Energy Considerations for Topology-Unaware TDMA MAC Protocols

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Abstract

Since the energy budget of mobile nodes is limited, the performance of a networking protocol for such users should be evaluated in terms of its energy efficiency, in addition to the more traditional metrics such as throughput. In this paper, two *topology-unaware* MAC protocols - in which the scheduling time slots are allocated irrespectively of the underline topology - are considered and their energy consumption is derived. It turns out that the *per frame power consumption* is lower for the less throughput-efficient protocol, suggesting that energy savings are achieved at the expense of throughput.

A finer energy consumption study is carried out in the sequel, focusing on the amount of energy consumed to *successfully* transmit a certain number of packets, or equivalently, on the *per successful transmission power consumption*. It is shown that the more throughput-efficient protocol consumes less energy per successful transmission under certain conditions (which are derived), due to the lower number of transmission attempts before a data packet is successfully transmitted. The same energy-efficiency relation is observed under certain conditions (which are derived) when data packets are *delay constrained* and, thus, may become *obsolete* if not transmitted successfully within a specific time interval. The conditions under which the *per successful transmission power consumption* is minimized for delay-constrained packets, are also established in this work and it is observed that when the system throughput is maximized, the power consumed is close to the minimum. Simulation results support the claims and the expectations of the aforementioned analysis.

Key words: Power Consumption, Ad-Hoc, TDMA, MAC, Topology-Unaware
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1 Introduction

The maximization of the *system throughput* is the main target for many Medium Access Control (MAC) protocols. In ad-hoc networks, where nodes play also the role of relays, the *network lifetime* is rather important and strongly affected by the *power consumed* by the nodes. Given the unknown, random or even changing underlying topology, the design of a MAC protocol capable of *maximizing* the system throughput and *minimizing* the *power consumption* is challenging in ad-hoc networks.

Contention-based MAC protocols are widely employed in ad-hoc networks, such as the CSMA/CA-based IEEE 802.11, [1]. In addition to the carrier sensing mechanism, MACA, [2], employs the *Ready-To-Send/Clear-To-Send* (RTS/CTS) handshake mechanism. This mechanism is mainly introduced to avoid the *hidden/exposed terminal* problem, which causes significant performance degradation in ad-hoc networks. Other protocols based on variations of the RTS/CTS mechanism have been proposed as well, [3], [4], [5]. TDMA-based MAC protocols have also been employed, [6], [7], [8], [9], [10], [11], where each node is allowed to transmit during a specific set of TDMA *scheduling time slots*. In general, optimal solutions to the problem of time slot assignment often result in NP-hard problems, [12], [13], which are similar to the n-coloring problem in graph theory.

Topology-unaware scheduling schemes assign the scheduling time slots to nodes irrespectively of the underlying topology. In [14], [15], a TDMA-based topology-unaware scheme was proposed guaranteeing that at least one time slot in a frame would be collision-free. In [16], the policy in [14], [15], - which does not allow nodes to access slots other than those assigned to them (referred to as the *Deterministic Policy*) - is extended by allowing access to non-assigned slots with some nonzero probability p (referred to as the *Probabilistic Policy*). It was shown that the Probabilistic Policy achieves a better performance under certain conditions when the benefit of utilizing otherwise idle slots outweighs the loss due to collisions induced by the introduced control interference. The issue of the maximization of the system throughput was also addressed and simplified bounds on the value of the access probability that maximizes the system throughput were derived. Further studies, [17], [18], have demonstrated the advantage of the Probabilistic Policy over the Deterministic Policy regarding the utilization of *unused* time slots either under *topology control* conditions or under *non-heavy traffic* conditions.

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As throughput performance may not be the only or main concern in energy-constrained ad-hoc networks, several mechanisms, algorithms and MAC protocols have been proposed that aim to reduce the power consumed for a particular transmission, [19], [20], [21], [22], [23], [24], [25], [26], [27]. A framework for energy-efficient communications with quality of service provisioning has been recently proposed as well, [28]. In this paper, the performance of the aforementioned policies is considered with respect to the consumed energy. Since, under the Probabilistic Policy, the nodes are expected to be attempting transmissions (and consume power) over non-assigned slots as well, it is conceivable that this policy is more energy demanding, despite the potentially higher throughput achieved. It is shown that on a *per frame* basis the Probabilistic Policy consumes more power than the Deterministic Policy. On the other hand, the *power consumed per successful transmission* may be smaller under the Probabilistic Policy under certain conditions that are studied here. This study also establishes the conditions under which the *power consumed per successful transmission of delay constrained packets* under the Probabilistic Policy is smaller than that under the Deterministic Policy. Finally, simulation results support the claims and the expectations of the aforementioned analysis. In accordance with the analytical results, it is also revealed the fact that the boundaries of the value of p, for which the system throughput is maximized, derived in [16], determine a range of values of p for which the power consumed under the Probabilistic Policy is smaller than the value consumed under the Deterministic Policy. In addition, it is shown that the aforementioned lower bound of the access probability p, is close to that value of p that minimizes the power consumption.

In Section 2, a description of the system and the policies is presented. In Section 3, it is shown that on a per frame basis the power consumed under the Probabilistic Policy is higher than the power consumed under the Deterministic Policy and therefore, the power consumed per successful transmission is analyzed. The case where data transmissions are subject to delay constraints is considered in Section 4. Section 5 contains simulation results for a variety of network topologies. The conclusions are drawn in Section 6.

2 System and Network Definition

An ad-hoc network may be viewed as a time varying multihop network and may be described in terms of a graph $G(V, E)$, where V denotes the set of nodes and E the set of (bidirectional) links between the nodes at a given time instance. Let $|X|$ denote the number of elements in set X and let $N = |V|$ denote the number of nodes in the network. Let S_u denote the set of neighbors of node u, $u \in V$. Let D denote the maximum number of neighbors for a node; clearly $|S_u| \leq D$, $\forall u \in V$. These are the nodes v to which a direct transmission

($transmission\ u \rightarrow v$) is possible.

In this work, omni-directional antennas are considered for transmission and reception purposes over the wireless medium. Time is divided into time slots with fixed duration (as it is the case in TDMA-based environments). Collisions with other transmissions are considered to be the only reason for a transmission not to be successful (*corrupted*).

Suppose that node u wants to transmit to a particular neighbor node v in a particular time slot i. Transmission $u \rightarrow v$ is corrupted in time slot i if at least one transmission $\chi \rightarrow \psi$, $\chi \in S_v \cup \{v\} - \{u\}$ and $\psi \in S_\chi$, takes place in time slot i, [16].

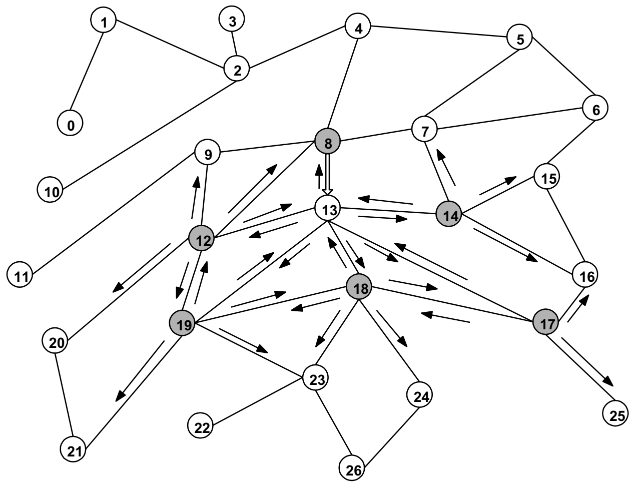


Fig. 1. Example network of 27 nodes.

Suppose that transmission $8 \rightarrow 13$, shown in Figure 1, takes place in time slot i. If a node χ, $\chi \in S_v \cup \{13\} - \{8\}$, transmits, then transmission $8 \rightarrow 13$ becomes corrupted. A possible mechanism to notify the transmitting node (node 8) of the successful reception of transmission $8 \rightarrow 13$ is by allowing part of the end of each time slot to be used for a positive acknowledgement (ACK) by the receiver (node 13) upon a successful reception, [11], [16]. It is assumed that the transmitting node is instantaneously aware of a successful transmission, [30].

Transmission power is responsible for most of the energy typically consumed by communicating nodes. In this work it is assumed that all nodes transmit with the same power and therefore, the power consumed for each individual transmission is the same for all transmissions.

Under the Deterministic Policy and the Probabilistic Policy, [14], [15], [16], each node $u \in V$ is randomly assigned a unique polynomial f_u of degree k with coefficients from a finite Galois field of order q ($GF(q)$). Polynomial f_u is represented as $f_u(x) = \sum_{i=0}^{k} a_i x^i$, [15], where $a_i \in \{0, 1, 2, ..., q-1\}$; parameters q and k are calculated based on N and D, according to the algorithm presented either in [14] or [15]. For both algorithms it is satisfied that $k \geq 1$ and $q > kD$ or $q \geq kD + 1$ (k and D are integers), to allow at least one transmission in one frame to be successful, and $q^{k+1} \geq N$ to satisfy that there exist enough unique polynomials for all nodes in the network, [14].

The access scheme considered is a TDMA scheme with a frame consisted of q^2 time slots. The frame is divided into q subframes s of size q and the time slot assigned to node u in subframe s, $(s = 0, 1, ..., q-1)$ is given by $f_u(s) \text{mod } q$, [15]. Consequently, one time slot is assigned for each node in each subframe. Let Ω_u be the the set of time slots assigned to node u. Given that the number of subframes is q and a node is allowed to transmit only during one time slot in a subframe, $|\Omega_u| = q$.

