Analysis of a Probabilistic Topology-Unaware TDMA MAC Policy for Ad-Hoc Networks

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Abstract

The design of an efficient Medium Access Control (MAC) for ad-hoc networks is challenging. Topology-unaware TDMA-based schemes, suitable for ad-hoc networks, that guarantee a minimum throughput, have already been proposed. These schemes consider a *deterministic policy* for the utilization of the assigned scheduling time slots that never utilizes non-assigned slots although in such slots collision-free transmissions are possible even under heavy traffic conditions. A simple *probabilistic policy*, capable of utilizing the non-assigned slots according to an access probability, fixed for all users in the network, is introduced and analyzed here. The conditions under which the system throughput under the probabilistic policy is higher than that under the deterministic policy are derived analytically. Further analysis of the system throughput is shown to be difficult or impossible for the general case and certain approximations have been considered whose accuracy is also investigated. The approximate analysis determines the value for the access probability that *maximizes* the system throughput as well as *simplified lower and upper bounds* that depend only on a topology density metric. Simulation results demonstrate the comparative advantage of the probabilistic policy over the deterministic policy and show that the approximate analysis successfully determines the range of values for the access probability for which the system throughput under the probabilistic policy is not only higher than that under the deterministic policy, but it is also close to the maximum.

Index Terms

Ad-Hoc, TDMA, MAC, Topology-Unaware, Probabilistic Policy.

I. Introduction

IDIOSYNCRATIC networks, like *ad-hoc networks*, make the design of an efficient Medium Access Control (MAC) a challenging problem. These networks require no infrastructure and nodes are free to enter, leave or move inside the network without prior configuration. This flexibility introduces new challenges and several MAC protocols have been proposed that is possible to categorize them into two different categories according to the *scheduling* of their transmissions.

The first category corresponds to MAC protocols that allow the users to content in order to transmit. Corrupted transmissions (*collisions*) are possible and the CSMA/CA-based IEEE 802.11, [1], is a very well known example. Additionally to the carrier sensing mechanism, MACA, [2], employs the *Ready-To-Send/Clear-To-Send* (RTS/CTS) handshake mechanism. This mechanism is mainly introduced to avoid the *hidden/exposed terminal* problem, which is a reason for significant performance

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degradation in ad-hoc networks. Specifically, a node that intends to transmit, sends a RTS message to to the intended receiver node. The intended receiver replies with a CTS message and the sender may start transmitting while all neighbor nodes that have heard this "dialogue" refrain from transmission. Other protocols based on variations of this mechanism also exist, [3], [4], [5].

The second category refers to TDMA-based MAC protocols where each node has been assigned a certain set of TDMA *scheduling time slots* that is allowed to transmit. S-TDMA - proposed by Kleinrock and Nelson, [6], is capable of providing *collision-free* scheduling based on the exploitation of noninterfering transmissions in the network. In general, optimal solutions to the problem of time slot assignment often result in NP-hard problems, [7], [8], which are similar to the n-coloring problem in graph theory.

Topology-unaware scheduling schemes determine the scheduling time slots irrespectively of the underlying topology. Chlamtac and Farago, [9], have proposed a TDMA-based topology-unaware scheme that exploits the mathematical properties of polynomials with coefficients from finite Galois fields to randomly assign scheduling time slot sets to each node of the network. For each node it is guaranteed that at least one time slot in a frame would be collision-free, [9]. Another scheme proposed by Ju and Li, [10], maximizes the minimum guaranteed throughput. However, both schemes employ a deterministic policy for the utilization of the assigned time slots that fails to utilize non-assigned time slots that could result in successful transmissions, as it is shown here.

Both the aforementioned approaches allow collisions in one frame and the sender node (being not aware which slot is collision-free) needs to transmit on all assigned time slot sets in one frame, [11]. Collision-free scheduling schemes were proposed to overcome this problem, [12], [13], [14], [15], while in [16] the use of *acknowledgments* (ACK) at the end of each time slot was proposed. The work presented in [14] is based on pseudo-random sequences creating the scheduling slots. This approach suffers from the need for frequent exchanges of the scheduling time slots especially when the nodes move fast in the network.

In this paper, the general approach proposed in [9] and [10] is considered and the idea of allowing the nodes to utilize (according to a common access probability) scheduling slots not originally assigned (according to the rules in [9], [10]) to them, is presented. As it is shown in this paper, this policy achieves a better performance under certain conditions (that are studied here), when the benefit of utilizing otherwise idle slots outweighs the loss due to collisions induced by the introduced controlled interference. The issue of the maximization of the *system throughput* is also addressed in this paper. Parts of this work have been presented in [17] and [18].

In Section II a general ad-hoc network is described and some key definitions are introduced. The proposed policy (to be referred to as the *Probabilistic Policy*) is motivated and introduced in Section III; the one introduced in [9], [10] is also described and is referred to as the *Deterministic Policy*. In Section IV the case of a specific transmission between two given neighbor nodes, is considered. A study is presented establishing the conditions under which the Probabilistic Policy achieves higher probability of success, for a specific transmission, than that under the Deterministic Policy. In Section V, a preliminary system throughput analysis shows that it is difficult or impossible, for the general case, to fully analyze it. An approximate analysis is presented in Section VI that establishes the conditions for the existence of an *efficient range* of values for the access probability (values of the access probability under which the Probabilistic Policy outperforms the Deterministic Policy).

Furthermore, this analysis determines the maximum value for the system throughput and the corresponding value for the access probability; bounds on the latter probability are determined analytically as a function of an appropriately defined topology density metric. In Section VII, the accuracy of the approximate analysis is studied. Simulation results, presented in Section VIII, show that a value for the access probability that falls within the bounds, as they are determined based on the topology density metric, results in a system throughput that is close to the maximum. Section IX presents the conclusions.

II. System Definition

An ad-hoc network may be viewed as a time varying multihop network and may be described in terms of a graph $G(V, E)$, where V denotes the set of nodes and E the set of links between the nodes at a given time instance. Let $|X|$ denote the number of elements in set X and let $N = |V|$ denote the number of nodes in the network.

Let S_u denote the set of neighbors of node u, $u \in V$. These are the nodes v to which a direct transmission from node u (*transmission* $u \rightarrow v$) is possible. Let D denote the maximum number of neighbors for a node; clearly $|S_u| \leq D$, $\forall u \in V$. In this work, omni-directional antennas are considered both for transmission and reception purposes over the wireless medium. Additionally, time is divided into time slots with fixed duration (as it is the case in TDMA-based environments) and collisions with other transmissions is considered to be the only reason for a transmission not to be successful (*corrupted*).

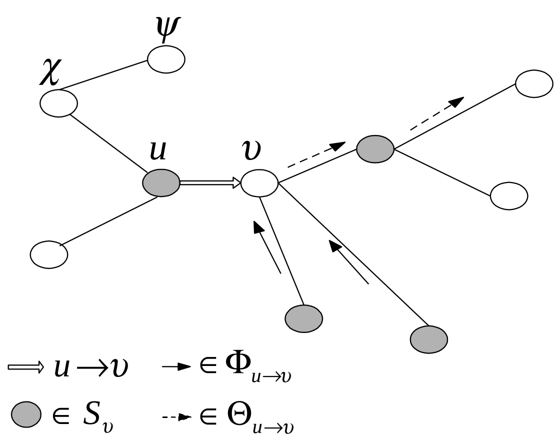

Fig. 1. Example transmission $u \rightarrow v$, set of nodes S_v and transmissions that belong in $\Phi_{u\rightarrow v}$ or $\Theta_{u\rightarrow v}$.

Suppose that node u wants to transmit to a particular neighbor node v in a particular time slot i. In order for the transmission $u \rightarrow v$, depicted in Figure 1, to be successful (*uncorrupted*), two conditions should be satisfied. First, node v should not transmit in the particular time slot i, or equivalently, no transmission $v \rightarrow \psi$, $\forall \psi \in S_v$ should take place in time slot i. Second, no neighbor of v - except u - should transmit in time slot i, or equivalently, no transmission $\zeta \rightarrow \chi$, $\forall \zeta \in S_v - \{u\}$ and $\chi \in S_\zeta$, should take place in time slot i. Consequently, transmission $u \rightarrow v$ is corrupted in time slot i if at least one transmission $\chi \rightarrow \psi$, $\chi \in S_v \cup \{v\} - \{u\}$ and $\psi \in S_\chi$, takes place in time slot i. In Figure 1, nodes colored gray belong in S_v. It is clear that if nodes that belong in $S_v \cup \{v\} - \{u\}$ transmit, transmission $u \rightarrow v$ becomes corrupted.

As it may be seen in Figure 1, the transmission(s) that corrupts transmission $u \rightarrow v$ may or may not be successful itself. Specifically, in the presence of transmission $u \rightarrow v$, transmission $\chi \rightarrow \psi$, $\chi \in S_v \cup \{v\} - \{u\}$ and $\psi \in S_\chi \cap (S_u \cup \{u\})$, is corrupted (dense black arrows). If $\psi \in S_\chi - (S_\chi \cap (S_u \cup \{u\}))$, then transmission $\chi \rightarrow \psi$ is not affected by transmission $u \rightarrow v$ (dotted black arrows).

Let $\Phi_{u\rightarrow v}$ be the set of transmissions which corrupt transmission $u \rightarrow v$ and at the same time transmission $u \rightarrow v$ corrupts them as well. Let $\Theta_{u\rightarrow v}$ be the set of transmissions which corrupt transmission $u \rightarrow v$ but are not corrupted themselves by it. Note that transmissions that belong in $\Theta_{u\rightarrow v}$ may still be corrupted by a transmission other than transmission $u \rightarrow v$.

Transmission sets $\Phi_{u\rightarrow v}$ and $\Theta_{u\rightarrow v}$ are given by equations (1) and (2) respectively.

$$\Phi_{u\rightarrow v} = \Big\{\chi \rightarrow \psi : \chi \in S_v \cup \{v\} - \{u\}, \psi \in S_\chi \cap (S_u \cup \{u\})\Big\} \quad (1)$$

$$\Theta_{u\rightarrow v} = \Big\{\chi \rightarrow \psi : \chi \in S_v \cup \{v\} - \{u\}, \psi \in S_\chi - (S_\chi \cap (S_u \cup \{u\}))\Big\}. \quad (2)$$

It is evident that $\Phi_{u\rightarrow v} \cup \Theta_{u\rightarrow v}$ is the set of transmissions that corrupts transmission $u \rightarrow v$. Obviously $\Phi_{u\rightarrow v} \cap \Theta_{u\rightarrow v} = \emptyset$. Figure 2 depicts an example topology of 27 nodes. Transmission $8 \rightarrow 13$ is denoted by a white arrow between nodes 8 and 13 and transmissions that belong in $\Phi_{8\rightarrow 13}$ ($\Theta_{8\rightarrow 13}$) are denoted by black dense (dotted black) arrows.

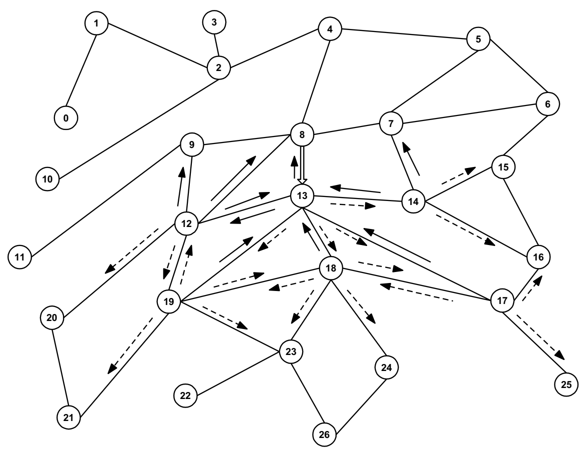

Fig. 2. Transmission sets $\Phi_{8\rightarrow 13}$ and $\Theta_{8\rightarrow 13}$ for an example network of 27 nodes.

