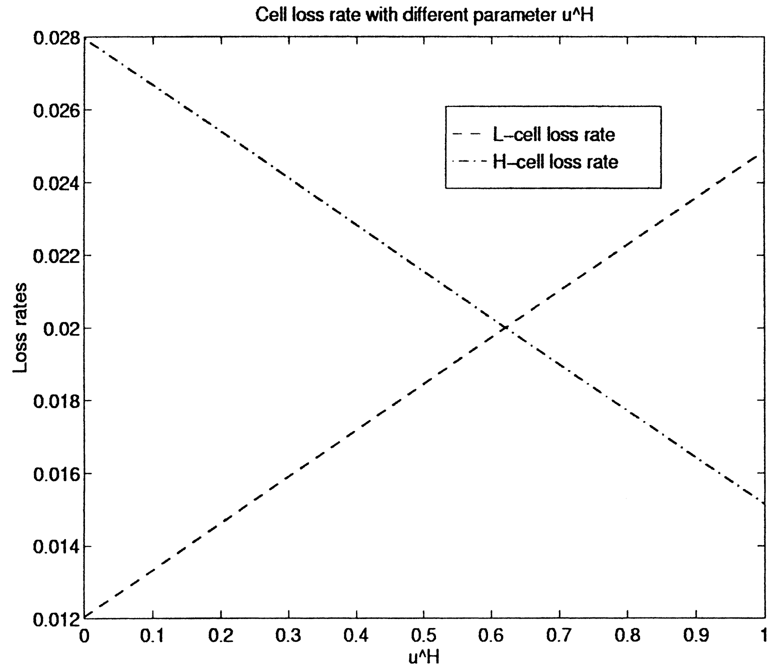

Fig. 7. L-cell loss rates for $D = 14$ $(P_{14,x})$ vs. $\mu^H = x : \lambda^H = \lambda^L = 0.45$; $T^H = 6$; $T^L = 20$.

Table 1
Total cell loss rate P_{loss} vs. parameter D

D	P_{loss}
0	2.44×10^{-6}
20	1.80×10^{-5}
45	2.02×10^{-7}
47	1.67×10^{-7}
49	1.44×10^{-7}
50	1.40×10^{-7}
51	1.44×10^{-7}
53	1.67×10^{-7}
55	2.22×10^{-7}
70	9.00×10^{-7}
80	2.44×10^{-6}
100	1.56×10^{-5}

indicate that this policy induces, in general, unequal cell losses — although in some cases biasing the service of qualified cells (parameter μ^H) can eliminate this inequality. Nevertheless, this policy is optimal in the sense that it minimizes the total cell loss probability. This is captured in Table 1 which illustrates that the STE policy ($D = 50$) minimizes the total losses.

Although the discrete-valued range of D could create a granularity problem regarding the achievable values for P^H_{loss} and P^L_{loss}, this is not the case due to the additional fine-tuning provided by the class-selection probabilities μ^H and μ^L. While in this work μ^H and μ^L have been selected to correspond to a random selection of the cell to be served next among all qualified cells, it is evident that these probabilities can be set arbitrarily (under the constraint $\mu^H + \mu^L = 1$) to bias according to the induced values of P^H_{loss} and P^L_{loss}. The discussion below clarifies this point.

Let $P^H_{k,x}$ denote the induced value of P^H_{loss} for $D = k$ and $\mu^H = x, 0 \le k \le T^L, 0 \le x \le 1$. By invoking simple sample path arguments, it can be shown that at any time the amount of service to H-cells under x_1 cannot be less than that under x_2 for $x_1 > x_2$, and thus, $P^H_{k,x_1} \le P^H_{k,x_2}$. Since $P^H_{k,x}$ is derived through a set of continuous mappings of simple continuous functions of x — see, for instance, Eqs. (13), (27) and (28) and note that these equations are simple continuous functions of $x = \mu^H$ — it is evident that the continuous range of values of x in the interval [0, 1] will generate a continuous range of values of $P^H_{k,x}$ in the interval $[P_{k,1}, P_{k,0}]$. The previous can be summarized in the following proposition.

Proposition 1. *For any value of k of the parameter D,* $0 \le k \le T^L$:
1. $P^H_{k,x}$ *is a monotonically decreasing function of* x, $0 \le x \le 1$.
2. $P^H_{k,x}$ *may assume any value in the interval* $[P^H_{k,1}, P_{k,0}]$ *by properly selecting the class-selection probability* $x = \mu^H$.