2.1 The Deterministic Policy

Under the Deterministic Policy each node u transmits in a slot i only if $i \in \Omega_u$, provided that it has data to transmit. The relation between N, D, q and k is important in order to explain the main property of the Deterministic Policy: there exists at least one time slot in a frame over which a specific transmission will remain uncorrupted, [14]. Suppose that two neighbor nodes u and v have been assigned two (unique) polynomials f_u and f_v of degree k, respectively. Given that the roots of each node's polynomial correspond to the assigned time slots to each node, k common time slots is possible to be assigned among two neighbor nodes. Given that D is the maximum number of neighbor nodes of any node, kD is the maximum number of time slots over which a transmission of any node is possible to become corrupted. Since the number of time slots that a node is allowed to transmit in a frame is q, if $q > kD$ or $q \geq kD + 1$ (k and D are integers) is satisfied, there will be at least one time slot in a frame in which a specific transmission will remain uncorrupted for any node in the network, [14].

The assignment of the unique polynomials, or equivalently the assignment of the time slot sets Ω_χ to any node χ, is random in the sense that neither node χ nor its neighbor nodes are taken into account in order to assign the polynomial. The polynomial assignment is similar to the MAC identification number (MAC ID) assignment: either it is already in the device or it is distributed by the time a node becomes part of the network. It is important to guarantee that the number of unique polynomials is enough for all nodes in the network, or $q^{k+1} \geq N$. If, for a given N and q, $q^{k+1} \geq N$ is not satisfied, then k has to increase until the number of unique polynomials is sufficient, resulting also to a new value for q that satisfies $q > kD$.

Depending on the particular random assignment of the polynomials, it is possible that two nodes be assigned overlapping time slots (i.e., $\Omega_u \cap \Omega_v \neq \emptyset$). Let $C_{u\to v}$ be the set of overlapping time slots between those assigned to node u and those assigned to any node $\chi \in S_v \cup \{v\} - \{u\}$. Therefore, $C_{u\to v} = \Omega_u \cap \left(\bigcup_{\chi\in S_v\cup\{v\}-\{u\}} \Omega_\chi\right)$. Given that $|S_v|$ is the number of nodes that influence transmission $u \to v$, it is concluded that $|C_{u\to v}| \leq k|S_v|$.

Heavy traffic conditions are assumed to characterize the traffic load in the network; each node always contains data for transmission, [14], [15], [16]. Let $P_{D,u\to v}$ denote the *probability of success* of a transmission $u \to v$ in a slot averaged over a frame under the Deterministic Policy. Let P_D denote the average value of $P_{D,u\to v}$ over all nodes (*system throughput*). Then, it is easily shown, [16], that

$$P_{D,u\to v} = \frac{q - |C_{u\to v}|}{q^2}, \tag{1}$$

$$P_D = \frac{1}{N} \sum_{u\in V} \frac{q - |C_{u\to v}|}{q^2}. \tag{2}$$

2.2 The Probabilistic Policy

Let $R_{u\to v}$ denote the set of time slots i, $i \notin \Omega_u$, over which transmission $u \to v$ would be successful. Equivalently, $R_{u\to v}$ contains those slots not included in set $\bigcup_{\chi\in S_v\cup\{v\}} \Omega_\chi$. Consequently, $|R_{u\to v}| = q^2 - \left|\bigcup_{\chi\in S_v\cup\{v\}} \Omega_\chi\right|$. Figure 2 depicts an example frame of size $q^2 = 121$ and both sets Ω_u and $R_{u\to v}$ for node u and transmission $u \to v$ are depicted respectively.

In general, the use of slots i, $i \in R_{u\to v}$, may increase the average number of successful transmissions Under the Probabilistic Policy, [16], each node u always transmits in slot i if $i \in \Omega_u$ and transmits with probability p in slot i if $i \notin \Omega_u$, provided it has data to transmit. The access probability p is a simple parameter common for all nodes. Under the Probabilistic Policy, all slots $i \notin \Omega_u$ are potentially utilized by node u: both, those that would be collision-free (which are in $R_{u\to v}$, for a given transmission $u \to v$, as well as those not in $\Omega_u \cup R_{u\to v}$ that may be left by neighboring nodes under non-heavy traffic conditions) and those that would interfere with otherwise collision-free transmissions (which are the remaining). The Probabilistic Policy does not require specific topology information (e.g., knowledge of $R_{u\to v}$, etc.) and, thus, induces no additional control overhead.

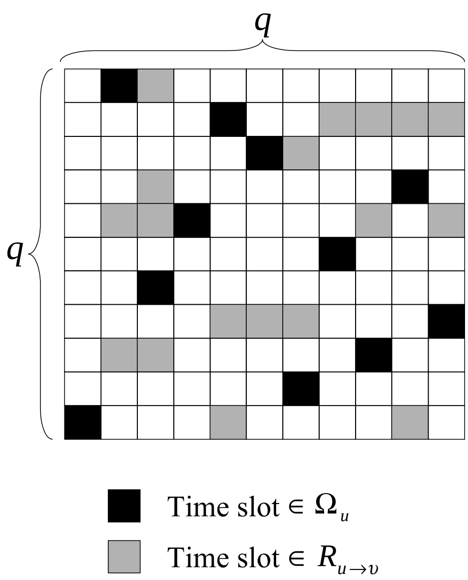


Fig. 2. Example sets Ω_u and $R_{u\to v}$ for node u and transmission $u \to v$ respectively (frame size: $q^2 = 121$).

Under the Probabilistic Policy, it was shown that the probability of success of transmission $u \to v$ in a frame $P_{P,u\to v}$, is given by,

$$P_{P,u\to v} = \frac{q - |C_{u\to v}| + p|R_{u\to v}|}{q^2}(1-p)^{|S_v|}. \tag{3}$$

A comparative analysis has shown that $P_{P,u\to v} > P_{D,u\to v}$, if $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$ is satisfied and a maximum value of $P_{P,u\to v}$ is assumed at $p_{0,u\to v} = \frac{|R_{u\to v}|-(q-|C_{u\to v}|)|S_v|}{|R_{u\to v}|(|S_v|+1)}$. This case is depicted in Figure 3. $p_{\max,u\to v}$ corresponds to that value of $p \neq 0$, for which $P_{P,u\to v} = P_{D,u\to v}$.

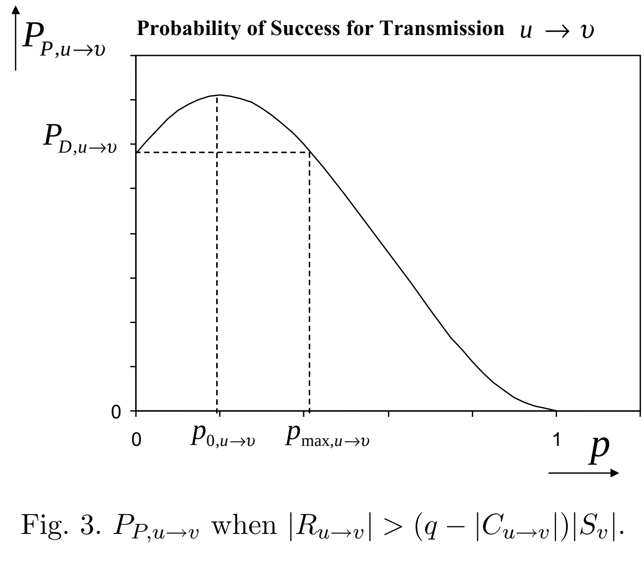


Fig. 3. $P_{P,u\to v}$ when $|R_{u\to v}| > (q - |C_{u\to v}|)|S_v|$.

The probability of success averaged over all transmissions in the network for the Probabilistic Policy, is denoted by P_P (*system throughput*) and it is given by the following equation.

$$P_P = \frac{1}{N} \sum_{\forall u \in V} \frac{q - |C_{u \to v}| + p|R_{u \to v}|}{q^2} (1-p)^{|S_v|}. \tag{4}$$

An approximate analysis, [16], determined lower and upper bounds on the value of p, for which P_P is maximized. In particular, it was shown that this particular value of p belongs in $[\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}}]$, where $\tilde{p}_{0_{min}} = \frac{q^2-(2\overline{|S|}+1)\left(q-\frac{2\overline{|S|}+1}{4}\right)}{\left(q^2-(\overline{|S|}+1)\left(q-\frac{2\overline{|S|}+1}{4}\right)\right)(\overline{|S|}+1)}$, $\tilde{p}_{0_{max}} = \frac{1}{\overline{|S|}+1}$ $\overline{|S|} = \frac{1}{N}\sum_{\forall u \in V} |S_u|$ corresponds to the *average number of neighbor nodes* in the network and $\overline{|S|}/D$ is referred to as the *topology density* of the network.

3 Power Consumption per Successful Transmission

Let β (assumed to be constant) denote the power consumed by a node transmitting over a time slot. For a specific transmission $u \to v$, the number of transmissions in a frame under the Deterministic Policy and heavy traffic conditions is q (corrupted and uncorrupted) and consequently, $q\beta$ is the power consumed in a frame for transmissions from node u to node v. Under the Probabilistic Policy, each node transmits during $q + p(q-1)q = (1 + p(q-1))q$ time slots in one frame and consequently, the power consumed in a frame is equal to $(1 + p(q-1))q\beta$. Obviously, $(1 + p(q-1))q\beta \geq q\beta$ (the equality holds for $p = 0$ where the Probabilistic Policy is reduced to the Deterministic Policy).