Suppose that transmission $u \rightarrow v$ takes place in time slot i. It is important for node u to have knowledge whether the transmission was successful. It is assumed that an acknowledge message (ACK) is returned by the receiver after the successful reception of a transmission. In particular, a fixed part at the end of each time slot may be used for this purpose (to be referred to as the *ACK part* of the time slot), [16]. Suppose that transmission $u \rightarrow v$, depicted in Figure 1, was uncorrupted in time slot i. Consequently, at the ACK part of time slot i, transmission $v \rightarrow u$ will take place in order to notify node u about the successful reception of transmission $u \rightarrow v$ (ACK message).

In general, transmission $v \rightarrow u$ may become corrupted if a node $\chi \in S_u - \{v\}$ transmits. In the aforementioned case, node χ may transmit during the ACK part of time slot i if and only if it has received a uncorrupted transmission (for example transmission $\psi \rightarrow \chi$ depicted in Figure 1) in time slot i and therefore, transmission of the ACK message is required during the ACK part of time slot i. The latter is not possible since transmission $\psi \rightarrow \chi$ would have been corrupted due to the presence of transmission $u \rightarrow v$. Consequently, such a mechanism for the acknowledgment of the uncorrupted transmissions is valid. For the rest, it is assumed that the transmitting node is instantaneously aware of an uncorrupted transmission, [19].

III. Scheduling Policies

Under the policy proposed in [9] and [10], each node $u \in V$ is randomly assigned a unique polynomial f_u of degree k with coefficients from a finite Galois field of order q ($GF(q)$). Polynomial f_u is represented as $f_u(x) = \sum_{i=0}^{k} a_i x^i$, [10], where $a_i \in \{0, 1, 2, ..., q-1\}$; parameters q and k are calculated based on N and D, according to the algorithm presented either in [9] or [10].

The access scheme considered is a TDMA scheme with a frame consisted of q^2 time slots. If the frame is divided into q subframes s of size q, then the time slot assigned to node u in subframe s, ($s = 0, 1, ..., q-1$) is given by $f_u(s)$mod q, [10]. Let the set of time slots assigned to node u be denoted as Ω_u. Consequently, $|\Omega_u| = q$. The deterministic transmission policy, proposed in [9] and [10], is the following.

The Deterministic Policy: Each node u transmits in a slot i only if $i \in \Omega_u$, provided that it has data to transmit.

The relation between N, D, q and k is important in order to explain the main property of the Deterministic Policy: there exists at least one time slot in a frame over which a specific transmission will remain uncorrupted, [9]. Suppose that two neighbor nodes u and v have been assigned two (unique) polynomials f_u and f_v of degree k, respectively. Given that the roots of each node's polynomial correspond to the assigned time slots to each node, k common time slots is possible to be assigned among two neighbor nodes. Given that D is the maximum number of neighbor nodes of any node, kD is the maximum number of time slots over which a transmission from any node is possible to become corrupted. Since the number of time slots that a node is allowed to transmit in a frame is q, if $q > kD$ or $q \geq kD + 1$ (k and D are integers) is satisfied, there will be at least one time slot in a frame in which a specific transmission will remain uncorrupted for any node in the network, [9].

The assigned unique polynomials may be considered as similar to MAC identification numbers (MAC IDs) and are assigned to each node accordingly: either they are included in the device or they are assigned using a control mechanism. In this work, it is assumed that nodes have been assigned a unique polynomial *randomly* (without taking into account the neighbor nodes of the node and/or their assigned polynomials) before the node enters the network. At this point is important to guarantee that the number of unique polynomials is enough for all nodes in the network, or $q^{k+1} \geq N$. If for a given N and q, $q^{k+1} \geq N$ is not satisfied, then k has to be increased (perhaps resulting to a new q that satisfies $q > kD$) until the number of unique polynomials is sufficient. Given that large values of k correspond to a larger value of q compared to D (linear increment) and even larger values of N (exponential increment), it can be concluded that $k > 1$ is the case of networks with large N and comparably small D, or "large" networks, [10].

Depending on the particular random assignment of the polynomials, it is possible that two nodes be assigned overlapping time slots (i.e., $\Omega_u \cap \Omega_v \neq \emptyset$). Let $C_{u \to v}$ be the set of overlapping time slots between those assigned to node u and those assigned to any node $\chi \in S_v \cup \{v\} - \{u\}$. $C_{u \to v}$ is given by (3).

$$C_{u \to v} = \Omega_u \cap \left(\bigcup_{\chi \in S_v \cup \{v\} - \{u\}} \Omega_\chi \right). \tag{3}$$

Let $R_{u \to v}$ denote the set of time slots i, $i \notin \Omega_u$, over which transmission $u \to v$ would be successful. Equivalently, $R_{u \to v}$ contains those slots not included in set $\bigcup_{\chi \in S_v \cup \{v\}} \Omega_\chi$. Consequently,

$$|R_{u \to v}| = q^2 - \left| \bigcup_{\chi \in S_v \cup \{v\}} \Omega_\chi \right|. \tag{4}$$

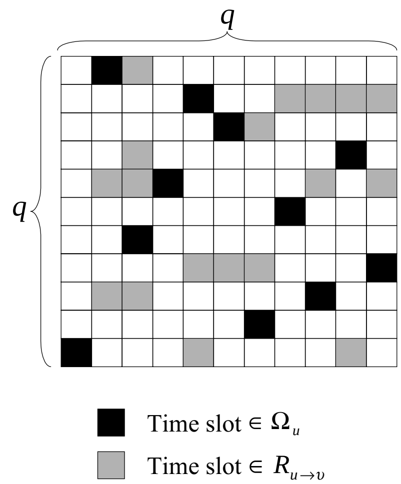

Fig. 3. Example sets Ω_u and $R_{u\to v}$ for node u and transmission $u \to v$ respectively (frame size: $q^2 = 121$).

$R_{u\to v}$ is the set of *non-assigned eligible* time slots for transmission $u \to v$, that if used by transmission $u \to v$, the probability of success for the particular transmission could be increased. Figure 3 depicts an example frame of size $q^2 = 121$ and both sets Ω_u and $R_{u\to v}$ for node u and transmission $u \to v$ are depicted respectively. The increased probability of success for transmission $u \to v$ does not necessarily increase the average probability of success of all transmissions in the network (throughput); the presence of transmission $u \to v$ in a slot i, $i \notin \Omega_u$, may corrupt another, otherwise successful, transmission $\chi \to \psi$, for which $i \in \Omega_\chi$ and transmission $u \to v \in \Theta_{\chi\to\psi}$. Then, transmission $\chi \to \psi$ will not be a successful one, even though $u \to v$ will be.

Theorem 1: It is satisfied that $|R_{u\to v}| \geq q(k-1)D$.

Proof: Notice that $\left|\bigcup_{\chi\in S_v\cup\{v\}} \Omega_\chi\right| \leq (|S_v|+1)q$ holds, since $|\Omega_\chi| = q$, $\forall \chi \in V$. From Equation (4) it is concluded that $|R_{u\to v}| \geq q^2 - (|S_v|+1)q$ is satisfied, or $|R_{u\to v}| \geq q(q - |S_v| - 1)$ and since $D \geq |S_v|$, $|R_{u\to v}| \geq q(q - D - 1)$. Since $q \geq kD + 1$ (see [9], [10]), $q - D - 1 \geq (k-1)D$ is satisfied. Consequently, $|R_{u\to v}| \geq q(k-1)D$. ∎

From Theorem 1 it is obvious that for $k > 1$, $|R_{u\to v}| > qD$. Consequently, the number of non-assigned eligible slots may be quite significant for the cases where $k > 1$ (this case corresponds to "large networks," [10]). Even for the case where $k = 1$, $|R_{u\to v}| \geq 0$, that is, $|R_{u\to v}|$ can still be greater than zero. For those nodes for which the set of overlapping slots is not the largest possible $\left(\text{i.e., } \left|\bigcup_{\chi\in S_v\cup\{v\}} \Omega_\chi\right| < (|S_v|+1)q\right)$, $|R_{u\to v}|$ is strictly greater than zero, even for $k = 1$. Furthermore, if the neighborhood of node v is not dense, or $|S_v|$ is small compared to D, then $|R_{u\to v}|$ is even higher (see Equation (4)).

In general, the use of slots i, $i \in R_{u\to v}$, may increase the average number of successful transmissions, as long as $R_{u\to v}$ is determined and time slots $i \in R_{u\to v}$ are used efficiently. The determination of $R_{u\to v}$ requires the existence of a mechanism for the extraction of sets Ω_χ, $\forall \chi \in S_v$. In addition, the efficient use of slots in $R_{u\to v}$ by node u, requires further coordination and control exchange with neighbor nodes χ, whose transmissions $\chi \to \psi$, with $R_{\chi\to\psi} \cap R_{u\to v} \neq \emptyset$, may utilize the same slots in $R_{\chi\to\psi} \cap R_{u\to v}$ and corrupt either transmission $u \to v$ or $\chi \to \psi$, or both.

Moreover, under non-heavy traffic conditions, there exist a number of idle slots, in addition to those in $R_{u\to v}$, not used by the node they are assigned to. In order to use all non-assigned time slots without the need for further coordination among the nodes, the following probabilistic transmission policy is proposed.

The Probabilistic Policy: Each node u always transmits in slot i if $i \in \Omega_u$ and transmits with probability p in slot i if

$i \notin \Omega_u$, provided it has data to transmit.

The Probabilistic Policy does not require specific topology information (e.g., knowledge of $R_{u\to v}$, etc.) and, thus, induces no additional control overhead. The access probability p is a simple parameter common for all nodes. Under the Probabilistic Policy, all slots $i \notin \Omega_u$ are potentially utilized by node u: both, those in $R_{u\to v}$, for a given transmission $u \to v$, as well as those not in $\Omega_u \cup R_{u\to v}$ that may be left by neighboring nodes under non-heavy traffic conditions. On the other hand, the probabilistic transmission attempts induce interference to otherwise collision-free transmissions. The following sections establish the conditions under which the loss due to the induced interference is more than compensated for by the utilization of the non-assigned time slots, for *heavy traffic* conditions.

IV. Specific Transmission Analysis

In this section both policies are analyzed for a specific transmission (transmission $u \to v$). The analysis assumes heavy traffic conditions; that is, there is always data available for transmission at each node, for every time slot.

Let $P_{i,u\to v}$ denote the *probability that transmission* $u \to v$ *in slot* i *is successful*. Let $P_{u\to v}$ be the average probability over a frame for transmission $u \to v$ to be successful during a time slot. That is, $P_{u\to v} = \frac{1}{q^2}\sum_{i=1}^{q^2} P_{i,u\to v}$, where q^2 is the frame size, in time slots. $P_{u\to v}$ may also be referred to as *throughput*.

Under the Deterministic Policy, $P_{i,u\to v} = 0$, $\forall i \notin \Omega_u$. For $i \in \Omega_u$ there are two distinct cases: for $i \in C_{u\to v}$, $P_{i,u\to v} = 0$, while for $i \notin C_{u\to v}$, $P_{i,u\to v} = 1$ (note the if $i \in C_{u\to v}$ then $i \in \Omega_u$ as well). Since $|\Omega_u| = q$, it is evident that under the Deterministic Policy the average over a frame probability of success for transmission $u \to v$ (denoted by $P_{D,u\to v}$), is given by

$$P_{D,u\to v} \quad = \quad \frac{q - |C_{u\to v}|}{q^2}. \tag{5}$$

Under the Probabilistic Policy, it is evident that $P_{i,u\to v} = 0$, for $i \in C_{u\to v}$, as well as for $i \notin \Omega_u$ and $i \notin R_{u\to v}$. On the other hand, $P_{i,u\to v} = (1-p)^{|S_v|}$, for $i \in \Omega_u$ and $i \notin C_{u\to v}$, whereas $P_{i,u\to v} = p(1-p)^{|S_v|}$, for $i \in R_{u\to v}$ (note that if $i \in R_{u\to v}$ then $i \notin \Omega_u$). Consequently, $P_{i,u\to v} = (1-p)^{|S_v|}$ for $q - |C_{u\to v}|$ time slots, while $P_{i,u\to v} = p(1-p)^{|S_v|}$ for $|R_{u\to v}|$ time slots. As a result, under the Probabilistic Policy the average over a frame probability of success for transmission $u \to v$ (denoted by $P_{P,u\to v}$), is given by

$$P_{P,u\to v} \quad = \quad \frac{q - |C_{u\to v}| + p|R_{u\to v}|}{q^2}(1-p)^{|S_v|}. \tag{6}$$

The term $\frac{q-|C_{u\to v}|}{q^2}(1-p)^{|S_v|}$ is equal to $P_{D,u\to v}$, decreased by the factor $(1-p)^{|S_v|}$ that is due to the interference introduced by the probabilistic transmission attempts. The term $\frac{p|R_{u\to v}|}{q^2}(1-p)^{|S_v|}$ is the gain due to the use of the non-assigned eligible slots $R_{u\to v}$. The aforementioned reduction of $P_{D,u\to v}$ due to the interference, is possible to be *more than compensated for* by the gain due to the use of the non-assigned eligible slots, resulting in $P_{P,u\to v} > P_{D,u\to v}$. This is investigated in the sequel.