By making simple sample path arguments as before for systems operating under $D = k_1$ and $D = k_2$, $k_1 < k_2$, it can be shown that $P^H_{k_1,x} \ge P^H_{k_2,x}$. That is, we arrive at the following proposition.

Proposition 2. *For any value x of the parameter* μ^H, $0 \le x \le 1$, $P^H_{k,x}$ *is a monotonically decreasing function of k*, $0 \le k \le T^L$.

The following corollary establishes upper and lower bounds on the achievable values of P^H_{loss}; its proof is obvious in view of the monotonicity of $P^H_{k,x}$ presented in Propositions 1 and 2.

Corollary 1. $P^H_{T^L,1} \le P^H_{k,x} \le P^H_{0,0}$.

Finally, the following theorem addresses the granularity issue regarding the achievable value of P^H_{loss}.

Theorem 1. P^H_{loss} *may assume any value in the interval* $[P^H_{T^L,1},\ P^H_{0,0}]$.

Proof. In view of the above propositions, a necessary and sufficient condition in order for this theorem to hold is

$$[P^H_{T^L,1},\ P^H_{0,0}] = \bigcup_{k=0}^{T^L} [P^H_{k,1},\ P^H_{k,0}], \tag{32}$$

or, equivalently, that

$$P^H_{k,1} \le P^H_{k+1,0}, \quad 0 \le k < T^L. \tag{33}$$

This condition holds with the equality — as shown in Lemma 1 — concluding the proof of the theorem. □

Lemma 1. $P^H_{k,1} = P^H_{k+1,0}$, $0 \le k < T^L$.

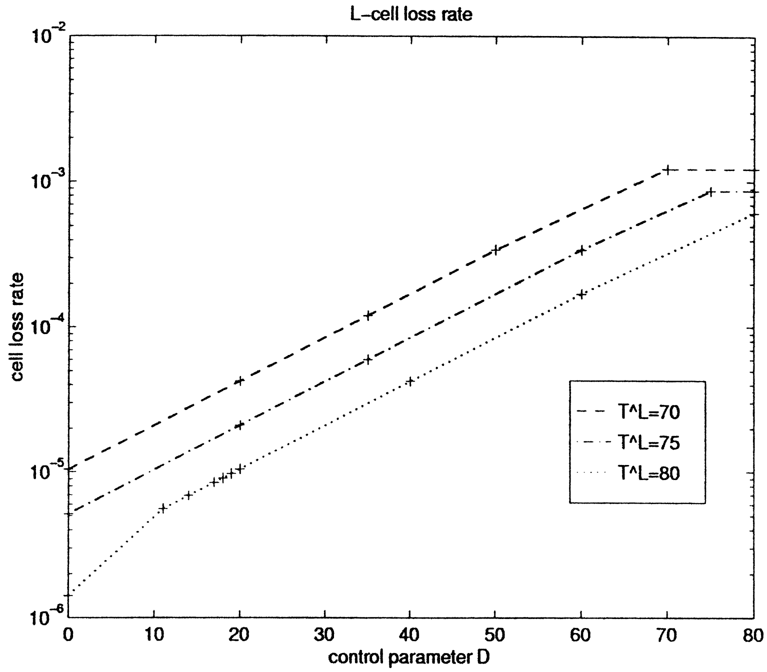

Fig. 8. L-cell loss probability vs. D under different L-cell deadlines: $\lambda^H = \lambda^L = 0.45$; $T^H = 80$.

Proof. In view of the service policy it is easy to show that H-cells receive identical service for all realizations under the policies with parameters $(D = k, \mu^H = 1)$ and $(D = k + 1, \mu^H = 0)$. A more rigorous proof can be provided by considering the recursive equations (for instance, the expected value of Eqs. (6)–(8) shown in Appendix C) under the above parameter setting and observing that they coincide. □

Fig. 7 presents some results for $P^H_{14,x}$ vs. $x = \mu^H, 0 \le x \le 1$; $\lambda^H = \lambda^L = 0.45$, $T^H = 6$ and $T^L = 20$. The results illustrate the points presented in Propositions 1 and 2. It was also found that $P^H_{14,1} = P^H_{15,0}$ and $P_{13,1} = P^H_{14,0}$ as Lemma 1 indicates. Results are shown for the L-cell loss probability for $D = 14$ and $\mu^H = x$ — denoted by $P^L_{14,x}$ — as well. Note that the above discussion regarding $P^H_{k,x}$ can be applied to $P^L_{k,x}$ in a straightforward fashion. Finally, by comparing the results in Figs. 5 and 6 and those in Fig. 7, it may be concluded that the parameter D can provide for a much greater QoS diversification compared to that achieved through parameter μ^H.