The above discussion shows that under heavy traffic conditions the Probabilistic Policy consumes more power than the Deterministic Policy in each frame and for each communicating pair of nodes. This is mainly due to the fact that the Probabilistic Policy attempts to transmit during more time slots (by $p(q-1)q$) than the Deterministic Policy in one frame. On the other hand, the throughput, under certain conditions, is higher under the Probabilistic Policy than under the Deterministic Policy. Consequently, it may be possible that a certain amount of data be transmitted in *fewer* frames under the Probabilistic Policy than under the Deterministic Policy, resulting, under certain conditions, in smaller power consumption. The analysis of the *power consumption per successful transmission* and the establishment of the aforementioned conditions is considered in the remaining of this section.

The power consumed during a frame under the Deterministic Policy is $q\beta$ for any transmission $u \to v$. The (average) number of successful transmissions in one frame is $q^2 P_{D,u\to v}$ and therefore, the (average) *power consumption during a frame per successful transmission* is $\frac{q\beta}{q^2 P_{D,u\to v}}$ (denoted by $B_{D,u\to v}$). Under the Probabilistic Policy, the (average) *power consumption during a frame per successful transmission* is equal to $\frac{1+p(q-1)}{q^2 P_{P,u\to v}} q\beta$ (denoted by $B_{P,u\to v}$). Thus,

$$B_{D,u\to v} = \frac{1}{q P_{D,u\to v}} \beta, \tag{5}$$

$$B_{P,u\to v} = \frac{1+p(q-1)}{q P_{P,u\to v}} \beta. \tag{6}$$

For $p = 0$, $B_{P,u\to v} = B_{D,u\to v}$, while for $p \to 1$, $B_{P,u\to v} \to +\infty$ ($P_{P,u\to v} \to 0$). The first objective of this analysis is to establish the conditions under which there exists a range of values for $p \in (0, 1)$ such that $B_{P,u\to v} \leq B_{D,u\to v}$, given that there exists a range of values for p such that $P_{P,u\to v} \geq P_{D,u\to v}$.

Theorem 1 *If* $|R_{u\to v}| \leq (q - |C_{u\to v}|)(q - 1 + |S_v|)$*,* $B_{P,u\to v} \geq B_{D,u\to v}$*, for any* $0 \leq p \leq 1$*.* □

The proof of Theorem 1 can be found in Appendix A.

According to Theorem 1, if $|R_{u\to v}|$ is relatively small, then the power consumed under the Probabilistic Policy is higher than that consumed under the Deterministic Policy. Consider, for example, the case for which $P_{P,u\to v} \leq P_{D,u\to v}$. According to Section 2, condition $|R_{u\to v}| \leq (q - |C_{u\to v}|)|S_v|$ is satisfied and, given that $q - 1 + |S_v| > |S_v|$, the condition of Theorem 1 is always satisfied.

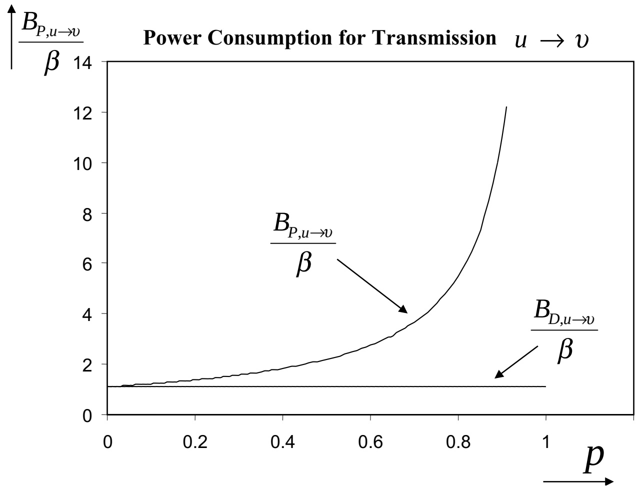


Fig. 4. $B_{D,u\to v}$ and $B_{P,u\to v}$ when the condition of Theorem 1 holds.

Figure 4 corresponds to the case when the condition of Theorem 1 is satisfied. It can be observed that $B_{P,u\to v}$ constantly increases as p increases. Given that

$B_{P,u\to v} = B_{D,u\to v}$ for $p = 0$, $B_{P,u\to v} > B_{D,u\to v}$ for any $0 < p \leq 1$.

Theorem 2 *If* $|R_{u\to v}| > (q - |C_{u\to v}|)(q - 1 + |S_v|)$, *there exists a range of values of* p, $(0, p^B_{\max,u\to v})$, *for some* $0 < p^B_{\max,u\to v} < 1$, *such that* $B_{P,u\to v} < B_{D,u\to v}$. $p^B_{\max,u\to v}$ *corresponds to that value of* $p \neq 0$, *for which* $B_{P,u\to v} = B_{D,u\to v}$. □

The proof of Theorem 2 can be found in Appendix B.

Figure 5 corresponds to the case that the condition of Theorem 2 is satisfied. Obviously, there exists a small range of values of p for which the power consumed per successful transmission under the Probabilistic Policy is smaller than the power consumed under the Deterministic Policy.

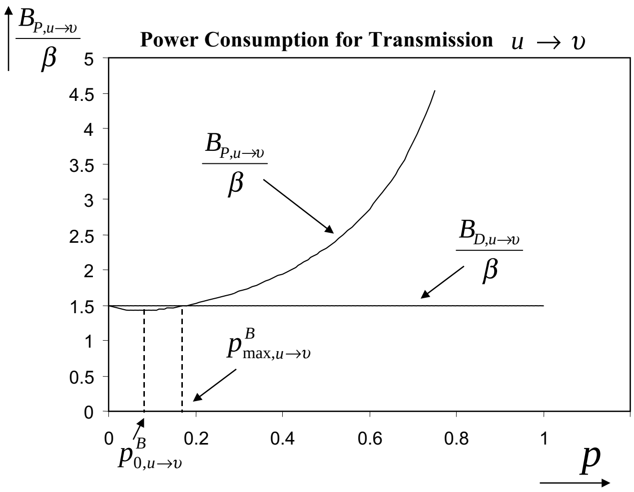


Fig. 5. $B_{D,u\to v}$ and $B_{P,u\to v}$ when the condition of Theorem 2 holds.

The following theorem shows that the value of p, for which the power consumed under the Probabilistic Policy is minimized (denoted by $p^B_{0,u\to v}$ and derived in Appendix B), is smaller than the value of p for which the throughput under the Probabilistic Policy is maximized (denoted by $p_{0,u\to v}$ as may be seen in Section 2).

Theorem 3 *If the condition of Theorem 2 is satisfied, then* $p^B_{0,u\to v} < p_{0,u\to v}$ *is also satisfied.* □

The proof of Theorem 3 can be found in Appendix C.

Theorems 1 and 2 establish the basic conditions for the efficient power consumption under the Probabilistic Policy. It is shown that these conditions are satisfied, by networks with certain characteristics.

Theorem 4 *If* $k = 1$, *then* $|R_{u\to v}| \leq (q - |C_{u\to v}|)(q - 1 + |S_v|)$ *and the condition of Theorem 1 is satisfied.*

Proof. $\left|\bigcup_{\chi\in S_v+\{v\}}\Omega_\chi\right|$ can be written as $\left|\bigcup_{j=1}^{|S_v|+1}\Omega_j\right|$ by assigning numbers, $j=1,...,|S_v|+1$, to each node $\chi\in S_v\cup\{v\}$. Without loss of generality, it is assumed that node u corresponds to $|S_v|+1$ or $\Omega_u\equiv\Omega_{|S_v|+1}$. The following equations are derived.

$$\left|\bigcup_{j=1}^{|S_v|+1}\Omega_j\right| = |\Omega_1| + \left|\bigcup_{j=2}^{|S_v|+1}\Omega_j\right| - \left|\Omega_1\cap\left(\bigcup_{j=2}^{|S_v|+1}\Omega_j\right)\right|$$

$$\vdots \qquad\qquad \vdots$$

$$\left|\bigcup_{j=|S_v|}^{|S_v|+1}\Omega_j\right| = |\Omega_{|S_v|}| + \left|\bigcup_{j=|S_v|+1}^{|S_v|+1}\Omega_j\right| - \left|\Omega_{|S_v|}\cap\left(\bigcup_{j=|S_v|+1}^{|S_v|+1}\Omega_j\right)\right|$$

$$\left|\bigcup_{j=|S_v|+1}^{|S_v|+1}\Omega_j\right| = |\Omega_{|S_v|+1}|.$$

Since $|\Omega_j|=q$, $\left|\bigcup_{j=1}^{|S_v|+1}\Omega_j\right| = (|S_v|+1)q - \sum_{j=1}^{|S_v|}\left|\Omega_j\cap\left(\bigcup_{l=j+1}^{|S_v|+1}\Omega_l\right)\right|$. Let $\theta_{u\to v} = \frac{\sum_{j=1}^{|S_v|}\left|\Omega_j\cap\left(\bigcup_{l=j+1}^{|S_v|+1}\Omega_l\right)\right|}{|S_v|+1}$. Consequently, $\left|\bigcup_{j=1}^{|S_v|+1}\Omega_j\right| = (|S_v|+1)q - (|S_v|+1)\theta_{u\to v} = (|S_v|+1)(q-\theta_{u\to v})$. Given that $|R_{u\to v}| = q^2 - \left|\bigcup_{\chi\in S_v\cup\{v\}}\Omega_\chi\right|$, $|R_{u\to v}| = q^2-(|S_v|+1)(q-\theta_{u\to v})$.

It is enough to prove that $q^2-(|S_v|+1)(q-\theta_{u\to v}) \le (q-|C_{u\to v}|)(q-1+|S_v|)$ or $q^2-q|S_v|-q+\theta_{u\to v}(|S_v|+1) \le q^2-q+q|S_v|-|C_{u\to v}|(q-1+|S_v|)$ or $\theta_{u\to v}(|S_v|+1)\le 2q|S_v|-|C_{u\to v}|(q-1+|S_v|)$ or $\theta_{u\to v}(|S_v|+1)+|C_{u\to v}|(q-1+|S_v|)\le 2q|S_v|$.