It is evident that $P_{P,u\to v} = P_{D,u\to v}$, when $p = 0$. For $p > 0$, $P_{P,u\to v}$ may or may not be greater than $P_{D,u\to v}$, depending on the values of p, q, $|S_v|$, $|R_{u\to v}|$ and $|C_{u\to v}|$. The scope of the following analysis is to determine the conditions under which $P_{P,u\to v} > P_{D,u\to v}$.

Theorem 2: $P_{P,u\to v} = P_{D,u\to v}$ for $p = 0$ and $P_{P,u\to v} < P_{D,u\to v}$ for $1 \geq p > 0$, provided that $|R_{u\to v}| \leq (q - |C_{u\to v}|)|S_v|$.

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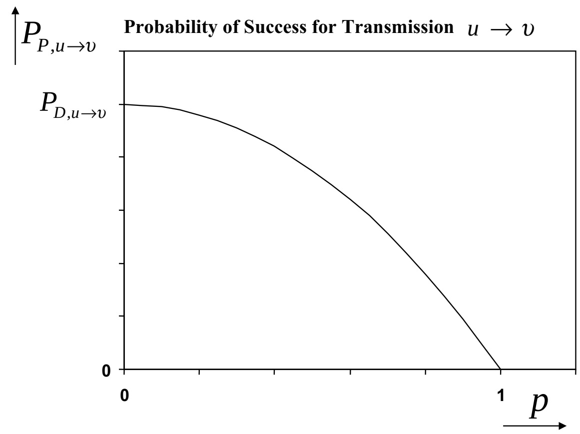

Fig. 4. $P_{P,u\to v}$ when $|R_{u\to v}| \le (q - |C_{u\to v}|)|S_v|$ and $2|R_{u\to v}| \le (q - |C_{u\to v}|)(|S_v| - 1)$.

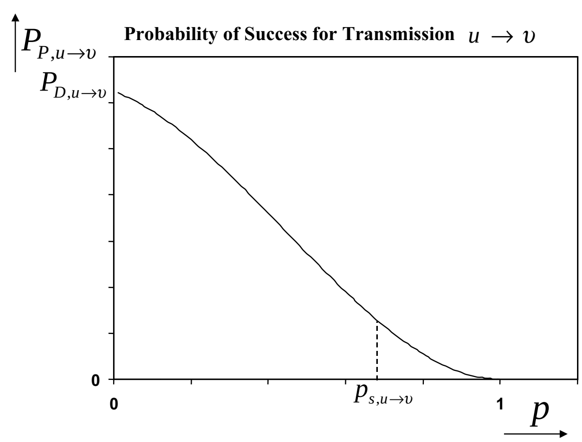

Fig. 5. $P_{P,u\to v}$ when $|R_{u\to v}| \le (q - |C_{u\to v}|)|S_v|$ and $2|R_{u\to v}| > (q - |C_{u\to v}|)(|S_v| - 1)$.

The proof of Theorem 2 can be seen in Appendix I.

Theorem 3: Provided that $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$ is satisfied, $P_{P,u\to v} > P_{D,u\to v}$ for $p \in (0, p_{max,u\to v})$, for some $0 < p_{max,u\to v} < 1$ such that $P_{P,u\to v} = P_{D,u\to v}$, for $p = p_{max,u\to v}$. ∎

The proof of Theorem 3 can be seen in Appendix II.

Figures 4 and 5 depict the generic behavior of $P_{P,u\to v}$ as a function of p, when $|R_{u\to v}| \le (q - |C_{u\to v}|)|S_v|$. If the second derivative of $P_{P,u\to v}$ with respect to p, $\frac{d^2 P_{P,u\to v}}{d^2 p}$, is not zero for any value p, $0 \le p < 1$, according to Appendix III, $2|R_{u\to v}| \le (q - |C_{u\to v}|)(|S_v| - 1)$ is satisfied. This is the case depicted in Figure 4. Figure 5 corresponds to the case where a value of p, $0 < p < 1$ exists such that $\frac{d^2 P_{P,u\to v}}{d^2 p} = 0$. According to Appendix III, this is true when $2|R_{u\to v}| > (q - |C_{u\to v}|)(|S_v| - 1)$. The value of p for which $\frac{d^2 P_{P,u\to v}}{d^2 p} = 0$ is denoted by $p_{s,u\to v}$ and $p_{s,u\to v} = \frac{2|R_{u\to v}| - (q - |C_{u\to v}|)(|S_v| - 1)}{|R_{u\to v}|(|S_v| + 1)}$.

The generic behavior of $P_{P,u\to v}$, as a function of p, when $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$, is shown in Figure 6. According to Appendix II there exists a maximum value for $P_{P,u\to v}$ for a particular value of $p \in (0, p_{max,u\to v})$ ($\equiv p_{0,u\to v} = \frac{|R_{u\to v}| - (q - |C_{u\to v}|)|S_v|}{|R_{u\to v}|(|S_v| + 1)}$). In Appendix III it is shown that the value of $p_{s,u\to v}$, for which $\frac{d^2 P_{P,u\to v}}{d^2 p} = 0$, satisfies $p_{0,u\to v} <$

$p_{s,u\to v} < 1$.

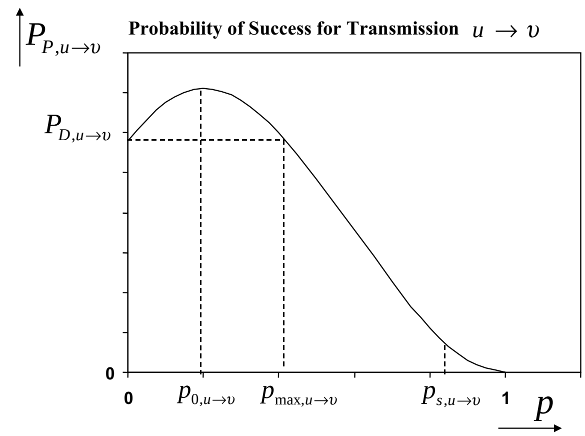

Fig. 6. $P_{P,u\to v}$ when $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$.

It should be noted that $p_{max,u\to v}$ is not easy to be calculated analytically from Equation (6). On the other hand, it can be calculated using numerical methods, such as the well-known Newton-Raphson method.

The analysis presented so far has established the conditions for which the probability of success under the Probabilistic Policy is higher than that under the Deterministic Policy, for a specific transmission. Since the same value of p is assumed to be adopted by all nodes under the Probabilistic Policy it is possible that this common value results in different comparative performance under the Deterministic and the Probabilistic Policies, for different transmissions. First, it may be that for some transmissions the condition of Theorem 2 holds and thus the Probabilistic Policy can never outperform the Deterministic one. Second, it may be that for some transmissions $\chi \to \psi$ the condition of Theorem 3 holds but the common p is outside the range $(0, p_{max,\chi\to\psi})$ and thus the Probabilistic Policy induces a lower probability of success. Finally, for some transmissions $\chi \to \psi$, the condition of Theorem 3 may hold and p is within the range $(0, p_{max,\chi\to\psi})$ and thus the Probabilistic Policy outperforms the Deterministic one. From the aforementioned three cases it is clear that the system throughput (averaged over all transmissions) under the Probabilistic Policy (denoted by P_P) may or may not outperform that under the Deterministic Policy (denoted by P_D) for a given value of p. The following sections focus on the system throughput analysis and in particular on the appropriate range of values of p.

V. SYSTEM THROUGHPUT

In this section the expressions for the *system throughput* under both policies are provided and the conditions under which the Probabilistic Policy outperforms the Deterministic Policy are derived. When these conditions are satisfied it is shown that there exists an *efficient* range of values for p (such that the system throughput under the Probabilistic Policy is higher than that under the Deterministic Policy).

The destination node v of a transmission $u \to v$ depends on the destination of the data and consequently, on the *application* as well as on the *routing protocol*. For the rest of this work it will be assumed that a node u transmits only towards a node v in one frame and node v will be a node randomly selected from S_u.

Let P_D (P_P) denote the probability of success of a transmission (averaged over all transmissions) under the Deterministic (Probabilistic) Policy (be referred to as the *system throughput* for both policies) assuming that each node u may transmit to only one node $v \in S_u$ in one frame. According to equations (5) and (6) it can be concluded that P_D and P_P are given by the following equations.

$$P_D = \frac{1}{N} \sum_{\forall u \in V} \frac{q - |C_{u \to v}|}{q^2}, \tag{7}$$

$$P_P = \frac{1}{N} \sum_{\forall u \in V} \frac{q - |C_{u \to v}| + p|R_{u \to v}|}{q^2} (1-p)^{|S_v|}, \tag{8}$$

where $v \in S_u$. From Equation (8) it can be seen that for $p = 0$, $P_P = P_D$, while for $p = 1$, $P_P = 0$. In general, P_P may or may not be greater than P_D. Consequently both equations have to be analyzed to establish the conditions under which $P_P \geq P_D$. The following theorem establishes the condition under which there exists an efficient range of values for p of the form $[0, p_{max}]$, for some $0 \leq p_{max} < 1$, such that $P_P \geq P_D$.

Theorem 4: Provided that $\sum_{\forall u \in V} \big(|R_{u \to v}| - (q - |C_{u \to v}|)|S_v|\big) \geq 0$ is satisfied, there exist an efficient range of values for p of the form $[0, p_{max}]$, for some $0 \leq p_{max} < 1$.

Proof: The first derivative $\frac{dP_P}{dp}$ is given by the following equation.

$$\begin{aligned} \frac{dP_P}{dp} &= \frac{1}{N} \sum_{\forall u \in V} \frac{|R_{u \to v}| - (q - |C_{u \to v}|)|S_v|}{q^2} (1-p)^{|S_v|-1} \\ &\quad - \frac{1}{N} \sum_{\forall u \in V} \frac{|R_{u \to v}|(|S_v|+1)p}{q^2} (1-p)^{|S_v|-1}. \end{aligned} \tag{9}$$

It is obvious that for small values of p, $\lim_{p \to 0} \frac{dP_P}{dp} = \frac{1}{N} \sum_{\forall u \in V} \frac{|R_{u \to v}| - (q - |C_{u \to v}|)|S_v|}{q^2}$. Consequently, if $\sum_{\forall u \in V} \Big(|R_{u \to v}| - (q - |C_{u \to v}|)|S_v|\Big) \geq 0$ there exists a range of values for p such that $\frac{dP_P}{dp} \geq 0$ and since $P_P = P_D$, for $p = 0$, it is concluded that $P_P \geq P_D$; since for some $p \geq 0$, P_P is a continuous function of p $\big($see Equation (8)$\big)$, and since $P_P = 0$, for $p = 1$, there exists a value for p, denoted by p_{max}, such that $P_P = P_D$ and $0 \leq p_{max} < 1$. Consequently, if the particular condition is satisfied then there exists an efficient range of values of the form $[0, p_{max}]$. ■

The following theorem is based on Theorem 4 for "large networks" ($k > 1$, see [10]).

Theorem 5: For $k > 1$, $|R_{u \to v}| \geq (q - |C_{u \to v}|)|S_v|$, for every transmission $u \to v$. For $k = 1$ $|R_{u \to v}| \geq (q - |C_{u \to v}|)|S_v|$, provided that $|S_v| \leq D/2$.