Figs. 8 and 9 present results for P^L_{loss} and P^H_{loss}, respectively, as a function of D for $0 \le D \le T^H = 80$ and for various values of T^L (70, 75 and 80); $\lambda^H = \lambda^L = 0.45$. These results can provide for a comparative study of the effectiveness of the proposed policy and the Nested Thresholds discarding policy identified for $D = 0$ and $T^H \ne T^L$. Assuming the above parameters and a buffer capacity equal to T^H, QoS diversification under the Nested Thresholds policy can be achieved by varying the threshold T^L. Under this policy, $P^L_{\text{loss}} < 10^{-5}$ can be achieved for $T^L \ge 70$, indicating a minimum value of $P^H_{\text{loss}} > 10^{-9}$. Under the proposed policy, $P^L_{\text{loss}} < 10^{-5}$ can be achieved under, for instance, $T^L = 80$ and $D = 20$, indicating a value of P^H_{loss} of about 10^{-12}.

By applying the technique outlined in Section 4.3, results for the induced average L-cell and H-cell delay were derived for various values of D and they are shown in Fig. 10; $T^L = 65$, $T^H = 40$, $\lambda^H = 0.3$

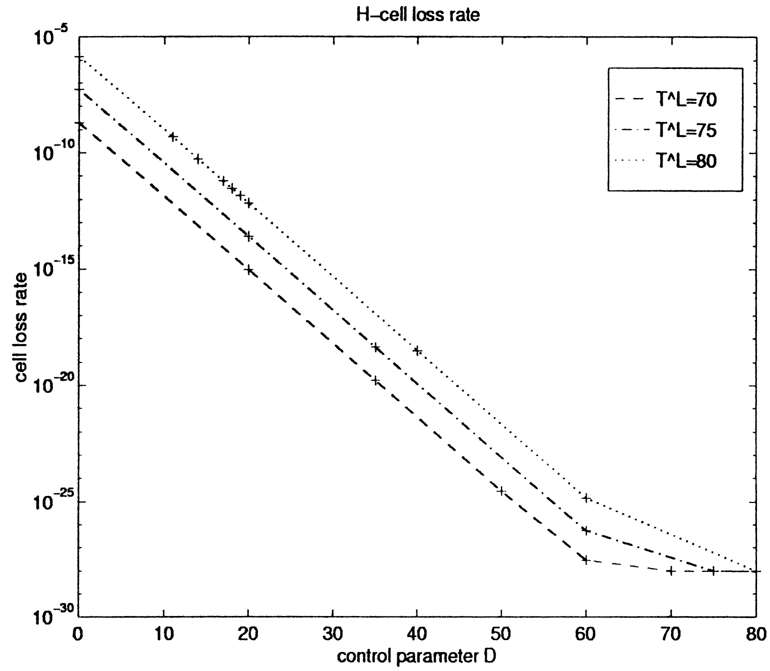

Fig. 9. H-cell loss probability vs. D under different L-cell deadlines: $\lambda^H = \lambda^L = 0.45$; $T^H = 80$.