It is satisfied that $\Omega_j\cap\left(\bigcup_{l=j+1}^{|S_v|+1}\Omega_l\right)\le kS_v$ (no more than k common roots with any of the S_v nodes in maximum). Consequently, $\frac{\sum_{j=1}^{|S_v|}|kS_v|}{|S_v|+1} < k|S_v|$. Consequently, it is satisfied that $\theta_{u\to v}<k|S_v|$. It is also satisfied that $|C_{u\to v}|\le k|S_v|$. Consequently, $\theta_{u\to v}(|S_v|+1)+|C_{u\to v}|(q-1+|S_v|)\le 2q|S_v|$ holds, if $k|S_v|(|S_v|+1)+k|S_v|(q-1+|S_v|)\le 2q|S_v|$ holds or $k(q+2|S_v|)\le 2q$ or $k\le\frac{2q}{q+2|S_v|}$. Given that $\frac{2q}{q+2|S_v|}<2$ ($q\ge kD+1$) and k is a positive integer, $k\le\frac{2q}{q+2|S_v|}$ is satisfied for $k=1$. □

Theorem 5 *If* $|C_{u\to v}|\ge\frac{2q|S_v|}{q-1+|S_v|}$, *then* $|R_{u\to v}|\ge(q-|C_{u\to v}|)(q-1+|S_v|)$ *is satisfied.* $k\ge 2$ *is a necessary (but not sufficient condition) for* $|C_{u\to v}|\ge\frac{2q|S_v|}{q-1+|S_v|}$.

Proof. $|R_{u\to v}|\ge(q-|C_{u\to v}|)(q-1+|S_v|)$ is satisfied, if $q^2-(|S_v|+1)(q-\theta_{u\to v})\ge(q-|C_{u\to v}|)(q-1+|S_v|)$ holds, or $\theta_{u\to v}(|S_v|+1)+|C_{u\to v}|(q-1+|S_v|)\ge 2q|S_v|$, as it may be seen from the proof of Theorem 4. Given that $\theta_{u\to v}\ge 0$, it is enough $|C_{u\to v}|(q-1+|S_v|)\ge 2q|S_v|$ to be satisfied, or $|C_{u\to v}|\ge\frac{2q|S_v|}{q-1+|S_v|}$.

Given that $|C_{u \to v}| \leq k|S_v|$, $\frac{2q|S_v|}{q-1+|S_v|} \leq k|S_v|$ should be satisfied, or $k \geq \frac{2q}{q-1+|S_v|}$. Given that $\frac{2q}{q-1+|S_v|} > 1$ and that k is a positive integer, $k \geq \frac{2q}{q-1+|S_v|}$ is satisfied if $k \geq 2$. □

From the previous two theorems it is evident that large values of k ($k > 1$) allow for smaller power consumption under the Probabilistic Policy. According to Section 2, $q \geq kD + 1$ and $q^{k+1} \geq N$ need to be satisfied. If N increases, it is possible that the number of unique polynomials be less than the required number, or $q^{k+1} < N$. If this is the case, k must increase. Given that q depends on k and D ($q \geq kD + 1$), it appears that if k increases, q also increases and therefore, q^{k+1} increases even faster than if q was independent of k. Consequently, $k > 1$ corresponds to those networks that contain a large number of nodes (large N) compared to the maximum number of neighbor nodes (small D). For example, 11^3 is the maximum value of N when $k = 2$, $D = 5$ and $q = 11$ ($q \geq kD + 1 = 11$). Networks like this one (11^3 nodes present in the network but no more than 5 neighbor nodes per node) cannot be considered as the typical case, [29], although not impossible. In general, it is expected the (average) number of neighbor nodes to increase as the number of nodes in the network increases, [29].

From the previous discussion it is evident that $B_{P,u \to v} \leq B_{D,u \to v}$, may not be satisfied for certain cases ($k = 1$), depending on the characteristics of the network. However, if the packets are delay constrained and not successfully transmitted by a certain time period, they waste energy and increase the average energy spent per successfully and timely delivered packet. The previous may affect the relative performance of the two policies with respect to the consumed power and it is investigated in the next section.

4 Power Consumption per Successful Transmission for Delay Constrainted Packets

Suppose that node χ wants to sent data to node ψ. If $\chi \notin S_\psi$, an efficient *routing protocol* is assumed to determine an efficient *path* from node χ to node ψ. A particular *data flow* between node χ and node ψ is denoted as $\chi \twoheadrightarrow \psi$ and an example is shown in Figure 6.

Data flows associated with real-time applications, such as voice and video, have delay constraints; if *delay constrained packets* are delayed by more than a certain upper bound, they become *obsolete* and they affect the quality of the application. As they consume resources, while not receiving satisfactory service, it may be better sometimes to not provide service to such applications at all. Let $K_{u \to v}$ denote the number of data flows whose path traverses the link between node u and node v (Figure 7). The scheduling of the transmissions is

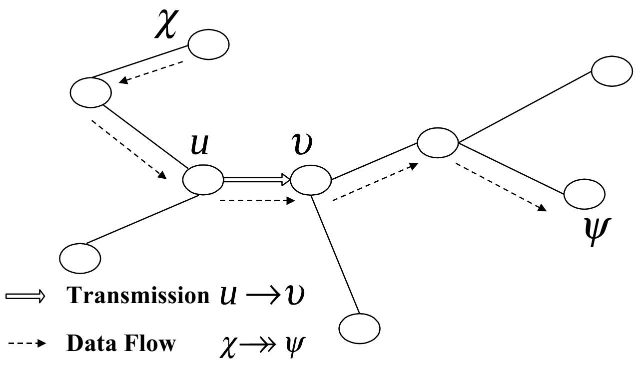


Fig. 6. Example data flow $\chi \twoheadrightarrow \psi$. Transmission $u \rightarrow v$ is required in order for data packets from node χ to arrive at node ψ.

assumed that is taking place in an efficient way (e.g., if the Earliest Deadline First scheduling scheme is employed). However, it is still possible that some packets may become obsolete.

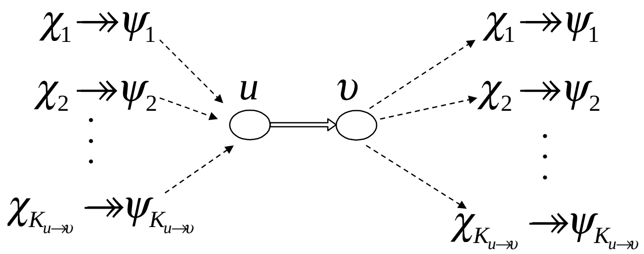


Fig. 7. Data flows $\chi_1 \twoheadrightarrow \psi_1, \ldots, \chi_{K_{u\rightarrow v}} \twoheadrightarrow \psi_{K_{u\rightarrow v}}$, requiring transmission $u \rightarrow v$ to take place.

Suppose that at a specific time instance t and immediately before the beginning of the next frame, packets queued at node u require $h_{u\rightarrow v}q^2$ transmissions $u \rightarrow v$. Suppose that all these packets have to be delivered in less than $d_{u\rightarrow v}$ frames to node v, otherwise they will become obsolete. After a packet has become obsolete either it is forwarded all the way till the destination or it is immediately dropped. In this work, it is assumed that obsolete packets are dropped, thus, saving power by avoiding unnecessary transmissions.

Under the Deterministic Policy, $\frac{h_{u\rightarrow v}}{P_{D,u\rightarrow v}}$ frames are required to achieve $h_{u\rightarrow v}q^2$ successful transmissions. If $\frac{h_{u\rightarrow v}}{P_{D,u\rightarrow v}} \leq d_{u\rightarrow v}$, it is obvious that no packets (out of the $h_{u\rightarrow v}$ packets) will become obsolete at node u. However, if $\frac{h_{u\rightarrow v}}{P_{D,u\rightarrow v}} \geq d_{u\rightarrow v}$, a number of packets will become obsolete. Clearly, $\frac{h_{u\rightarrow v}}{P_{D,u\rightarrow v}} - d_{u\rightarrow v}$ corresponds to the number of frames required to transmit the obsolete packets. Given that in every frame, $P_{D,u\rightarrow v}q^2$ successful transmissions may take place, the number of successful transmissions consumed by obsolete packets under the Deterministic Policy is equal to $\left(\frac{h_{u\rightarrow v}}{P_{D,u\rightarrow v}} - d_{u\rightarrow v}\right) P_{D,u\rightarrow v}q^2$.

Under the Probabilistic Policy, $\frac{h_{u\rightarrow v}}{P_{P,u\rightarrow v}}$ is the (average) number of frames required for the transmission of $h_{u\to v}q^2$ transmissions $u \to v$. Provided that $d_{u\to v} \leq \frac{h_{u\to v}}{P_{P,u\to v}}$, the average number of frames consumed for the transmission of obsolete packets is equal to $\frac{h_{u\to v}}{P_{P,u\to v}} - d_{u\to v}$. For the rest of this analysis - and in order to make point in a simple way - it is assumed that obsolete packets exist under both policies or that $d_{u\to v} \leq \frac{h_{u\to v}}{P_{P,u\to v}}$ and $d_{u\to v} \leq \frac{h_{u\to v}}{P_{D,u\to v}}$ are satisfied.