Proof: From Theorem 1 and for any transmission $u \to v$, it is concluded that $|R_{u \to v}| \geq qD$, for $k > 1$. Since $q \geq q - |C_{u \to v}|$ and $|S_v| \leq D$, it is concluded that $(q - |C_{u \to v}|)|S_v| \leq qD$, and, consequently, $|R_{u \to v}| \geq (q - |C_{u \to v}|)|S_v|$ holds for $k > 1$.

In [9], [10] it is established that $q \geq D + 1$ for $k = 1$. From Theorem 1 and for any transmission $u \to v$, $|R_{u \to v}| \geq q(q - |S_v| - 1)$ or $|R_{u \to v}| \geq (q - |C_{u \to v}|)(q - |S_v| - 1)$, since $q \geq q - |C_{u \to v}|$. To show that $|R_{u \to v}| \geq (q - |C_{u \to v}|)|S_v|$ it suffices to show that $q - |S_v| - 1 \geq |S_v|$ or $q \geq 2|S_v| + 1$. Since $q \geq D + 1$, it suffices to show that $D + 1 \geq 2|S_v| + 1$ or $|S_v| \leq D/2$. ■

The above analysis shows an obvious connection between the number of the neighbor nodes of nodes in the network and the system throughput for all transmissions $u \to v$. From equations (3) and (4) it is concluded that $|C_{u \to v}|$ increases and $|R_{u \to v}|$ decreases as $|S_v|$ increases for any node v. Under the Deterministic Policy $\Big($see Equation (7)$\Big)$ the system throughput for transmission $u \to v$ decreases linearly as $|C_{u\to v}|$ increases (as $|S_v|$ increases). Under the Probabilistic Policy $\Big($see Equation (8)$\Big)$ as $|S_v|$ increases the system throughput: (a) decreases linearly as $|C_{u\to v}|$ increases; (b) increases linearly as $|R_{u\to v}|$ decreases; and (c) decreases exponentially $\Big($term $(1-p)^{|S_v|}\Big)$ as $|S_v|$ increases. Thus, it seems that an increase in $|S_v|$ has a more negative impact on the probability of success under the Probabilistic Policy than under the Deterministic.

Theorem 4 and Theorem 5 establish the conditions for the existence of an efficient range of values for the access probability p of the form $[0, p_{max}]$, for some $0 \leq p_{max} < 1$. Given that Equation (8) and Equation (9) are difficult or impossible to be analyzed for $D > 1$ and $D > 2$ (correspond to polynomials with degree $D+1$ and D) respectively, the maximum value of P_P cannot be determined because the value of p that achieves it cannot be determined either. In addition, the range $[0, p_{\ max}]$ cannot be determined analytically. In order to avoid the aforementioned problems an approximate analysis is considered and presented in the following section. This analysis establishes the appropriate conditions for the existence of an efficient range of values, based on a topology density metric that will represent the topology density of the network. The maximum value for the system throughput is possible to be calculated, since the corresponding value of p ($\tilde{p}_{\,0}$) is analytically determined. Easy to compute boundaries of the region containing $\tilde{p}_{\,0}$ are also determined analytically as a function of an introduced topology density metric.

VI. Approximate Analysis

The problems mentioned in the previous section arise from the polynomial nature of Equation (8) that is difficult or impossible to be analyzed for $D > 2$. The approximate analysis presented in this section is based on a polynomial that is more tractable than that in Equation (8). Let the system throughput P_P, be approximated by $\tilde{P}_P$:

$$\tilde{P}_P \;=\; \frac{1}{N}\sum_{\forall u\in V}\frac{q-|C_{u\to v}|+p|R_{u\to v}|}{q^2}(1-p)^{\overline{|S|}}, \tag{10}$$

where $\overline{|S|} = \frac{1}{N}\sum_{\forall u\in V}|S_u|$, denotes the *average number of neighbor nodes*. Let $\overline{|S|}/D$ be referred to as the *topology density*. For a given pair of N and D, numerous topologies exist that can be categorized according to the average number of neighbor nodes. Certainly, several different networks correspond to the same values of N, D and $\overline{|S|}/D$

In the sequel, the conditions under which $\tilde{P}_P \geq P_D$, are established and the value for p (denoted by $\tilde{p}_{\,0}$) that maximizes $\tilde{P}_P$ is determined as well. Let $\phi_{u\to v} = \frac{\sum_{\chi\in S_v\cup\{v\}-\{u\}}|\Omega_\chi\cap\Omega_u|}{|S_v|+1}$ denote the *average number of overlapping slots* of node u with each node $\chi \in \Big\{S_v \cup \{v\} - \{u\}\Big\}$. As it can be seen from Appendix V, the following inequality holds.

$$|R_{u\to v}| \geq q^2 - (|S_v|+1)(q-\phi_{u\to v}). \tag{11}$$

Let $\overline{\phi} = \frac{1}{N}\sum_{\forall u\in V}\phi_{u\to v}$.

Theorem 6: Provided that $\sum_{\forall u\in V}\Big(|R_{u\to v}| - (q-|C_{u\to v}|)\overline{|S|}\Big) \geq 0$ is satisfied, there exists a range of efficient values of p of the form $[0, \tilde{p}_{max}]$, for some $0 \leq \tilde{p}_{max} < 1$. $\tilde{P}_P$ assumes a maximum for $p = \frac{\sum_{\forall u\in V}\Big(|R_{u\to v}|-(q-|C_{u\to v}|)\overline{|S|}\Big)}{\sum_{\forall u\in V}\Big(|R_{u\to v}|(\overline{|S|}+1)\Big)}$ $(\equiv \tilde{p}_0)$. ■
The proof of Theorem 6 can be found in Appendix VI.

Theorem 6 not only establishes the conditions for the existence of an efficient range of values for p, but also determines the value of p ($\tilde{p}_0$) that maximizes $\tilde{P}_P$. This is rather useful, but in the general case, knowledge of $C_{u\to v}$ and $R_{u\to v}$ for all possible transmissions in the network, is not available. Theorem 7 establishes a condition equivalent to that of Theorem 6, based on the average number of overlapping slots $\overline{\phi}$ and $\overline{|S|}$. In addition, Theorem 8 determines lower and upper bounds on $\tilde{p}_0$ as a function of $\overline{|S|}$ only.

Theorem 7: There exists an efficient range of values for p, provided that $\overline{\phi} \geq \frac{2\overline{|S|}+1}{4}$. ■

The proof of Theorem 7 can be found in Appendix VII.

The condition of Theorem 7 (or Theorem 6) is sufficient but not necessary in order for $\tilde{P}_P \geq P_D$. Notice also that these theorems do not provide for a way to derive $\tilde{p}_{max}$. In addition, $\tilde{p}_0$ depends on parameters that are difficult to know for the entire network. Theorem 8 not only provides for a range of efficient values for p but also determines simple bounds ($\tilde{p}_{0_{max}}$, $\tilde{p}_{0_{min}} : \tilde{p}_{0_{max}} \leq \tilde{p}_0 \leq \tilde{p}_{0_{min}}$) on the values of $\tilde{p}_0$ (that maximizes $\tilde{P}_P$) as a function of $\overline{|S|}$ only.

Theorem 8: $\tilde{p}_{0_{max}} = \frac{1}{\overline{|S|}+1}$, and $\tilde{p}_{0_{min}} = \frac{q^2-(2\overline{|S|}+1)\left(q-\frac{2\overline{|S|}+1}{4}\right)}{\left(q^2-(\overline{|S|}+1)\left(q-\frac{2\overline{|S|}+1}{4}\right)\right)(\overline{|S|}+1)}$, provided that there exists an efficient range of values for p. ■

The proof of Theorem 8 can be found in Appendix VIII.

Both Theorems 7 and 8 are important for the realization of a system that efficiently implements the Probabilistic Policy. Given a polynomial assignment that satisfies Theorem 7, for a value of p between $\tilde{p}_{0_{min}}$ and $\tilde{p}_{0_{max}}$, the achievable system throughput is expected to be close to the maximum. For the determination of $\tilde{p}_{0_{min}}$ and $\tilde{p}_{0_{max}}$ it is enough to have knowledge of the density of the topology that is captured by the topology density metric $\overline{|S|}/D$.

In this work N and D are assumed to be known before the *set-up* of the network in order to allow the derivation of the corresponding scheduling time slot sets. $\overline{|S|}$ is possible to be available at the same time in order to derive a suitable value for p (p is assumed to be calculated once and for all at the set-up time). This value of p will remain valid as long as there is no movement (for example in sensor networks) or if the movement does not result to significant changes to the average topology density.

Specifically, if the movement of the nodes is such that the topology density decreases then $P_P > P_D$ will remain valid, even though P_P will not be maximized. If the movement of the nodes is such that the topology density increases, it is possible that $P_P < P_D$. Consequently, even if $\overline{|S|}$ is known at the set-up time but nothing is known about the movement of the nodes, a suitable choice is to select a value of p corresponding to the most dense topology or $\overline{|S|} = D$. It may easily be concluded from Theorem 8 that the upper bound, $\tilde{p}_{0_{max}}$, decreases as $\overline{|S|}$ increases and consequently, for $\overline{|S|} = D$ the range of values $(\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}})$ contains smaller values for p. Therefore, for example, $p = \tilde{p}_{0_{min}}\big|_{\overline{|S|}=D}$ is a suitable value for p, when $\overline{|S|}$ is not available and even though it does not lead to P_P maximization, $P_P > P_D$ is satisfied.

VII. On the Accuracy of the Approximation

The analysis presented in Section VI has established the conditions under which $\tilde{P}_P \geq P_D$, as well as the range of values of p for which $\tilde{P}_P$ is maximized. This section investigates the conditions under which $\tilde{P}_P$ is *close* to P_P. In addition, if the condition for which $\tilde{P}_P \geq P_D$ holds, is satisfied, it is investigated whether the condition $P_P \geq P_D$ holds as well. According to Theorem 6, there exists an efficient range of values such that $\tilde{P}_P \geq P_D$, if $\sum_{\forall u \in V}\left(|R_{u\to v}| - (q - |C_{u\to v}|)\overline{|S|}\right) \geq 0$ is satisfied and according to Theorem 4, there exists an efficient range of values such that $P_P \geq P_D$, if $\sum_{\forall u \in V}\left(|R_{u\to v}| - (q - |C_{u\to v}|)|S_v|\right) \geq 0$ is also satisfied. This section investigates the conditions under which the aforementioned conditions are *close*.

In Appendix XI, it is shown that the difference $|P_P - \tilde{P}_P| \leq \frac{1}{N}\sum_{\forall u \in V}\left(\frac{q-|C_{u\to v}|+p|R_{u\to v}|}{q^2}(1-p)^{\overline{|S|}}\left|(1-p)^{|S_v|-\overline{|S|}} - 1\right|\right) \equiv$

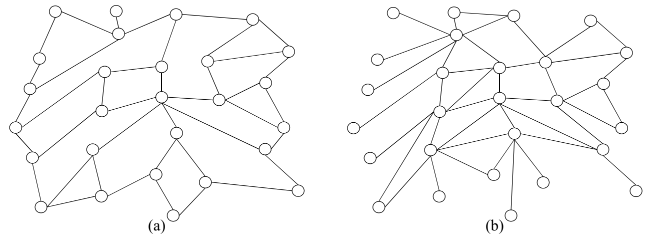

Fig. 7. Networks A (a) and B (b) with the same topology density value ($\overline{|S|}/D = 0.494$) with Network C (Figure 2) but different topology density variation ($Var\{|S|\} = 0.037$, 0.198 and 0.136, respectively).

ε_1. Let $Var\{|S|\} = \frac{1}{ND}\sum_{v\in V}\left|\overline{|S|} - |S_v|\right|$ be defined as the *topology density variation*. It is evident that as $Var\{|S|\}$ increases, ε_1 increases exponentially. Consequently, $\tilde{P}_P$ is a good approximation of P_P, for rather small values of $Var\{|S|\}$ ($\varepsilon_1 \to 0$). In the exceptional case of a network for which all nodes have the same number of neighbor nodes (except the node that has D neighbor nodes), $P_P \approx \tilde{P}_P$ ($Var\{|S|\} \to 0$). For the case that all nodes have equal number of neighbor nodes and equal to D, then $P_P = \tilde{P}_P$ ($Var\{|S|\} = 0$). An example of the latter case is a network with uniformly distributed nodes on a sphere's surface.