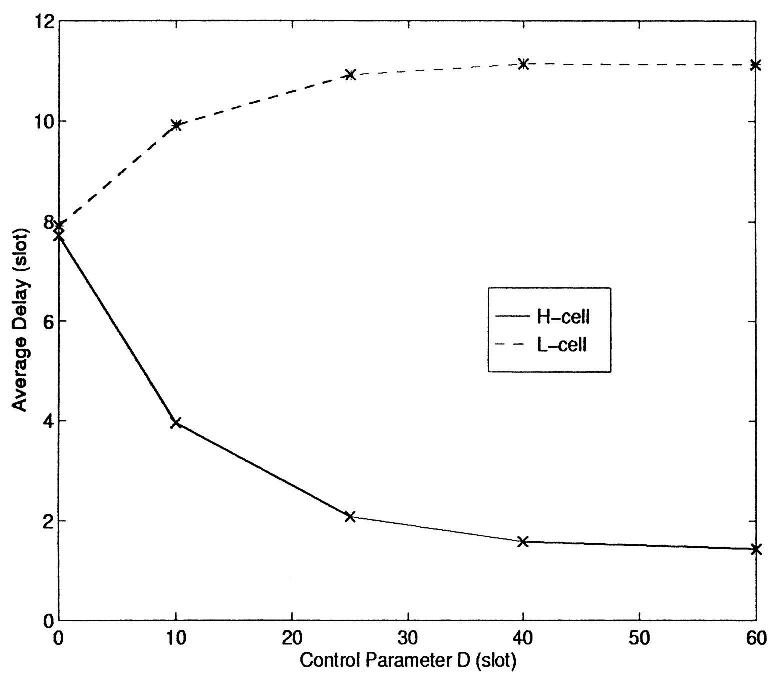

Fig. 10. Average delays: $\lambda^H = 0.3$; $\lambda^L = 0.6$; $T^H = 40$; $T^L = 65$.

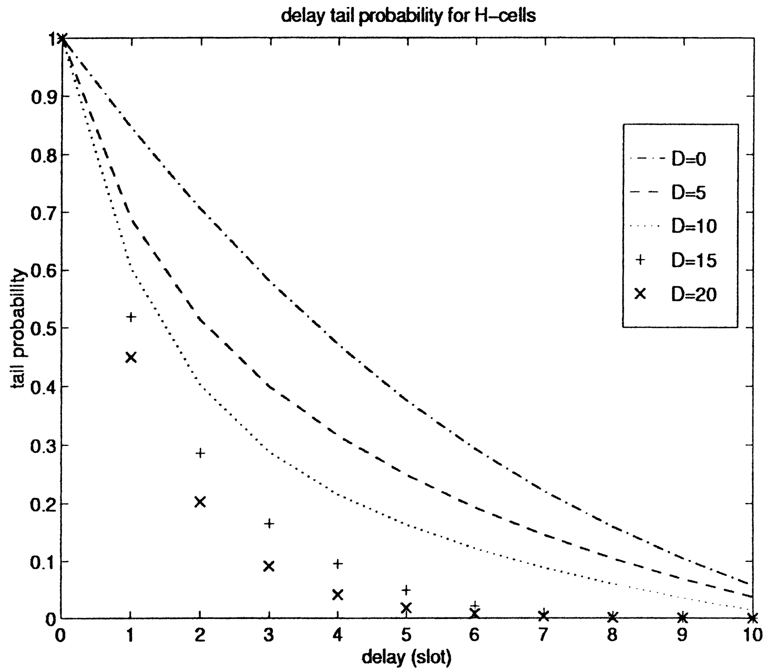

Fig. 11. H-cell tail delay probability: $\lambda^H = \lambda^L = 0.45$; $T^H = 10$; $T^L = 20$.

and $\lambda^L = 0.6$. Notice that under the FCFS policy ($D = 0$), the L-cell average delay is slightly greater than that of the H-cells, since $T^L > T^H$, and thus, cells with larger delays contribute to the average delay of L-cells (discarded cells are not considered in the delay calculation).

Figs. 11 and 12 present some results for the tail of the L-cell and H-cell delay distribution for $D = 0, 5, 10, 15, 20$; $T^H = 10$, $T^L = 20$, $\lambda^H = \lambda^L = 0.45$.

Figs. 13 and 14 present a comparison between the induced deadline violation probabilities under the proposed policy (which does not serve expired cells) and an otherwise identical policy which serves all cells; $\lambda^H = \lambda^L = 0.50$. The deadline violation probability under the latter policy is equal to the tail of the delay distribution beyond the value 50 of a system operating under the proposed policy with $T^H = T^L = \infty$. Since the latter system is of infinite dimensionality, a truncated system at $T^H = T^L = 120$ is solved providing a very tight lower bound to the tail of the infinite dimensional system. The results clearly indicate the benefit obtained by not serving expired cells if deadline violation probability is to be minimized.