Suppose that a packet has been forwarded $x_{u\to v}$ hops (on average) before becoming obsolete at node u. The (average) number of successful transmissions that have already taken place under the Deterministic Policy, before these data packets arrive at node u, is equal to $x_{u\to v}\left(\frac{h_{u\to v}}{P_{D,u\to v}} - d_{u\to v}\right) P_{D,u\to v}q^2$ and the average power consumed is given by $x_{u\to v}\left(\frac{h_{u\to v}}{P_{D,u\to v}} - d_{u\to v}\right) P_{D,u\to v}q^2\beta$. Under the Probabilistic Policy, the average power consumed by the obsolete packets before their arrival at node u, is given by $x_{u\to v}\left(\frac{h_{u\to v}}{P_{P,u\to v}} - d_{u\to v}\right) P_{P,u\to v}q^2\beta$.

To calculate the average power consumed by a successful transmission $u \to v$, the average power consumed by all the packets (discarded and transmitted) over a frame needs to be considered and be divided by the average number of successfully transmitted packets over a frame. Under the Deterministic Policy the average power consumed by the discarded (obsolete) packets is $x_{u\to v}\left(\frac{h_{u\to v}}{P_{D,u\to v}} - d_{u\to v}\right) P_{D,u\to v}q^2\beta$ and that of the transmitted $q\beta$, while the mean number of successfully transmitted packets is $q^2 P_{D,u\to v}$. Thus, the *power consumed per successful transmission of delay constrained packets*, denoted by $\hat{B}_{D,u\to v}$, under the Deterministic Policy, is given by,

$$\hat{B}_{D,u\to v} = \frac{q + x_{u\to v}\left(\frac{h_{u\to v}}{P_{D,u\to v}} - d_{u\to v}\right) P_{D,u\to v}q^2}{q^2 P_{D,u\to v}}\beta. \tag{7}$$

Similarly, and using the corresponding quantities obtained earlier, the power consumed per successful transmission of delay constrained packets under the Probabilistic Policy, denoted by $\hat{B}_{P,u\to v}$, is given by,

$$\hat{B}_{P,u\to v} = \frac{(1 + p(q-1))q + x_{u\to v}\left(\frac{h_{u\to v}}{P_{P,u\to v}} - d_{u\to v}\right) P_{P,u\to v}q^2}{q^2 P_{P,u\to v}}\beta. \tag{8}$$

For $x_{u\to v} = 0$, $\hat{B}_{D,u\to v} = B_{D,u\to v}$ and $\hat{B}_{P,u\to v} = B_{P,u\to v}$. For $p = 0$, $\hat{B}_{P,u\to v} = \hat{B}_{D,u\to v}$, while for $p \to 1$, $\hat{B}_{P,u\to v} \to +\infty$ ($P_{P,u\to v} \to 0$).

Theorem 6 *There exists a range of values of p such that $\hat{B}_{P,u\to v} \leq \hat{B}_{D,u\to v}$ is satisfied, if $|R_{u\to v}| \geq (q - |C_{u\to v}|)\frac{x_{u\to v}h_{u\to v}q|S_v| - q - |S_v| + 1}{x_{u\to v}h_{u\to v}q - 1}$. The range of values is of the form $[0, p^{\hat{B}}_{\max,u\to v}]$, where $p^{\hat{B}}_{\max,u\to v}$ ($0 < p^{\hat{B}}_{\max,u\to v} < 1$) corresponds to that value of p for which $\hat{B}_{P,u\to v} = \hat{B}_{D,u\to v}$.* □

The proof of Theorem 6 can be found in Appendix D.

Theorem 6 establishes the conditions regarding the existence of a range of values of p for which $\hat{B}_{P,u\to v} \le \hat{B}_{D,u\to v}$ is satisfied. Theorem 7 ensures that if $x_{u\to v}h_{u\to v} > \frac{1}{q}$ is satisfied, then, whenever the throughput under the Probabilistic Policy is higher than that under the Deterministic Policy, the power consumption is smaller. Theorem 7 establishes also a lower bound for $P_{P,u\to v}$ in order for $\hat{B}_{P,u\to v} \le \hat{B}_{D,u\to v}$ to be satisfied.

Theorem 7 *If $P_{P,u\to v} \ge P_{D,u\to v}$, there exists a range of values of p such that $\hat{B}_{P,u\to v} \le \hat{B}_{D,u\to v}$, provided that $x_{u\to v}h_{u\to v} > \frac{1}{q}$. $x_{u\to v}h_{u\to v} > \frac{1}{q}$ is satisfied if $P_{P,u\to v} > \frac{1}{qx_{u\to v}d_{u\to v}}$ $(x_{u\to v} > 0,\ h_{u\to v} > 0,\ d_{u\to v} > 0)$.* □

The proof of Theorem 7 can be found in Appendix E.

Theorem 8 *If the condition of Theorem 6 is satisfied, then there exists a certain value of p, $p^{\hat{B}}_{0,u\to v}$, such that $\hat{B}_{P,u\to v}$ is minimized. $p^{\hat{B}}_{0,u\to v} < p_{0,u\to v}$ is also satisfied.* □

The proof of Theorem 8 and an analytical expression for $p^{\hat{B}}_{0,u\to v}$ can be found in Appendix F.

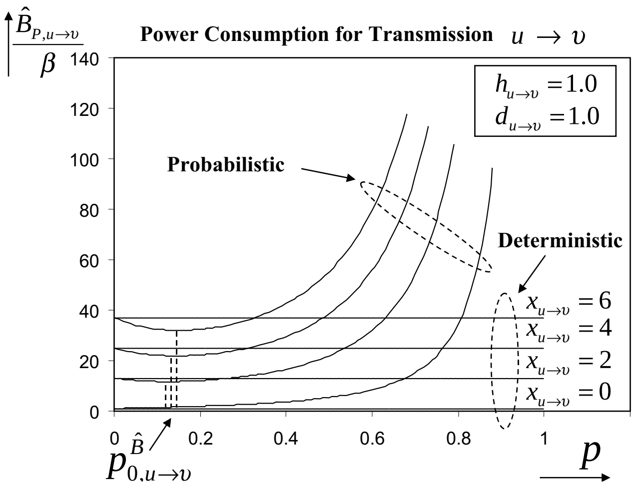


Fig. 8. $\hat{B}_{P,u\to v}$ as a function of p, for different values of $x_{u\to v}$.

Figures 8 and 9 depict $\hat{B}_{P,u\to v}$ as a function of p ($\hat{B}_{D,u\to v}$ is also depicted but remains constant as it is not a function of p). For both cases the condition of Theorem 6 is satisfied. Clearly, for $p = 0$, $\hat{B}_{P,u\to v} = \hat{B}_{D,u\to v}$, while for $p \to 1$, $\hat{B}_{P,u\to v} \to +\infty$.

In Figure 8, for $x_{u\to v} = 0$, it may be observed that the curve corresponding to $\hat{B}_{P,u\to v}$ is similar to the curve corresponding to $B_{P,u\to v}$, depicted in Figure 4. It may be also observed that as $x_{u\to v}$ increases, $p^{\hat{B}}_{0,u\to v}$ slightly increases. On the other hand, in Figure 9, it is obvious that as $d_{u\to v}$ increases, $p^{\hat{B}}_{0,u\to v}$

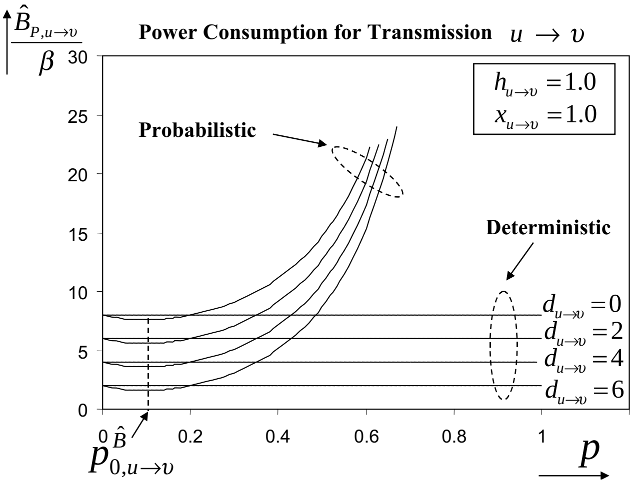


Fig. 9. $\hat{B}_{P,u\to v}$ as a function of p, for different values of $d_{u\to v}$.

remains the same. This can be explained by Equation (8) where the factor of the equation depending on $d_{u\to v}$ does not depend on p. Consequently, different values of $d_{u\to v}$ (graphically) correspond to movement of the $B_{P,u\to v}$ curve along the y-axis.

The fact that power consumption is smaller under the Probabilistic Policy when delay constraints of multi-hop flows are considered, has been analytically investigated in this section. In the following section, simulation results support the results of this analysis and allow for further observations and conclusions. In particular, Theorem 8 and the relation between $p^{\hat{B}}_{0,u\to v}$ and $p_{0,u\to v}$ allow for the use of the results of the analysis concerning the system throughput maximization, presented in [16], to provide for the minimization of the power consumption per successful transmission when delay constrained packets are considered.

5 Simulation Results

Let the power consumption per successful transmission when delay constraints are considered, averaged over all nodes, be denoted as $\hat{B}_D$ ($\hat{B}_P$) under the Deterministic (Probabilistic) Policy. Let $\overline{x}$ be the mean number of hops for all obsolete packets in the network, $\overline{d}$ the average number of frames before a packet becomes obsolete at a node and $\overline{h}q^2$ the average number of packets available for transmissions in a frame. For simulation purposes, $\overline{h} = 1$ to reflect the fact that the network operates under heavy traffic conditions and therefore, q^2 transmissions are available per frame.