The absolute difference between the left-hand terms in the previously mentioned conditions $\left|\sum_{\forall u\in V}\left(|R_{u\to v}| - (q - |C_{u\to v}|)\overline{|S|}\right) - \sum_{\forall u\in V}\left(|R_{u\to v}| - (q - |C_{u\to v}|)|S_v|\right)\right|$, is calculated to be equal to $\left|\sum_{\forall u\in V}(q - |C_{u\to v}|)(\overline{|S|} - |S_v|)\right| \leq \sum_{\forall u\in V}(q - |C_{u\to v}|)\left|\overline{|S|} - |S_v|\right| \equiv \varepsilon_2$. Consequently, as $Var\{|S|\}$ approaches zero, ε_2 approaches zero linearly and consequently, the condition under which $\tilde{P}_P \geq P_D$ holds, approaches linearly the condition under which $P_P \geq P_D$ holds.

From the above discussion is obvious that as $Var\{|S|\}$ increases, ε_2 increases linearly but ε_1 increases exponentially. Consequently, for the case where ε_1 is not small, it is possible that ε_2 and $P_P \geq P_D$ holds if the condition corresponding to $\tilde{P}_P \geq P_D$ is satisfied.

Let p_0 denote that value for p that maximizes P_P. Obviously, $\left.\frac{dP_P}{dp}\right|_{p=p_0} = 0$. Equation (9) is a polynomial of degree D and it is difficult or impossible to be solved to obtain an analytical form for p_0. It is obvious that for $Var\{|S|\} = 0$, $\tilde{p}_0 \equiv p_0$ and therefore, $p_0 \in (\tilde{p}_{0,min}, \tilde{p}_{0,max})$. In general p_0 may or may not belong in $(\tilde{p}_{0,min}, \tilde{p}_{0,max})$ but any value $p \in (\tilde{p}_{0,min}, \tilde{p}_{0,max})$ for which $\tilde{P}_P \geq P_D$ holds and $\tilde{P}_P$ is close to its maximum value, possibly (depending on the value of ε_2) leads to $P_P \geq P_D$ and it is also possible (depending on the value of ε_1) that P_P is close to its maximum value as well.

In order to provide an example let the networks depicted in figures 7(a), 7(b) and 2 be referred to as network A, B and C, respectively. All three network have the same number of nodes ($N = 27$), the same number of maximum neighbor nodes ($D = 6$) and the same topology density ($\overline{|S|}/D = 0.494$).

Figure 8 depicts $P_{P,A}$, $P_{P,B}$ and $P_{P,C}$ for the three aforementioned networks as well as $\tilde{P}_P$ as a function of p. These are simulation results and were obtained as it is described in Section VIII. Let $p_{0,A}$, $p_{0,B}$ and $p_{0,C}$ those values of p for which $P_{P,A}$, $P_{P,B}$ and $P_{P,C}$ are maximized, respectively. It is observed from Figure 8 that as $Var\{|S|\}$ increases the difference between P_P (for any of the networks) and $\tilde{P}_P$ increases, while the difference between the value of p that maximizes P_P and the value of p that maximizes $\tilde{P}_P$ ($\tilde{p}_0$), increases but not dramatically.

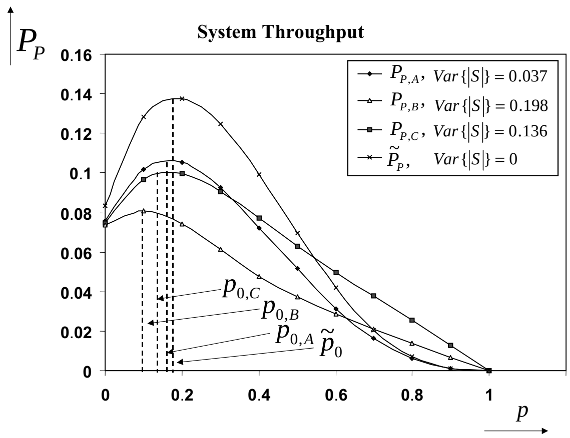

Fig. 8. System throughput for network A, B and C.

VIII. Simulation Results

Networks of 100 nodes are considered in the simulation for various values of D and the topology density $\overline{|S|}/D$. The aim is to demonstrate the applicability of the analytical results for a variety of topologies with different characteristics. In particular, four different topology categories are considered. The number of nodes in each topology category is $N = 100$, while D is 5, 10, 15 and 20. These four topology categories are denoted as D5N100, D10N100, D15N100 and D20N100 respectively. From the numerous topologies that correspond to a pair of N and D, three different topologies that correspond to different topology density values $\overline{|S|}/D$ are considered for each topology category.

Figures 9, 10 and 11 depict simulation results for the system throughput P_P for different topology density values. In particular, in Figure 9, $\overline{|S|}/D$ is small (around 0.2), in Figure 10, $\overline{|S|}/D$ is around 0.6, while in Figure 11, $\overline{|S|}/D$ is high (around 0.85). For all cases the number of neighbor nodes for each node is not the same; this leads to nonzero values for the topology density variation $Var\{|S|\}$.

The algorithm presented in [10] is used to derive the sets of scheduling slots and the system throughput is calculated averaging the simulation results over 100 frames. Unique polynomials, that correspond to time slot sets Ω_{χ}, are assigned randomly to each node χ, for each particular topology. The particular assignment is kept the same for each topology category throughout the simulations. Heavy traffic conditions have also been assumed in the sense that data are always available for transmission at each node in the network, for each time slot,

According to Theorem 1, for $k > 1$ the number of non-assigned eligible time slots is expected to be higher for a higher value of k and there exists an efficient range of values for p, as it has been proved in Theorem 5. Consequently, it is expected that for $k > 1$ all transmissions will achieve a higher system throughput under the Probabilistic Policy for any value of p within the efficient range. When $k = 1$, the Probabilistic Policy will outperform the Deterministic Policy if the condition of Theorem 7 is satisfied and p belongs in the efficient range of values. If $p \in (\tilde{p}_{0,min}, \tilde{p}_{0,max})$ then the achieved system throughput is possible to be close to the maximum, as it appears from Theorem 8.

The simulation results presented demonstrate the performance for $k = 1$ (the resulting value for k is equal to 1 for the four topology categories, [10]), that is the case that the number of non-assigned eligible time slots is expected to be rather small

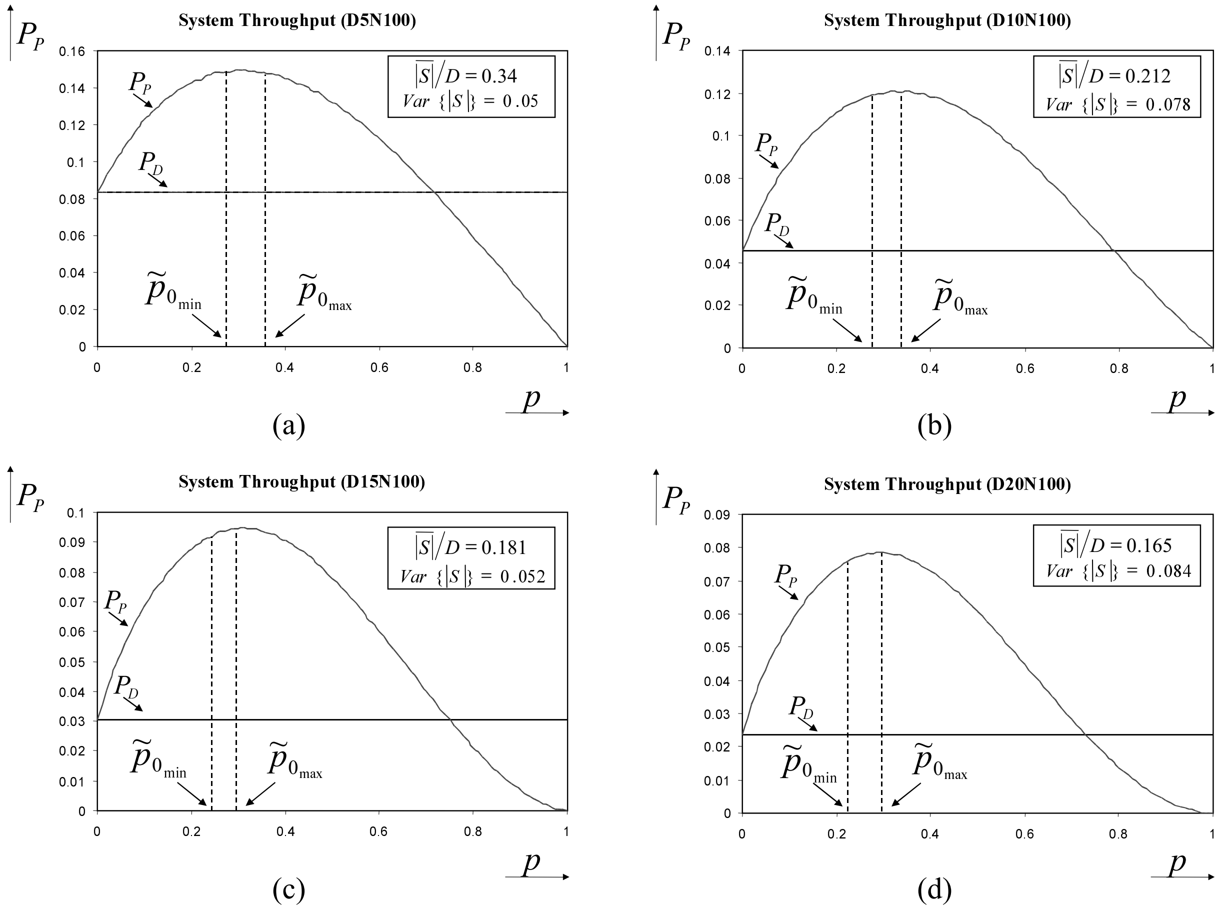

Fig. 9. System throughput simulation results for different values of p (small topology density values $\overline{|S|}/D$), for both the Deterministic Policy and the Probabilistic Policy.

and, thus, the effectiveness of the Probabilistic Policy rather low. The value of $\overline{\phi}$ calculated for each topology satisfies the condition of Theorem 7 for all cases.

In all three sets of simulations it can be observed, as expected, that the system throughput achieved under the Deterministic Policy is constant with respect to p. On the other hand, the system throughput under the Probabilistic Policy is equal to that under the Deterministic Policy for $p = 0$ and equal to zero for $p = 1$, as it may also be concluded from Equation (8). It can be observed that there exists an efficient range of values for the access probability p for all cases. The range of values $(\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}})$, as it is determined by Theorem 8, is shown as well. Obviously, $(\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}})$ determines a range of the values of p for which $P_P > P_D$ and it appears that P_P is close to its maximum value.

Table I summarizes the the values of $\overline{|S|}/D$, $Var\{|S|\}$, $(\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}})$ and $(\tilde{p}_{0_{min}}|_{\overline{|S|}=D}, \tilde{p}_{0_{max}}|_{\overline{|S|}=D})$ for each topology. It is clear that the range of values of p, $(\tilde{p}_{0_{min}}|_{\overline{|S|}=D}, \tilde{p}_{0_{max}}|_{\overline{|S|}=D})$ is smaller than $(\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}})$ which would have resulted in smaller system throughput but it would have been a suitable choice when knowledge of $\overline{|S|}$ is not available.