Finally, Fig. 15 presents results similar to those shown in Figs. 5 and 6 but derived, instead, under correlated arrival processes and obtained via simulations. Notice that the analysis can be modified and can be applied to this simple case of correlated arrival process as suggested earlier in the paper. The parameters considered are: $\lambda^H = \lambda^L = 0.45$, $T^H = 50$ and $T^L = 100$. The arrival process is governed by a two-state (states 1 and 0) underlying Markov process. No packets are generated when the Markov process is in state 0; packets are generated following the geometric distribution given in Eq. (1) when the Markov process is in state 1. The Markov process remains in each state for a geometrically distributed time with parameter $p = 0.8$. The results shown in Fig. 15 suggest that the proposed policy does provide significant diversification in the loss rates under correlated arrivals as well.

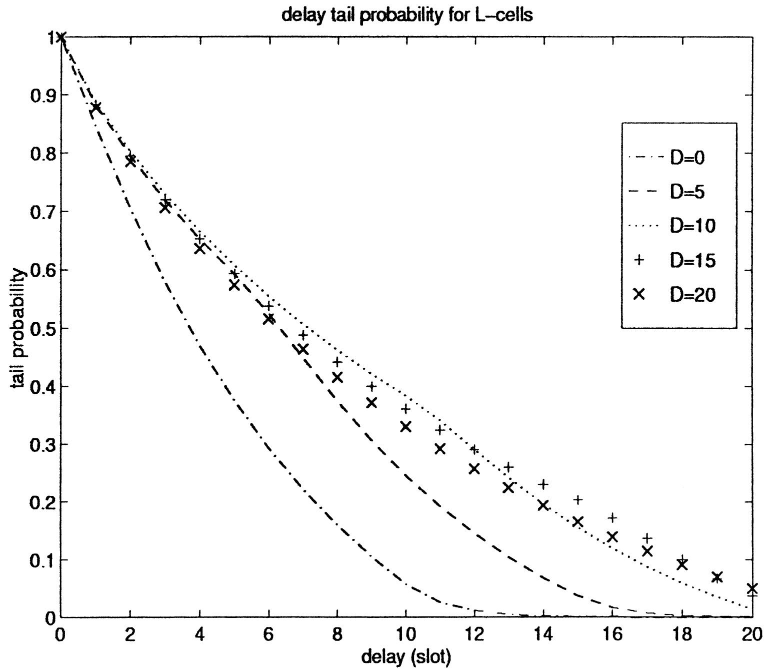

Fig. 12. L-cell tail delay probability: $\lambda^H = \lambda^L = 0.45$; $T^H = 10$; $T^L = 20$.

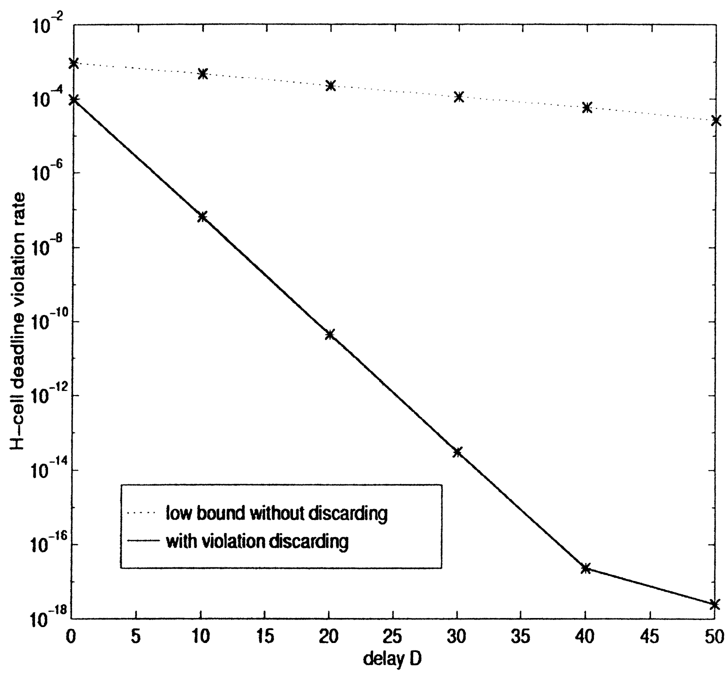

Fig. 13. H-cell deadline violation with and without discarding: $\lambda^H = \lambda^L = 0.50$; $T^H = 50$; $T^L = 50$.