The aim of the simulation results is to show that for four different network topologies ($N = 100$, $D = 10$ and four different values of the topology density

$\overline{|S|}/D$), $p = \tilde{p}_{0_{min}}$ may be used for not only achieving system throughput close to the maximum but also for achieving power consumption close to the minimum. This is a direct conclusion from Theorem 8 where the close relation between the value of p, for which the throughput under the Probabilistic Policy is maximized, and the value of p, for which the power consumption under the Probabilistic Policy is minimized, is established.

The algorithm presented in [15] is used to derive the sets of scheduling slots and the system throughput is calculated averaging the simulation results over 100 frames. Unique polynomials, that correspond to time slot sets Ω_χ, are assigned randomly to each node χ, for each particular topology. The particular assignment is kept the same for each topology throughout the simulations.

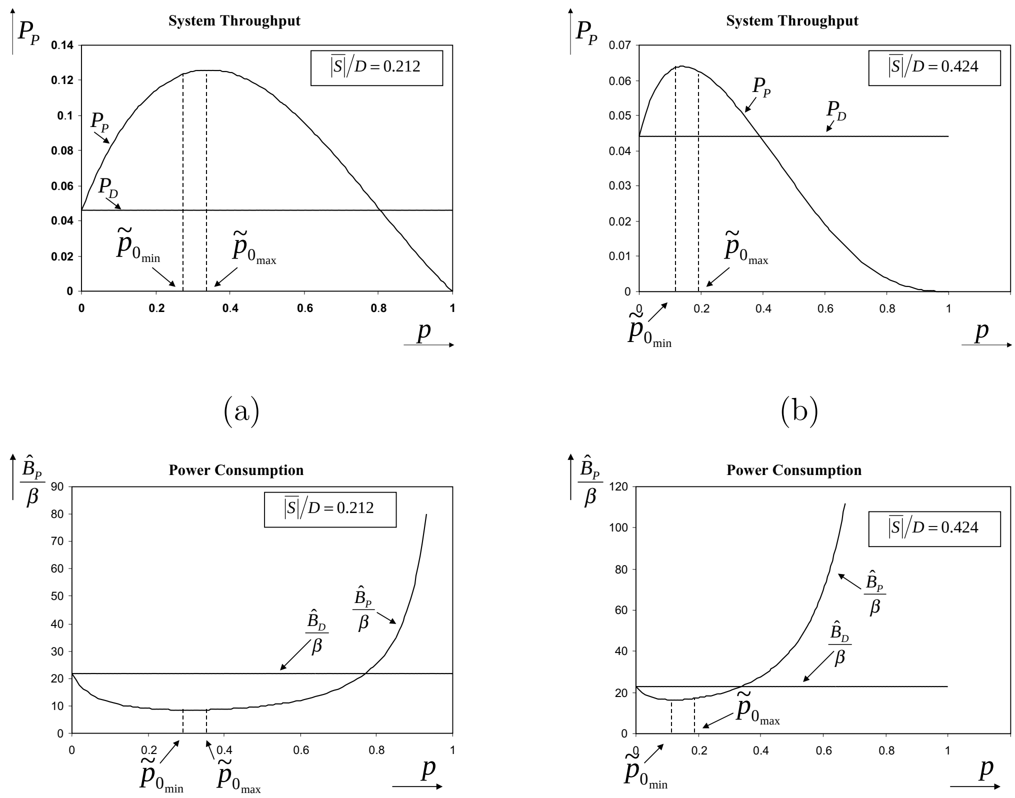
The simulation results presented demonstrate the performance for $k = 1$ (the resulting value for k is equal to 1 for the four topologies, [15]), that is the case that the number of non-assigned eligible time slots is expected to be rather small and, thus, the effectiveness of the Probabilistic Policy to be low, while the power consumed to be high as it may be concluded from Theorem 5. The values of $\tilde{p}_{0_{min}}$ and $\tilde{p}_{0_{max}}$ corresponding to the particular simulation scenarios are summarized in Table 1.

Table 1
$\tilde{p}_{0_{min}}$ and $\tilde{p}_{0_{max}}$ for different values of $\overline{|S|}/D$.

$\overline{\|S\|}/D$	$\tilde{p}_{0_{min}}$	$\tilde{p}_{0_{max}}$
0.212	0.283345	0.320513
0.424	0.144466	0.19084
0.614	0.088993	0.140056
0.866	0.048454	0.10352

In Figures 10(a) and 10(b), P_P is depicted as a function of p for two topologies corresponding to $\overline{|S|}/D = 0.212$ and $\overline{|S|}/D = 0.412$, respectively and $\overline{x} = 1$, $\overline{d} = 1$. For the same network topologies, $\hat{B}_P$ is depicted in Figures 10(c) and 10(d). It is clear that the range of values $[\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}}]$, determines a range of values for p such that $\hat{B}_P < \hat{B}_D$. Note that $\tilde{p}_{0_{min}}$ appears to be closer to the minimum of $\hat{B}_P$ than $\tilde{p}_{0_{max}}$. This can be explained by Theorem 8 where the value of p that minimizes $\hat{B}_{P,u\to v}$ is less than the value of p that maximizes $P_{P,u\to v}$.

As the topology density $\overline{|S|}/D$ increases, the system throughput P_P exponentially decreases as it may be concluded from Equation (4) (as $\overline{|S|}/D$ increases - D is known - $|S_v|$ increases on average). In Figures 11(a) and 11(b), P_P is depicted as a function of p for two topologies corresponding to $\overline{|S|}/D = 0.614$ and $\overline{|S|}/D = 0.866$, respectively and $\overline{x} = 1$, $\overline{d} = 1$. $\hat{B}_P$ is also depicted in Figures 11(c) and 11(d). It is clear that P_P is small for large values of $\overline{|S|}/D$



(a) (b)

(c) (d)

Fig. 10. Simulation results for $\overline{|S|}/D = 0.212$ and 0.412. $\overline{x} = 1$, $\overline{d} = 1$.

and that the range of values for which $P_P > P_D$ is satisfied, is small. The same applies for $\hat{B}_P$. It is interesting to observe that even for large values of $\overline{|S|}/D$, $p \in [\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}}]$ allows for $P_P > P_D$ and $\hat{B}_P < \hat{B}_D$, and for $p = \tilde{p}_{0_{min}}$, $\hat{B}_P$ is close to the minimum.

In Figure 12, simulation results for the topology corresponding to $\overline{|S|}/D = 0.424$, are presented. Obviously, as $\overline{x}$ increases the range of values of p, for which the power consumed under the Probabilistic Policy is smaller than the power consumed under the Deterministic Policy, increases as it may be seen from Figure 12(a) ($\frac{\hat{B}_P}{\beta} = \frac{\hat{B}_D}{\beta}$ for larger values of p as $\overline{x}$ increases). Consequently, if $\overline{x}$ is that large that the condition of Theorem 7 is satisfied, then $p \in [\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}}]$ is a suitable choice.

Note that according to Equation (8), different values of $d_{u \to v}$ result to different values of the left-hand part of the equation irrespectively of p, which can graphically be reflected as a movement of the $\hat{B}_P$ curve along the y-axis direction. Consequently, $\overline{d}$ has no impact on the range of values of p for which $\hat{B}_P < \hat{B}_D$. This can be observed from Figure 12(b), where $\frac{\hat{B}_P}{\beta}$ is depicted for various values of $\overline{d}$. It can be seen that the corresponding range of values of p

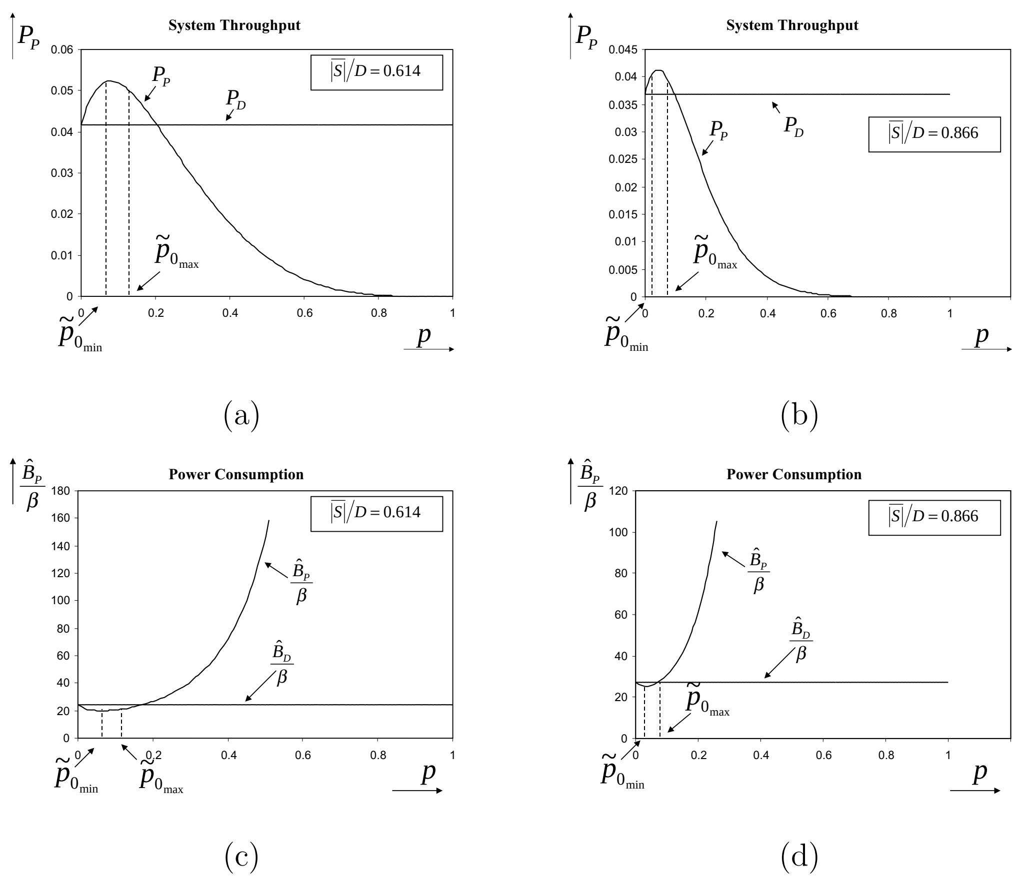


Fig. 11. Simulation results for $\overline{|S|}/D = 0.614$ and 0.866. $\overline{x} = 1$, $\overline{d} = 1$.