For the comparison between the two schemes, it is set $p = \tilde{p}_{0_{min}}$. From Figure 12, it can be seen that the achieved system throughput under the Probabilistic Policy is higher than that under the Deterministic Policy. In particular, for small values of the topology density $\overline{|S|}/D$, the system throughput is almost double compared to that of the Deterministic Policy. As $\overline{|S|}/D$

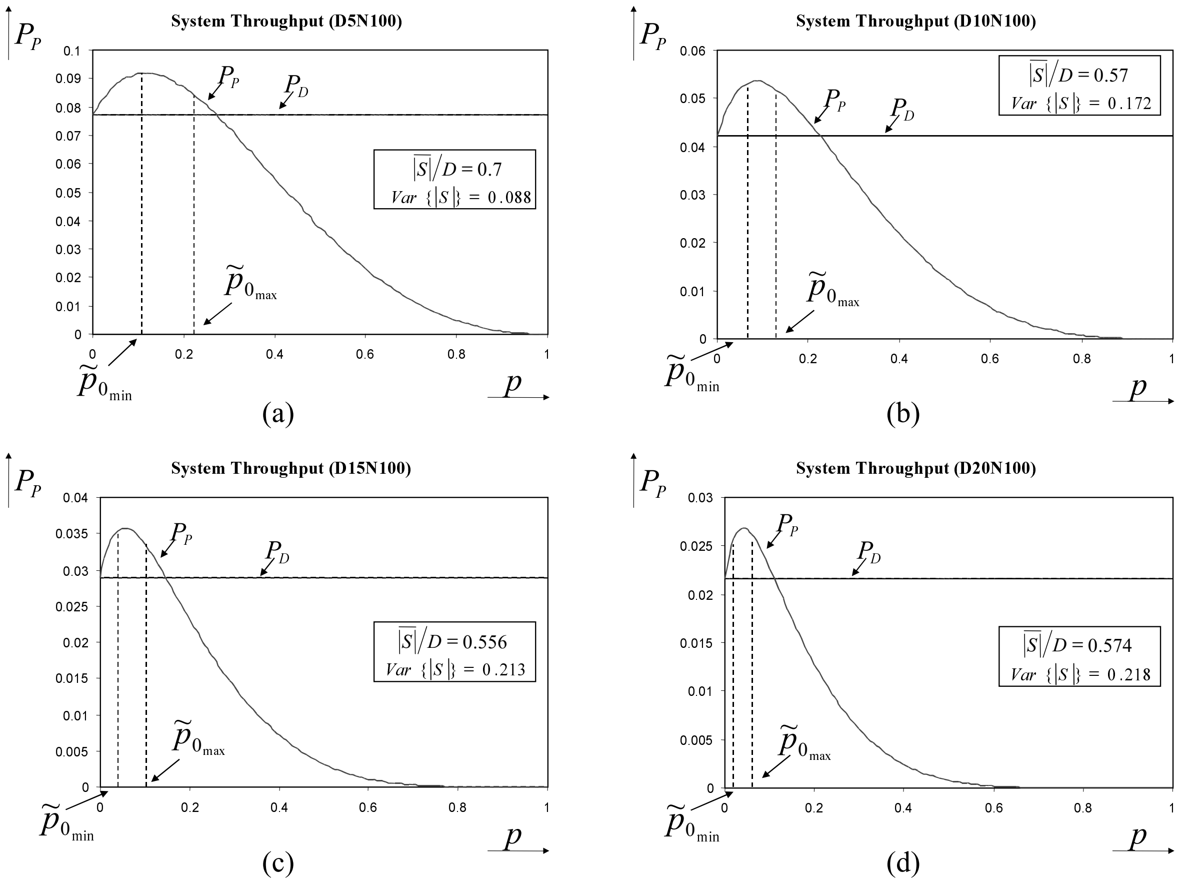

Fig. 10. System throughput simulation results for different values of p (medium topology density values $\overline{|S|}/D$), for both the Deterministic Policy and the Probabilistic Policy.

TABLE I

SUMMARY OF VALUES.

Density	Topology	$\overline{\lvert S\rvert}/D$	$Var\{\lvert S\rvert\}$	$\tilde{p}0_{min}$	$\tilde{p}0_{max}$	$\tilde{p}0_{min}\rvert_{\overline{\lvert S\rvert}=D}$	$\tilde{p}0_{max}\rvert_{\overline{\lvert S\rvert}=D}$
Low	$D5N100$	0.34	0.05	0.296608	0.37037	0.059735	0.166667
	$D10N100$	0.212	0.078	0.283345	0.320513	0.34509	0.090909
	$D15N100$	0.181	0.052	0.242686	0.268817	0.024156	0.0625
	$D20N100$	0.165	0.084	0.212199	0.232558	0.018567	0.047619
Medium	$D5N100$	0.7	0.088	0.125	0.222222	0.059735	0.166667
	$D10N100$	0.57	0.172	0.099117	0.149254	0.34509	0.090909
	$D15N100$	0.556	0.213	0.072515	0.107066	0.024156	0.0625
	$D20N100$	0.574	0.218	0.053515	0.080128	0.018567	0.047619
High	$D5N100$	0.912	0.016	0.075071	0.179856	0.059735	0.166667
	$D10N100$	0.87	0.079	0.04798	0.103093	0.34509	0.090909
	$D15N100$	0.875	0.079	0.033187	0.033187	0.024156	0.0625
	$D20N100$	0.866	0.103	0.02606	0.054585	0.018567	0.047619

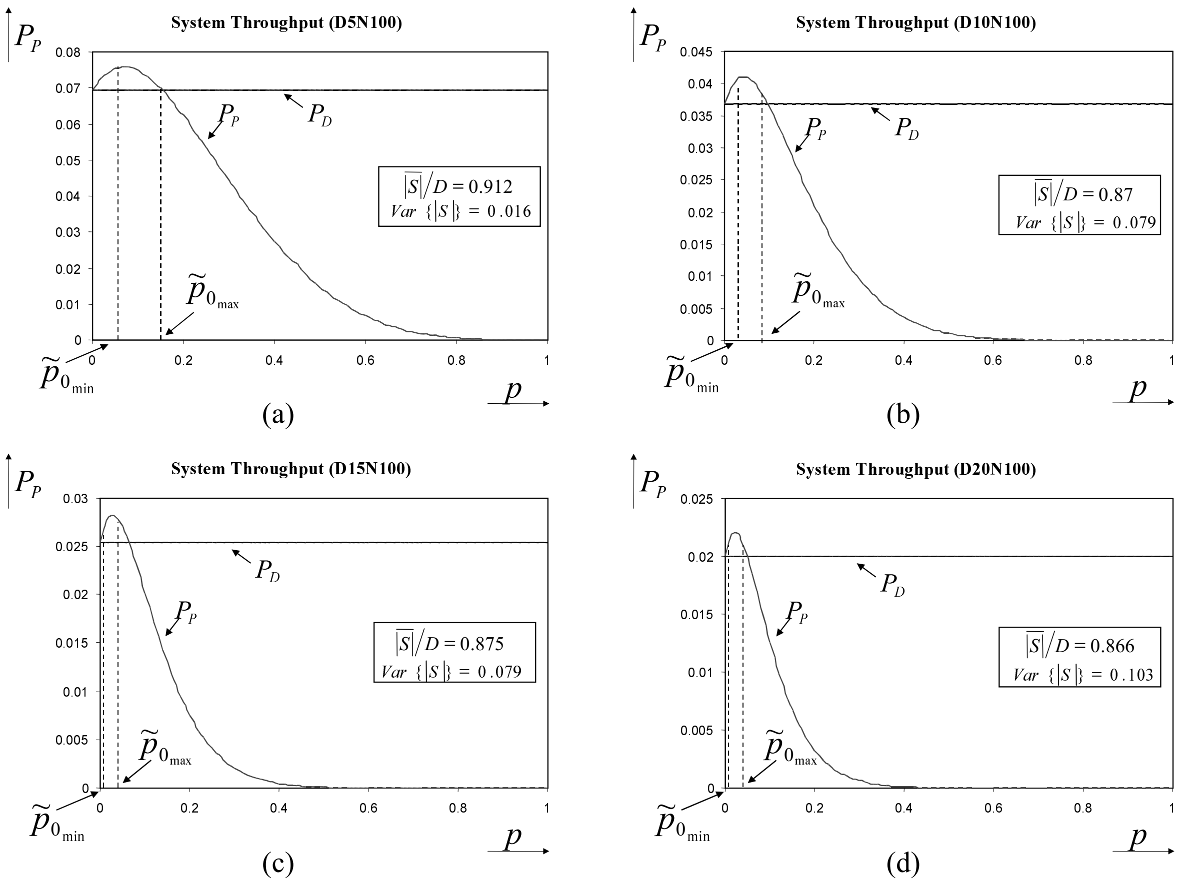

Fig. 11. System throughput simulation results for different values of p (high topology density values $\overline{|S|}/D$), for both the Deterministic Policy and the Probabilistic Policy.

increases the system throughput under the Probabilistic Policy converges to that under the Deterministic Policy. From Figure 12 it can also be observed that the achievable system throughput under the Probabilistic Policy decreases exponentially as the topology density $\overline{|S|}/D$ increases. This is also concluded from Equation (8). It is a fact that for high topology density values and small networks ($k = 1$) the gain of the Probabilistic Policy is negligible but for any other case (small topology density values and $k = 1$ or any topology density values and $k > 1$) the gain is significantly high.

IX. Conclusion

In this paper the inherent inefficiencies of the Deterministic (slot assignment) Policy in an ad-hoc network, proposed in [9] and [10], are investigated and the Probabilistic (slot assignment) Policy is introduced in an effort to improve the achieved network throughput. The basic idea behind the proposed policy is to use (with some probability p) slots not assigned to a node under the assignment scheme in [9], [10]. The study in this paper has been carried out under heavy traffic conditions, which are expected to minimize the benefits of the Probabilistic Policy that eventually tries to utilize slots non-assigned to anybody or not used by others.

A specific generic transmission $u \to v$ is considered and it is shown that under certain conditions the Probabilistic Policy can

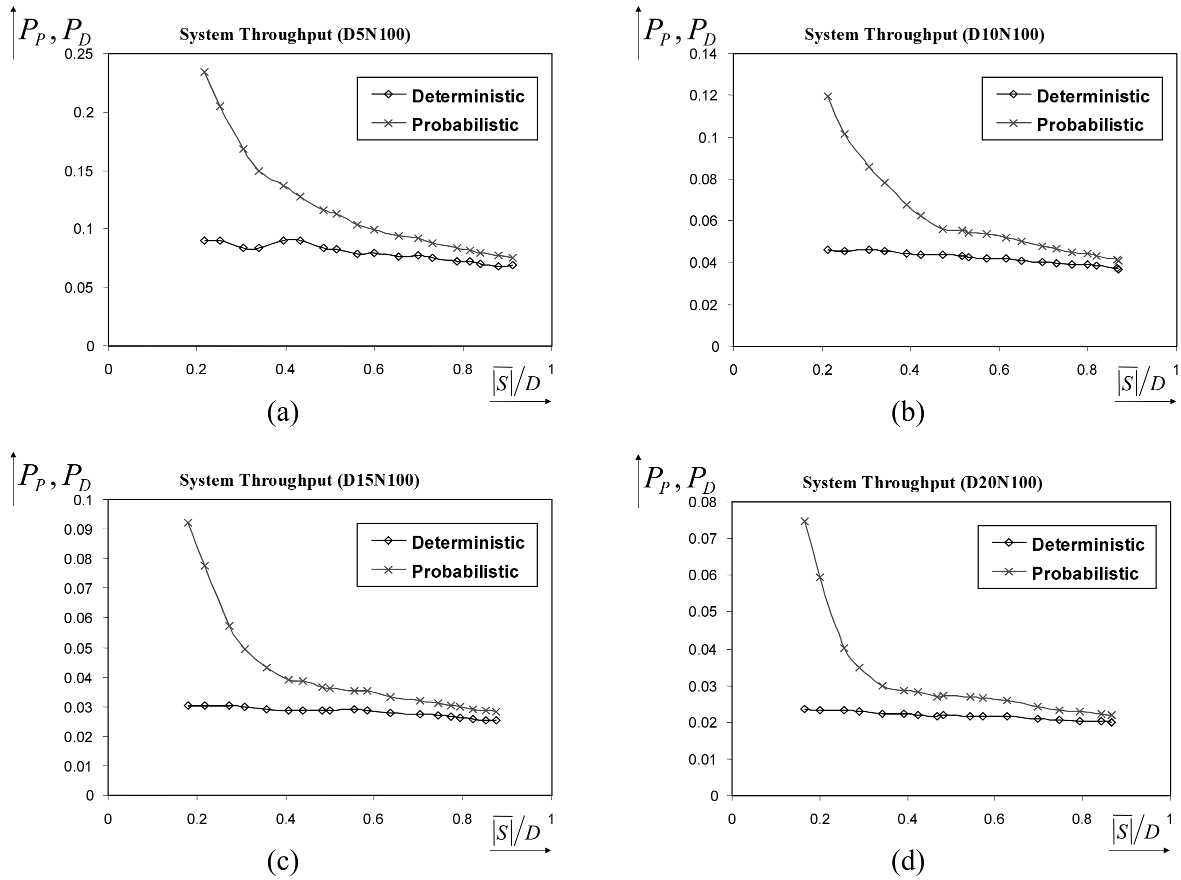

Fig. 12. System throughput simulation results, for both policies, for different values of the topology density $\overline{|S|}/D$.

outperform the Deterministic Policy for access probabilities p in a range $[0, p_{max,u\to v}]$. In the sequel, a common probability p is assumed for all transmissions in the network - as it would practically be the case - and the system throughput is considered. An approximate system throughput analysis is presented that (a) identifies the existence of a suitable range of values for the access probability p for which the Probabilistic Policy outperforms the Deterministic Policy, $[0, p_{max}]$; (b) identifies the value of the access probability that maximizes the system throughput; (c) determines simple bounds on the access probability that maximize the system throughput as a function of the topology density. The accuracy of the approximations is also analytically investigated.