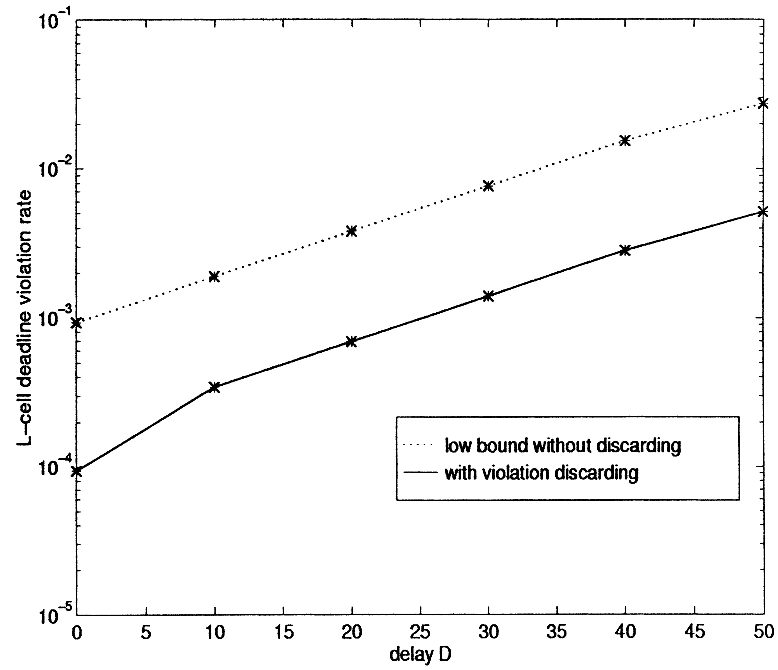

Fig. 14. L-cell deadline violation with and without discarding: $\lambda^H = \lambda^L = 0.50$; $T^H = 50$; $T^L = 50$.

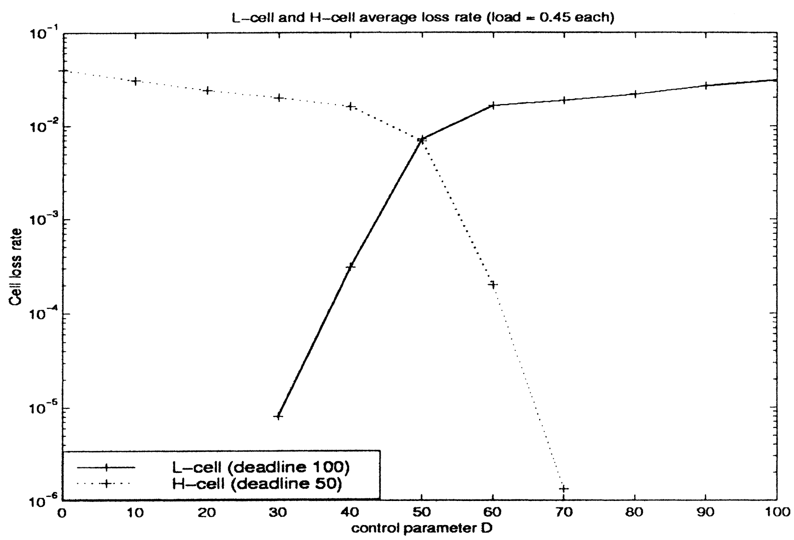

Fig. 15. L-cell and H-cell loss probability vs. D under correlated arrivals: $\lambda^H = \lambda^L = 0.45$; $T^H = 50$; $T^L = 100$.

Appendix A. Derivation of parameters μ^H and μ^L

In this appendix, the probabilities μ^H and μ^L (that the H-axis or L-axis is examined next when $i = D$) are derived when all cells having the same priority are equally likely to be selected for service (randomly selected).

Let $\hat{P}^H(\hat{P}^L)$ denote the probability that the cell transmitted next when $i = D$ is an H-cell (L-cell). These probabilities are given by

$$\hat{P}^H = \sum_{h=1}^{\infty}\sum_{l=0}^{\infty}\frac{h}{h+l}\,\Pr\{N^H = h, N^L = l\} = \sum_{h=1}^{\infty}\sum_{l=0}^{\infty}\frac{h}{h+l}(1-q^H)(q^H)^h(1-q^L)(q^L)^l$$
$$\left(\hat{P}^L = \sum_{l=1}^{\infty}\sum_{h=0}^{\infty}\frac{l}{h+l}(1-q^H)(q^H)^h(1-q^L)(q^L)^l\right), \tag{A.1}$$

where N^H and N^L are denoted in Section 4.1.