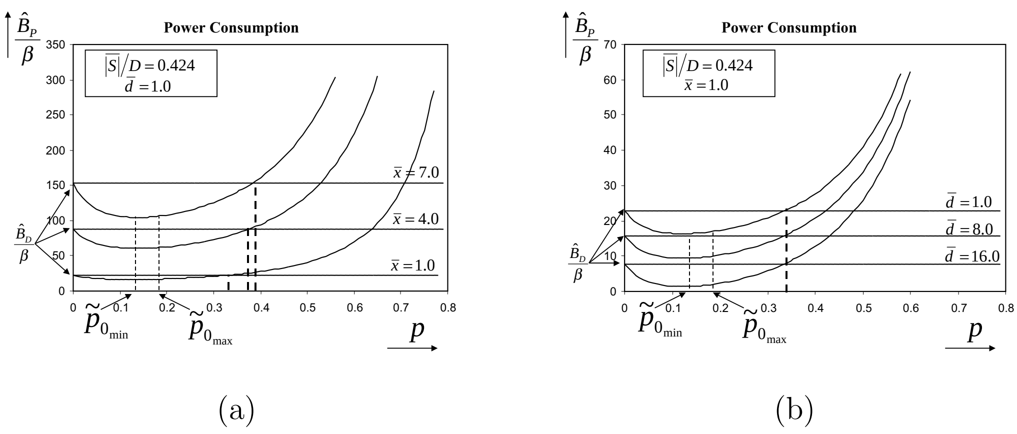


Fig. 12. Simulation results for $\overline{|S|}/D = 0.424$ for different values of (a) $\overline{x}$ and (b) $\overline{d}$.

does not change ($\frac{\hat{B}_P}{\beta} = \frac{\hat{B}_D}{\beta}$ for the same value of p, $p \neq 0$).

Eventually, the range of value of p, $[\tilde{p}_{0_{min}}, \tilde{p}_{0_{max}}]$, is suitable in order for $\hat{B}_P < \hat{B}_D$ (as long as the condition of Theorem 7 is satisfied). In addition, $p = \tilde{p}_{0_{min}}$ is a suitable choice for the minimization of $\hat{B}_P$.

6 Conclusions

The increased number of transmissions (both successful and corrupted) under the Probabilistic Policy increases the power consumed per frame. In this work the power consumed under the Deterministic Policy and the Probabilistic Policy is studied and certain conditions are established under which the power efficiency under the latter policy is higher than that under the former policy.

The system is presented and a brief description of the Deterministic Policy and the Probabilistic Policy reveals the fact that the power consumed under the Probabilistic Policy over a frame is higher than the power consumed under the Deterministic Policy. On the other hand, the higher throughput under the Probabilistic Policy motivates an analysis of the system with respect to the power consumed per successful transmission.

The first step towards this analysis is to establish the conditions under which the power consumed per successful transmission is smaller under the Probabilistic Policy. The analysis shows that this is possible for $k \geq 2$, which corresponds to networks with large number of nodes present but with small number of neighbor nodes for every node.

The analysis continues by considering delay constrained packets and it establishes the conditions under which the power efficiency under the Probabilistic Policy is higher than that under the Deterministic Policy. Simulation results support the claims and the expectations of the aforementioned analysis and reveal the fact that the boundaries of p, for which the system throughput is maximized, determine a range of values of p that minimizes the power consumption. In addition, it is shown that the lower bound of this range of values is close to that value of p for which the power consumed is minimized under the Probabilistic Policy.

A Proof of Theorem 1

The first derivative of $P_{P,u\to v}$ with respect to p is the following, $\frac{dP_{P,u\to v}}{dp} = \frac{|R_{u\to v}|-(q-|C_{u\to v}|)|S_v|-|R_{u\to v}|(|S_v|+1)p}{q^2}(1-p)^{|S_v|-1}$. Consequently,

$$\begin{aligned}\frac{dB_{P,u\to v}}{dp} &= \frac{\beta}{q}\frac{(q-1)P_{P,u\to v} - \Big(1+p(q-1)\Big)\frac{dP_{P,u\to v}}{dp}}{P^2_{P,u\to v}} \\ &= \frac{\beta}{P^2_{P,u\to v}q^3}\Big((q-|C_{u\to v}|)(q-1+|S_v|)-|R_{u\to v}|\Big)(1-p)^{|S_v|-1}\end{aligned}$$

$$+\frac{\beta}{P^2_{P,u\to v}q^3}p\Big((q-1)(q-|C_{u\to v}|)(|S_v|-1)+|R_{u\to v}|(|S_v|+1)\Big)(1-p)^{|S_v|-1}$$
$$+\frac{\beta}{P^2_{P,u\to v}q^3}p^2(q-1)|R_{u\to v}||S_v|(1-p)^{|S_v|-1}.$$

Given that $\frac{\beta}{P^2_{P,u\to v}q^3}>0$, $(q-1)(q-|C_{u\to v}|)(|S_v|-1)+|R_{u\to v}|(|S_v|+1)\geq 0$, $(q-1)|R_{u\to v}||S_v|\geq 0$ and $(1-p)^{|S_v|-1}\geq 0$, it is evident that if $(q-|C_{u\to v}|)(q-1+|S_v|)-|R_{u\to v}|\geq 0$, $\frac{dB_{P,u\to v}}{dp}\geq 0$, for any $0\leq p\leq 1$.

B Proof of Theorem 2

If $|R_{u\to v}|>(q-|C_{u\to v}|)(q-1+|S_v|)$ then, according to Appendix A, it is calculated that $\lim_{p\to 0^+}\frac{dB_{P,u\to v}}{dp}<0$. Let $f(p)$ denote the polynomial $f(p)=(q-1)|R_{u\to v}||S_v|p^2+\Big((q-1)(q-|C_{u\to v}|)(|S_v|-1)+|R_{u\to v}|(|S_v|+1)\Big)p+(q-|C_{u\to v}|)(q-1+|S_v|)-|R_{u\to v}|$. According to Appendix A, $\frac{dB_{P,u\to v}}{dp}=\frac{\beta}{P^2_{P,u\to v}q^3}f(p)(1-p)^{|S_v|-1}$.

It can be calculated that $\lim_{p\to 1}f(p)q|S_v|(q-|C_{u\to v}|+|R_{u\to v}|)$. Obviously, $\lim_{p\to 1}f(p)>0$ and therefore, $\lim_{p\to 1}\frac{dB_{P,u\to v}}{dp}>0$. Consequently, there must exist an odd number of values of p such that $\frac{dB_{P,u\to v}}{dp}=0$, in the range $(0,1)$. Given that $f(p)$ is a second degree polynomial, there exists only one value of $p\in(0,1)$, denoted by $p^B_{0,u\to v}$, which corresponds to the positive root of $f(p)$.

Let $\alpha=(q-1)|R_{u\to v}||S_v|$, $\beta=(q-1)(q-|C_{u\to v}|)(|S_v|-1)+|R_{u\to v}|(|S_v|+1)$ and $\gamma=(q-|C_{u\to v}|)(q-1+|S_v|)-|R_{u\to v}|$. $f(p)$ can be written as $f(p)=\alpha p^2+\beta p+\gamma$. It is derived that $p^B_{0,u\to v}=\frac{-\beta+\sqrt{\beta^2-4\alpha\gamma}}{2\alpha}$.

Consequently, for $p=0$, $B_{P,u\to v}=B_{D,u\to v}$, and as p increases, $B_{P,u\to v}$ decreases until $p=p^B_{0,u\to v}$. For $p>p^B_{0,u\to v}$, as p increases, $B_{P,u\to v}$ increases. Given that for $p\to 1$, $B_{P,u\to v}\to+\infty$, it is evident that there exists a value of p, denoted by $p^B_{\max,u\to v}$ ($p^B_{\max,u\to v}\neq 0$), such that $B_{P,u\to v}=B_{D,u\to v}$.

C Proof of Theorem 3

As it can be seen from the proof of Theorem 1 in Appendix A, $\frac{dB_{P,u\to v}}{dp} = q\beta\frac{(q-1)P_{P,u\to v}-\left(1+p(q-1)\right)\frac{dP_{P,u\to v}}{dp}}{P^2_{P,u\to v}}$. For $p = p^B_{0,u\to v}$, $\frac{dB_{P,u\to v}}{dp} = 0$ and therefore, $\frac{dP_{P,u\to v}}{dp} = \frac{q-1}{1+p^B_{0,u\to v}(q-1)}P_{P,u\to v} > 0$. It is evident that $\frac{dP_{P,u\to v}}{dp} > 0$ is satisfied only for $p < p_{0,u\to v}$. Consequently, $p^B_{0,u\to v} < p_{0,u\to v}$.