Simulation results have been derived for four network topology categories $\big($for four pairs $(N, D)\big)$ and for three values of the topology density ($\overline{|S|}/D$) for each topology category; $k = 1$ and heavy traffic conditions have been assumed, both of which are expected to induce only a small advantage of the Probabilistic Policy over the Deterministic Policy, compared to the cases of networks with $k > 1$ and non-heavy traffic conditions. The derived results have supported the claims and expectations regarding the comparative advantage of the Probabilistic Policy over the Deterministic Policy and that the approximate analysis determines the range of values that, under certain conditions, maximize the system throughput under the Probabilistic Policy, or induce a system throughput close to the maximum.

APPENDIX I

PROOF OF THEOREM 2

From Equation (6) it is concluded that $P_{P,u\to v} = P_{D,u\to v}$, for $p = 0$.

The first derivative of $P_{P,u\to v}$ with respect to p is calculated in Appendix IV. For $|R_{u\to v}| \leq (q - |C_{u\to v}|)|S_v|$, the first derivative is zero for two values of p: $p = 0$ and $p = 1$. For any other value $p \in (0, 1)$ the first derivative is always negative (see Appendix IV) and therefore, the global maximum corresponds to $p = 0$ ($P_{P,u\to v} = P_{D,u\to v}$) whereas, the global minimum corresponds to $p = 1$ ($P_{P,u\to v} = 0$). Consequently, for any value of $p \in (0, 1]$, $P_{P,u\to v} < P_{D,u\to v}$.

APPENDIX II

PROOF OF THEOREM 3

From Equation (6) it is concluded that $P_{P,u\to v} = P_{D,u\to v}$ for $p = 0$ and $P_{P,u\to v} = 0$ for $p = 1$. Consequently, the range of values for which $P_{P,u\to v} > P_{D,u\to v}$ includes neither 0 nor 1.

For $0 < p < 1$ and $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$ the first derivative of $P_{P,u\to v}$, with respect to p, is zero (see Appendix IV), when $p = \frac{|R_{u\to v}| - (q - |C_{u\to v}|)|S_v|}{|R_{u\to v}|(|S_v|+1)}$ ($\equiv p_{0,u\to v}$). For $p = p_{0,u\to v}$ (the first derivative is zero) the second derivative is negative (see Appendix IV). Consequently, $p_{0,u\to v}$ corresponds to a maximum value for $P_{P,u\to v}$. For every value p, $0 < p \leq p_{0,u\to v}$, the first derivative is always positive and consequently, $P_{P,u\to v} > P_{D,u\to v}$.

On the other hand, for every value $p_{0,u\to v} < p < 1$ the first derivative is always negative. For $p \to 1$, $P_{P,u\to v} \to 0$ and given that $P_{P,u\to v}$ is a continuous function of p, there exists a value $p_{0,u\to v} < p < 1$, such that $P_{P,u\to v} = P_{D,u\to v}$. Let $p_{max,u\to v}$ denote that value of p.

Finally, it is evident that for any value $p \in (0, p_{max,u\to v})$, $P_{P,u\to v} > P_{D,u\to v}$, provided that $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$.

APPENDIX III

ON THE EXISTENCE OF $p_{s,u\to v}$

$\frac{d^2 g(p)}{d^2 p} = 0$ for $p = 1$ and $p = \frac{2|R_{u\to v}| - (q - |C_{u\to v}|)(|S_v|-1)}{|R_{u\to v}|(|S_v|+1)} \equiv p_{s,u\to v}$. Since $0 < p \leq 1$ (0 is not included since it is the case for which $P_{P,u\to v} = P_{D,u\to v}$), in order for $p_{s,u\to v}$ to be a valid root it is required that $0 < p_{s,u\to v} \leq 1$. $p_{s,u\to v} > 0$ when $2|R_{u\to v}| > (q - |C_{u\to v}|)(|S_v| - 1)$. Note that $p_{s,u\to v} \leq 1$, when $p_{s,u\to v} > 0$. Notice that $2|R_{u\to v}| > (q - |C_{u\to v}|)(|S_v| - 1)$ is always met if $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$ and, in this case, $p_{s,u\to v} > p_{0,u\to v}$. To show the latter it suffices to show that $\frac{2|R_{u\to v}| - (q - |C_{u\to v}|)(|S_v|-1)}{|R_{u\to v}|(|S_v|+1)} > \frac{|R_{u\to v}| - (q - |C_{u\to v}|)|S_v|}{|R_{u\to v}|(|S_v|+1)}$ or $2|R_{u\to v}| - (q - |C_{u\to v}|)(|S_v| - 1) > |R_{u\to v}| - (q - |C_{u\to v}|)|S_v|$ or $2|R_{u\to v}| - (q - |C_{u\to v}|)|S_v| + q - |C_{u\to v}| > |R_{u\to v}| - (q - |C_{u\to v}|)|S_v|$ or $2|R_{u\to v}| + q - |C_{u\to v}| > |R_{u\to v}|$ or $|R_{u\to v}| + q - |C_{u\to v}| > 0$. The latter always holds.

APPENDIX IV

COMPLEMENT OF PROOFS OF THEOREMS 2 AND 3

For convenience, the following function $g(p)$ is considered $g(p) = (q - |C_{u\to v}| + |R_{u\to v}|p)(1 - p)^{|S_v|} = P_{P,u\to v} q^2$. The first and second derivative of $g(p)$ with respect to p are given by: $\frac{dg(p)}{dp} = \Big(|R_{u\to v}| - (q - |C_{u\to v}|)|S_v| - |R_{u\to v}|(|S_v| + 1)p\Big)(1 - p)^{|S_v|-1}$ and $\frac{d^2 g(p)}{d^2 p} = -|S_v|\Big(2|R_{u\to v}| - (q - |C_{u\to v}|)(|S_v| - 1) - |R_{u\to v}|(|S_v| + 1)p\Big)(1 - p)^{|S_v|-2}$.

$\frac{dg(p)}{dp} = 0$ for $p = 1$ and $p = \frac{|R_{u\to v}| - (q - |C_{u\to v}|)|S_v|}{|R_{u\to v}|(|S_v|+1)} \equiv p_{0,u\to v}$. Since $0 < p \leq 1$, in order for $p_{0,u\to v}$ to be a valid root it is required that $0 < p_{0,u\to v} \leq 1$. $p_{0,u\to v} > 0$ when $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$. Note that $p_{0,u\to v} \leq 1$, when $p_{0,u\to v} > 0$.

If $|R_{u\to v}| < (q - |C_{u\to v}|)|S_v|$ then $\frac{dg(p)}{dp} < 0$ for any value of p. Therefore the maximum value for $g(p)$ is assumed for $p = 0$ and it is $g(0) = q - |C_{u\to v}|$.

If $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$, $\frac{d^2g(p)}{d^2p} = -|S_v|(q - |C_{u\to v}| + |R_{u\to v}|)(1-p)^{|S_v|-2}$, for $p = p_{0,u\to v}$. Since $(q - |C_{u\to v}|) + |R_{u\to v}| > 0$ and $(1-p)^{|S_v|-2} > 0$ for $p = p_{0,u\to v}$, it is concluded that $\frac{d^2g(p)}{d^2p} < 0$. As a result $g(p)$ assumes a maximum value at $p_{0,u\to v}$.

Appendix V

Proof of the inequality: $|R_{u\to v}| \geq q^2 - (|S_v|+1)(q - \phi_{u\to v})$

$\left|\bigcup_{\chi\in S_v+\{v\}} \Omega_\chi\right|$ can be written as $\left|\bigcup_{j=1}^{|S_v|+1} \Omega_j\right|$ by assigning numbers, $j = 1, ..., |S_v|+1$, to each node $\chi \in S_v \cup \{v\}$. Without loss of generality, it is assumed that node u corresponds to number $|S_v|+1$ or $\Omega_u \equiv \Omega_{|S_v|+1}$.

$$\left|\bigcup_{j=1}^{|S_v|+1} \Omega_j\right| = |\Omega_1| + \left|\bigcup_{j=2}^{|S_v|+1} \Omega_j\right| - \left|\Omega_1 \cap \left(\bigcup_{j=2}^{|S_v|+1} \Omega_j\right)\right|$$

$$\vdots \qquad\qquad \vdots$$

$$\left|\bigcup_{j=|S_v|}^{|S_v|+1} \Omega_j\right| = |\Omega_{|S_v|}| + \left|\bigcup_{j=|S_v|+1}^{|S_v|+1} \Omega_j\right| - \left|\Omega_{|S_v|} \cap \left(\bigcup_{j=|S_v|+1}^{|S_v|+1} \Omega_j\right)\right|$$

$$\left|\bigcup_{j=|S_v|+1}^{|S_v|+1} \Omega_j\right| = |\Omega_{|S_v|+1}|.$$

Since $|\Omega_j| = q$, by adding all lines: $\left|\bigcup_{j=1}^{|S_v|+1} \Omega_j\right| = (|S_v|+1)q - \sum_{j=1}^{|S_v|}\left|\Omega_j \cap \left(\bigcup_{l=j+1}^{|S_v|+1} \Omega_l\right)\right|$.

Let $\theta_{u\to v} = \frac{\sum_{j=1}^{|S_v|}\left|\Omega_j \cap \left(\bigcup_{l=j+1}^{|S_v|+1} \Omega_l\right)\right|}{|S_v|+1}$. The latter expression can be written as $\left|\bigcup_{j=1}^{|S_v|+1} \Omega_j\right| = (|S_v|+1)q - (|S_v|+1)\theta_{u\to v} = (|S_v|+1)(q - \theta_{u\to v})$, and therefore, given Equation (4), $|R_{u\to v}| = q^2 - (|S_v|+1)(q - \theta_{u\to v})$.

$\phi_{u\to v}$ can be written as follows: $\phi_{u\to v} = \frac{\sum_{j=1}^{|S_v|}|\Omega_j \cap \Omega_{|S_v|+1}|}{|S_v|+1}$ or $\phi_{u\to v} = \frac{\sum_{j=1}^{|S_v|}|\Omega_j \cap \Omega_u|}{|S_v|+1}$. Given that $\Omega_u \equiv \Omega_{|S_v|+1}$, it is concluded that $\theta_{u\to v} \geq \phi_{u\to v}$ and consequently, Equation (11) is proved.

Appendix VI

Proof of Theorem 6

From Equation (10) it is concluded that $\tilde{P}_P = 0$ for $p = 1$. Consequently, the range of values for which $P_P \geq P_D$ does not include 1. On the other hand, for $p = 0$, $\tilde{P}_P = P_D$ and therefore, 0 is included.