From the policy description, it is easy to establish that the class-selection probabilities μ^H and μ^L are related to the cell service probabilities $\hat{P}^H$ and $\hat{P}^L$ as follows:

$$\hat{P}^H = \mu^H \Pr\{N^H > 0\} + \mu^L \Pr\{N^L = 0\}\Pr\{N^H > 0\} = \mu^H q^H + (1-\mu^H)(1-q^L)q^H,$$
$$\hat{P}^L = \mu^L q^L + (1-\mu^L)(1-q^H)q^L. \tag{A.2}$$

From these equations, μ^H and μ^L can be derived as follows:

$$\mu^H = \frac{\hat{P}^H - (1-q^L)q^H}{q^H q^L}, \qquad \mu^L = \frac{\hat{P}^L - (1-q^H)q^L}{q^H q^L}. \tag{A.3}$$

The class-selection probabilities μ^H and μ^L can be alternatively set by considering arbitrary values for $\hat{P}^H$ and $\hat{P}^L$ instead of the values given by (A.1).

Appendix B. Equations in Case B: $T^L < T^H$.

Case B.1. $T^L \geq j > 0, \min(T^L - j, D+1) \geq i \geq m$.

1. $i < D$:

$$Y^H(i,j) = I^H + Y^H([i+\hat{I}^H, j-2\hat{I}^H+1]^*),$$
$$Y^L(i,j) = Y^L([i+\hat{I}^H, j-2\hat{I}^H+1]^*). \tag{B.1}$$

2. $i = D+1$:

$$Y^H(D+1,j) = Y^H([i-\hat{I}^L, j-\hat{I}^L+1]^*),$$
$$Y^L(D+1,j) = I^L + Y^L([i-\hat{I}^L, j-\hat{I}^L+1]^*). \tag{B.2}$$

3. $i = D$

$$Y^H(D,j) = \{I^H + Y^H([i+\hat{I}^H, j-2\hat{I}^H+1]^*)\}I_{\{H\}} + Y^H([i-\hat{I}^L, j-\hat{I}^L+1]^*)I_{\{L\}},$$
$$Y^L(D,j) = Y^L([i+\hat{I}^H, j-2\hat{I}^H+1]^*)I_{\{H\}} + \{I^L + Y^L([i-\hat{I}^L, j-\hat{I}^L+1]^*)\}I_{\{L\}}. \tag{B.3}$$

Case B.2. $T^H > j > T^L, i = T^L - j$.

$$Y^H(i,j) = I^H + Y^H([i+\hat{I}^H, j-2\hat{I}^H+1]^*),$$
$$Y^L(i,j) = Y^L([i+\hat{I}^H, j-2\hat{I}^H+1]^*). \tag{B.4}$$

Case B.3. $j = T^H, i = T^L - T^H$.

$$Y^H(i, T^H) = I^H + Y^H([i+1, T^H - \hat{I}^H]^*), \qquad Y^L(i, T^H) = Y^L([i+1, T^H - \hat{I}^H]^*). \quad \text{(B.5)}$$

Case B.4 . $j = 0, \min(T^L, D+1) \geq i \geq 1$.

$$Y^H(i, 0) = Y^H([i - \hat{I}^L, 1 - \hat{I}^L]^*), \qquad Y^L(i, 0) = I^L + Y^L([i - \hat{I}^L, 1 - \hat{I}^L]^*). \quad \text{(B.6)}$$

In Case B.3, i may be less than 0.