The first derivative of $\tilde{P}_P$, with respect to p, is

$$\begin{aligned}\frac{d\tilde{P}_P}{dp} &= \frac{1}{N}\sum_{\forall u\in V}\frac{|R_{u\to v}| - (q - |C_{u\to v}|)\overline{|S|}}{q^2}(1-p)^{\overline{|S|}-1} \\ &\quad -\frac{1}{N}\sum_{\forall u\in V}\frac{|R_{u\to v}|(\overline{|S|}+1)p}{q^2}(1-p)^{\overline{|S|}-1}.\end{aligned}$$

For $0 < p < 1$ and $\sum_{\forall u\in V}\left(|R_{u\to v}| - (q - |C_{u\to v}|)\overline{|S|}\right) \geq 0$, the first derivative of $\tilde{P}_P$ with respect to p $\left(\frac{d\tilde{P}_P}{dp}\right)$ is zero when $p = \frac{\sum_{\forall u\in V}\left(|R_{u\to v}| - (q - |C_{u\to v}|)\overline{|S|}\right)}{\sum_{\forall u\in V}\left(|R_{u\to v}|(\overline{|S|}+1)\right)}$ $(\equiv \tilde{p}_0)$. It is obvious that $\frac{d\tilde{P}_P}{dp} > 0$ for $p < \tilde{p}_0$ and $\frac{d\tilde{P}_P}{dp} < 0$ for $p > \tilde{p}_0$. Consequently, $P_{P,u\to v}$ assumes a maximum at $\tilde{p}_0$. Additionally, for every value p, $0 < p \leq \tilde{p}_0$, $\tilde{P}_P > P_D$.

For $p \to 1$, $\tilde{P}_P \to 0$ and given that $\tilde{P}_P$ is a continuous function of p, there exists a value $\tilde{p}_{max}$ ($\tilde{p}_0 < \tilde{p}_{max} < 1$) such that $\tilde{P}_P = P_D$. Finally, it is evident that for any value $p \in [0, \tilde{p}_{max}]$, $\tilde{P}_P \geq P_D$, provided that $\sum_{\forall u\in V}\left(|R_{u\to v}| - (q - |C_{u\to v}|)\overline{|S|}\right) \geq 0$.

APPENDIX VII

PROOF OF THEOREM 7

According to Theorem 6, there exists an efficient range of values of p if $\sum_{\forall u\in V}\left(|R_{u\to v}|-(q-|C_{u\to v}|)\overline{|S|}\right)\geq 0$ holds. From Equation (11), $|R_{u\to v}|\geq q^2-(|S_v|+1)(q-\phi_{u\to v})$. Therefore, $\sum_{\forall u\in V}\left(|R_{u\to v}|-(q-|C_{u\to v}|)\overline{|S|}\right)\geq 0$ always holds if $\sum_{\forall u\in V}\left(q^2-(|S_v|+1)(q-\phi_{u\to v})-(q-|C_{u\to v}|)\overline{|S|}\right)\geq 0$. Given that $|C_{u\to v}|\geq\phi_{u\to v}$ (see Appendix IX), it is enough to show that $\sum_{\forall u\in V}\left(q^2-(|S_v|+1)(q-\phi_{u\to v})-(q-\phi_{u\to v})\overline{|S|}\right)\geq 0$ or $\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+|S_v|+1)(q-\phi_{u\to v})\right)\geq 0$ or $\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+|S_v|+1)q+(\overline{|S|}+|S_v|+1)\phi_{u\to v})\right)\geq 0$ or $Nq^2-N(\overline{|S|}+\sum_{\forall u\in V}|S_v|+1)q+(\overline{|S|}+\sum_{\forall u\in V}|S_v|+1)\sum_{\forall u\in V}\phi_{u\to v}\geq 0$ or $q^2-(2\overline{|S|}+1)q+(2\overline{|S|}+1)\overline{\phi}\geq 0$.

Let Δ be equal to $(2\overline{|S|}+1)^2-4(2\overline{|S|}+1)\overline{\phi}=(2\overline{|S|}+1)(2\overline{|S|}+1-4\overline{\phi})$. For $\Delta\leq 0$, $q^2-(2\overline{|S|}+1)q+(2\overline{|S|}+1)\overline{\phi}\geq 0$ should hold. Since $2\overline{|S|}+1>0$, in order for $\Delta\leq 0$ to be satisfied, $2\overline{|S|}+1-4\overline{\phi}\leq 0$ should hold as well, or $\overline{\phi}\geq\frac{2\overline{|S|}+1}{4}$.

APPENDIX VIII

PROOF OF THEOREM 8

Proof: According to Theorem 6, there exists an efficient range of values of p, if condition $\sum_{\forall u\in V}\left(|R_{u\to v}|-(q-|C_{u\to v}|)\overline{|S|}\right)\geq 0$ holds. Then, $\tilde{p}_0=\frac{\sum_{\forall u\in V}\left(|R_{u\to v}|-(q-|C_{u\to v}|)\overline{|S|}\right)}{\sum_{\forall u\in V}\left(|R_{u\to v}|(\overline{|S|}+1)\right)}=\frac{1}{(\overline{|S|}+1)}\left(1-\frac{\sum_{\forall u\in V}(q-|C_{u\to v}|)\overline{|S|}}{\sum_{\forall u\in V}|R_{u\to v}|}\right)\leq\frac{1}{\overline{|S|}+1}$. Therefore, $\tilde{p}_{0_{max}}=\frac{1}{\overline{|S|}+1}$.

From Equation (11), $|R_{u\to v}|\geq q^2-(\overline{|S|}+1)(q-\phi_{u\to v})$ is satisfied and given that $\tilde{p}_0=\frac{1}{(\overline{|S|}+1)}\left(1-\frac{\sum_{\forall u\in V}(q-|C_{u\to v}|)\overline{|S|}}{\sum_{\forall u\in V}|R_{u\to v}|}\right)$, it is concluded that $\tilde{p}_0\geq\frac{1}{(\overline{|S|}+1)}\left(1-\frac{\sum_{\forall u\in V}(q-|C_{u\to v}|)\overline{|S|}}{\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+1)(q-\phi_{u\to v})\right)}\right)=\frac{\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+1)(q-\phi_{u\to v})-(q-|C_{u\to v}|)\overline{|S|}\right)}{\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+1)(q-\phi_{u\to v})\right)(\overline{|S|}+1)}$. Therefore, $\tilde{p}_0\geq\frac{\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+1)(q-\phi_{u\to v})-(q-|C_{u\to v}|)\overline{|S|}\right)}{\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+1)(q-\phi_{u\to v})\right)(\overline{|S|}+1)}\geq\frac{\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+1)(q-\phi_{u\to v})-(q-\phi_{u\to v})\overline{|S|}\right)}{\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+1)(q-\phi_{u\to v})\right)(\overline{|S|}+1)}$, since $|C_{u\to v}|\geq\phi_{u\to v}$ (Appendix IX). The latter expression can been written as $\frac{\sum_{\forall u\in V}\left(q^2-(2\overline{|S|}+1)(q-\phi_{u\to v})\right)}{\sum_{\forall u\in V}\left(q^2-(\overline{|S|}+1)(q-\phi_{u\to v})\right)(\overline{|S|}+1)}$. Finally, it is concluded that $\tilde{p}_0\geq\frac{q^2-(2\overline{|S|}+1)(q-\frac{\sum_{\forall u\in V}\phi_{u\to v}}{N})}{\left(q^2-(\overline{|S|}+1)(q-\frac{\sum_{\forall u\in V}\phi_{u\to v}}{N})\right)(\overline{|S|}+1)}=\frac{q^2-(2\overline{|S|}+1)(q-\overline{\phi})}{\left(q^2-(\overline{|S|}+1)(q-\overline{\phi})\right)(\overline{|S|}+1)}=\tilde{p}'_{min}(\overline{\phi})$. From Appendix X it is concluded that $\frac{d\tilde{p}'_{min}(\overline{\phi})}{d\overline{\phi}}$, is always positive and therefore, $\tilde{p}'_{min}(\overline{\phi})$ increases as $\overline{\phi}$ increases. Consequently, the minimum value for $\tilde{p}_0$, $\tilde{p}_{0_{min}}$, corresponds to the minimum value of $\overline{\phi}$ for which there exists an efficient range of values for p. This value according to Theorem 7, corresponds to $\overline{\phi}=\frac{2\overline{|S|}+1}{4}$ and as a result, $\tilde{p}_{0_{min}}=\frac{q^2-(2\overline{|S|}+1)\left(q-\frac{2\overline{|S|}+1}{4}\right)}{\left(q^2-(\overline{|S|}+1)\left(q-\frac{2\overline{|S|}+1}{4}\right)\right)(\overline{|S|}+1)}$.

APPENDIX IX

PROOF OF INEQUALITY: $|C_{u\to v}|\geq\phi_{u\to v}$

From Equation (3), $|C_{u\to v}|=\left|\Omega_u\cap\left(\bigcup_{\chi\in S_v\cup\{v\}-\{u\}}\Omega_\chi\right)\right|\geq|\Omega_u\cap\Omega_j|$, for all nodes $\chi\in(S_v\cup\{v\}-\{u\})$, denoted by numbers $j=1,...,|S_v|$, while node u is denoted by $|S_v|+1$. Consequently, $|S_v||C_{u\to v}|\geq\sum_{j=1}^{|S_v|}|\Omega_u\cap\Omega_j|$, or $|C_{u\to v}|\geq\frac{\sum_{j=1}^{|S_v|}|\Omega_u\cap\Omega_j|}{|S_v|}>\frac{\sum_{j=1}^{|S_v|}|\Omega_u\cap\Omega_j|}{|S_v|+1}$, or $|C_{u\to v}|>\phi_{u\to v}$. Consequently, $|C_{u\to v}|\geq\phi_{u\to v}$ (the equality holds when $\Omega_u\cap\left(\bigcup_{\chi\in S_v\cup\{v\}-\{u\}}\Omega_\chi\right)=\emptyset$ in which case $|C_{u\to v}|=\phi_{u\to v}=0$).

APPENDIX X
STRICTLY POSITIVE FIRST DERIVATIVE: $\frac{d\tilde{p}'_{min}(\overline{\phi})}{d\overline{\phi}} > 0$

$$\frac{d\tilde{p}'_{min}(\overline{\phi})}{d\overline{\phi}} = \frac{\overline{|S|}q^2}{(\overline{|S|}+1)\Big(q^2-(\overline{|S|}+1)(q-\overline{\phi})\Big)^2} > 0.$$

APPENDIX XI
DIFFERENCE: $|P_P - \tilde{P}_P|$

From Equation (8) and Equation (10), $|P_P-\tilde{P}_P| = \Bigg| \frac{1}{N}\sum_{\forall u\in V} \frac{q-|C_{u\to v}|+p|R_{u\to v}|}{q^2}(1-p)^{|S_v|} - \frac{1}{N}\sum_{\forall u\in V} \frac{q-|C_{u\to v}|+p|R_{u\to v}|}{q^2}(1-p)^{\overline{|S|}} \Bigg| = \Bigg| \frac{1}{N}\sum_{\forall u\in V} \frac{q-|C_{u\to v}|+p|R_{u\to v}|}{q^2}\Big((1-p)^{|S_v|}-(1-p)^{\overline{|S|}}\Big) \Bigg| \leq \frac{1}{N}\sum_{\forall u\in V} \Bigg| \frac{q-|C_{u\to v}|+p|R_{u\to v}|}{q^2}\Big((1-p)^{|S_v|}-(1-p)^{\overline{|S|}}\Big) \Bigg| = \frac{1}{N}\sum_{\forall u\in V} \frac{q-|C_{u\to v}|+p|R_{u\to v}|}{q^2}\Bigg|(1-p)^{|S_v|}-(1-p)^{\overline{|S|}}\Bigg| = \frac{1}{N}\sum_{\forall u\in V} \frac{q-|C_{u\to v}|+p|R_{u\to v}|}{q^2}(1-p)^{\overline{|S|}}\Bigg|(1-p)^{|S_v|-\overline{|S|}}-1\Bigg|$

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