Appendix C. The systems of linear equations

Case A. $T^L \geq T^H$.

$$\bar{Y}^H(0, 0) = 0, \qquad \bar{Y}^L(0, 0) = 0. \quad \text{(C.1)}$$

By taking expectation operation on both sides of Eqs. (5)–(12), the following are obtained:
Case A.1. $T^H > j > 0, \min(T^L - j, D+1) \geq i \geq m$.

$$\tilde{i} = \begin{cases} i & \text{if } i + j < T^L, \\ i - 1 & \text{if } i + j = T^L. \end{cases}$$

1. $i < D$:

$$\bar{Y}^H(i, j) = q^H + (1 - q^H)\bar{Y}^H(i+1, j-1) + q^H \bar{Y}^H(\tilde{i}, j+1),$$
$$\bar{Y}^L(i, j) = (1 - q^H)\bar{Y}^L(i+1, j-1) + q^H \bar{Y}^L(\tilde{i}, j+1). \quad \text{(C.2)}$$

2. $i = D + 1$:

$$\bar{Y}^H(D+1, j) = (1 - q^L)\bar{Y}^H(i-1, j) + q^L \bar{Y}^H(\tilde{i}, j+1),$$
$$\bar{Y}^L(D+1, j) = q^L + (1 - q^L)\bar{Y}^L(i-1, j) + q^L \bar{Y}^L(\tilde{i}, j+1). \quad \text{(C.3)}$$

3. $i = D$:

$$\bar{Y}^H(D, j) = \mu^H q^H + \mu^H(1 - q^H)\bar{Y}^H(i+1, j-1) + \mu^L(1 - q^L)\bar{Y}^H(i-1, j)$$
$$+(\mu^H q^H + \mu^L q^L)\bar{Y}^H(\tilde{i}, j+1),$$
$$\bar{Y}^L(D, j) = \mu^L q^L + \mu^L(1 - q^L)\bar{Y}^L(i-1, j) + \mu^H(1 - q^H)\bar{Y}^L(i+1, j-1)$$
$$+(\mu^H q^H + \mu^L q^L)\bar{Y}^L(\tilde{i}, j+1). \quad \text{(C.4)}$$

Case A.2. $j = T^H, T^L - T^H \geq i \geq m$.

$$\tilde{i} = \begin{cases} i + 1 & \text{if } i + T^H < T^L, \\ i & \text{if } i + T^H = T^L. \end{cases}$$

1. $i < D$:

$$\bar{Y}^H(i, T^H) = q^H + (1 - q^H)\bar{Y}^H(i+1, T^H - 1) + q^H \bar{Y}^H(\tilde{i}, T^H),$$
$$\bar{Y}^L(i, T^H) = (1 - q^H)\bar{Y}^L(i+1, T^H - 1) + q^H \bar{Y}^L(\tilde{i}, T^H). \quad \text{(C.5)}$$

2. $i \geq D+1$:

$$\begin{aligned}\bar{Y}^H(i,T^H) &= (1-q^L)\bar{Y}^H(i-1,T^H) + q^L\bar{Y}^H(\tilde{i},T^H),\\ \bar{Y}^L(i,T^H) &= q^L + (1-q^L)\bar{Y}^L(i-1,T^H) + q^L\bar{Y}^L(\tilde{i},T^H).\end{aligned} \tag{C.6}$$

3. $i = D$:

$$\begin{aligned}\bar{Y}^H(D,T^H) &= \mu^H q^H + \mu^H(1-q^H)\bar{Y}^H(i+1,T^H-1) + \mu^L(1-q^L)\bar{Y}^H(i-1,T^H)\\ &\quad +(q^H\mu^H + q^L\mu^L)\bar{Y}^H(\tilde{i},T^H),\\ \bar{Y}^L(D,T^H) &= \mu^L q^L + \mu^H(1-q^H)\bar{Y}^L(i+1,T^H-1) + \mu^L(1-q^L)\bar{Y}^L(i-1,T^H)\\ &\quad +(q^H\mu^H + q^L\mu^L)\bar{Y}^L(\tilde{i},T^H).\end{aligned} \tag{C.7}$$

Case A.3 . $j=0, \min(T^L, D+1) \geq i \geq 1$.

$$\tilde{i} = \begin{cases} i & \text{if } i < T^L,\\ i-1 & \text{if } i = T^L.\end{cases}$$

$$\begin{aligned}\bar{Y}^H(i,0) &= (1-q^L)\bar{Y}^H(i-1,0) + q^L\bar{Y}^H(\tilde{i},1),\\ \bar{Y}^L(i,0) &= q^L + (1-q^L)\bar{Y}^L(i-1,0) + q^L\bar{Y}^L(\tilde{i},1).\end{aligned} \tag{C.8}$$

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