D Proof of Theorem 6

The first derivative of $\hat{B}_{P,u\to v}$ with respect to p, $\frac{d\hat{B}_{P,u\to v}}{dp}$, is calculated to be equal to $\frac{dB_{P,u\to v}}{dp} - \frac{x_{u\to v}h_{u\to v}\frac{dP_{P,u\to v}}{dp}}{P^2_{P,u\to v}}\beta$. It can be observed that $x_{u\to v}h_{u\to v}\frac{dP_{P,u\to v}}{dp}\beta = x_{u\to v}h_{u\to v}\beta\frac{|R_{u\to v}|-(q-|C_{u\to v}|)|S_v|-|R_{u\to v}|(|S_v|+1)p}{q^2}(1-p)^{|S_v|-1} = \frac{x_{u\to v}h_{u\to v}q\beta}{q^3}\Big(|R_{u\to v}| - (q-|C_{u\to v}|)|S_v|\Big)(1-p)^{|S_v|-1} - \frac{x_{u\to v}h_{u\to v}q\beta}{q^3}|R_{u\to v}|(|S_v|+1)p(1-p)^{|S_v|-1}q^2(1-p)^{|S_v|-1}$.

Using the results of Appendix A, it can be calculated that,

$$\begin{aligned}\frac{d\hat{B}_{P,u\to v}}{dp} = &\frac{\beta}{P^2_{P,u\to v}q^3}\Big(x_{u\to v}h_{u\to v}q\Big(|R_{u\to v}| - (q-|C_{u\to v}|)|S_v|\Big)\Big)(1-p)^{|S_v|-1}\\ &+\frac{\beta}{P^2_{P,u\to v}q^3}\Big((q-|C_{u\to v}|)(q-1+|S_v|) - |R_{u\to v}|\Big)(1-p)^{|S_v|-1}\\ &+\frac{\beta}{P^2_{P,u\to v}q^3}p\Big(x_{u\to v}h_{u\to v}q|R_{u\to v}|(|S_v|+1)\Big)(1-p)^{|S_v|-1}\\ &+\frac{\beta}{P^2_{P,u\to v}q^3}p\Big((q-1)(q-|C_{u\to v}|)(|S_v|-1) + |R_{u\to v}|(|S_v|+1)\Big)(1-p)^{|S_v|-1}\\ &+\frac{\beta}{P^2_{P,u\to v}q^3}p^2(q-1)|R_{u\to v}||S_v|(1-p)^{|S_v|-1}.\end{aligned}$$

Obviously, $\lim_{p\to 0}\frac{d\hat{B}_{P,u\to v}}{dp} = \frac{\beta}{P^2_{P,u\to v}q^3}\Big(x_{u\to v}h_{u\to v}q\Big(|R_{u\to v}| - (q-|C_{u\to v}|)|S_v|\Big)\Big)(1-p)^{|S_v|-1} + \frac{\beta}{P^2_{P,u\to v}q^3}\Big((q-|C_{u\to v}|)(q-1+|S_v|)-|R_{u\to v}|\Big)(1-p)^{|S_v|-1}$ and $\lim_{p\to 0}\frac{d\hat{B}_{P,u\to v}}{dp} \geq 0$ if $|R_{u\to v}| \geq (q-|C_{u\to v}|)\frac{x_{u\to v}h_{u\to v}q|S_v|-q-|S_v|+1}{x_{u\to v}h_{u\to v}q-1}$.

It is known that for $p \to 1$, $P_{P,u\to v} \to 0$ and therefore, $\lim_{p\to 1}\hat{B}_{P,u\to v} = +\infty$. Consequently, if $|R_{u\to v}| \geq (q-|C_{u\to v}|)\frac{x_{u\to v}h_{u\to v}q|S_v|-q-|S_v|+1}{x_{u\to v}h_{u\to v}q-1}$ is satisfied, then, apart from $p = 0$, there exists another value of $0 < p < 1$ such that $\hat{B}_{P,u\to v} = \hat{B}_{D,u\to v}$. This value is denoted by $p^{\hat{B}}_{\max,u\to v}$. Consequently, for any $p \in [0, p^{\hat{B}}_{\max,u\to v}]$ and $|R_{u\to v}| \geq (q - |C_{u\to v}|)\frac{x_{u\to v}h_{u\to v}q|S_v|-q-|S_v|+1}{x_{u\to v}h_{u\to v}q-1}$, $\hat{B}_{P,u\to v} \leq \hat{B}_{D,u\to v}$ is satisfied.

E Proof of Theorem 7

$P_{P,u\to v} \geq P_{D,u\to v}$ is satisfied if $|R_{u\to v}| \geq (q - |C_{u\to v}|)|S_v|$, as it may be seen from Section 2. It is enough to show that $(q - |C_{u\to v}|)\frac{x_{u\to v}h_{u\to v}q|S_v|-q-|S_v|+1}{x_{u\to v}h_{u\to v}q-1} \leq (q - |C_{u\to v}|)|S_v|$ or $\frac{x_{u\to v}h_{u\to v}q|S_v|-q-|S_v|+1}{x_{u\to v}h_{u\to v}q-1} \leq |S_v|$. If $x_{u\to v}h_{u\to v}q - 1 > 0$ or $x_{u\to v}h_{u\to v} > \frac{1}{q}$, it is enough that $x_{u\to v}h_{u\to v}q|S_v| - q - |S_v| + 1 \leq (x_{u\to v}h_{u\to v}q - 1)|S_v|$ is satisfied or $-q+1 \leq 0$ which is always satisfied. If $x_{u\to v}h_{u\to v}q - 1 > 0$, then $-q + 1 \geq 0$, which is not satisfied.

Given that $h_{u\to v} \geq d_{u\to v}P_{P,u\to v}$, it is enough that $x_{u\to v}d_{u\to v}P_{P,u\to v} > \frac{1}{q}$ is satisfied, or $P_{P,u\to v} > \frac{1}{qx_{u\to v}d_{u\to v}}$.

F Proof of Theorem 8

If $|R_{u\to v}| < (q - |C_{u\to v}|)\frac{x_{u\to v}h_{u\to v}q|S_v|-q-|S_v|+1}{x_{u\to v}h_{u\to v}q-1}$ then, according to Appendix D, $\lim_{p\to 0}\frac{d\hat{B}_{P,u\to v}}{dp} < 0$. Let the $\hat{f}(p)$ denote the polynomial $\hat{f}(p) = (q-1)|R_{u\to v}||S_v|p^2 + \Big((q-1)(q-|C_{u\to v}|)(|S_v|-1) + |R_{u\to v}|(|S_v|+1) + x_{u\to v}h_{u\to v}q|R_{u\to v}|(|S_v|+1)\Big)p + (q-|C_{u\to v}|)(q-1+|S_v|) - |R_{u\to v}| + x_{u\to v}h_{u\to v}q\Big(|R_{u\to v}| - (q-|C_{u\to v}|)|S_v|\Big)$.
It is clear that $\lim_{p\to 1}\hat{f}(p) = q|S_V|(q - |C_{u\to v}| + |R_{u\to v}|) + x_{u\to v}h_{u\to v}q\Big(|R_{u\to v}|(|S_v|+2) - (q - |C_{u\to v}|)|S_v|\Big)$.

Obviously, $\lim_{p\to 1}\hat{f}(p) > 0$ and therefore, $\lim_{p\to 1}\frac{d\hat{B}_{P,u\to v}}{dp} > 0$. Consequently, there exist an odd number of values of p such that $\frac{d\hat{B}_{P,u\to v}}{dp} = 0$ in the range $(0, 1)$. Given that $\hat{f}(p)$ is a second degree polynomial, there exists only one value of $p \in (0, 1)$, denoted by $p^{\hat{B}}_{0,u\to v}$ which corresponds to the positive root of $\hat{f}(p)$.

Let $\hat{\alpha} = (q-1)|R_{u\to v}||S_v|$, $\hat{\beta} = (q-1)(q-|C_{u\to v}|)(|S_v|-1) + |R_{u\to v}|(|S_v|+1) + x_{u\to v}h_{u\to v}q|R_{u\to v}|(|S_v|+1)$ and $\hat{\gamma} = (q - |C_{u\to v}|)(q-1+|S_v|) - |R_{u\to v}| + x_{u\to v}h_{u\to v}q\Big(|R_{u\to v}| - (q - |C_{u\to v}|)|S_v|\Big)$. $\hat{f}(p)$ can be written as $\hat{f}(p) = \hat{\alpha}p^2 +$

$\hat{\beta}p + \hat{\gamma}$. It is derived that $p^{\hat{B}}_{0,u\to v} = \frac{-\hat{\beta}+\sqrt{\hat{\beta}^2-4\hat{\alpha}\hat{\gamma}}}{2\hat{\alpha}}$.

For $p = p^{\hat{B}}_{0,u\to v}$, $\frac{d\hat{B}_{P,u\to v}}{dp} = 0$. Equivalently, $\frac{dB_{P,u\to v}}{dp} - \frac{x_{u\to v}h_{u\to v}\frac{dP_{P,u\to v}}{dp}}{P^2_{P,u\to v}}\beta = 0$ or $\frac{\beta}{q}\frac{(q-1)P_{P,u\to v}-\left(1+p(q-1)\right)\frac{dP_{P,u\to v}}{dp}}{P^2_{P,u\to v}} - \frac{x_{u\to v}h_{u\to v}\frac{dP_{P,u\to v}}{dp}}{P^2_{P,u\to v}}\beta = 0$ or $(q-1)P_{P,u\to v} - \Big(1+p(q-1)\Big)\frac{dP_{P,u\to v}}{dp} = qx_{u\to v}h_{u\to v}\frac{dP_{P,u\to v}}{dp}$ or $\Big(qx_{u\to v}h_{u\to v}+1+p(q-1)\Big)\frac{dP_{P,u\to v}}{dp} = (q-1)P_{P,u\to v}$ or $\frac{dP_{P,u\to v}}{dp} = \frac{q-1}{qx_{u\to v}h_{u\to v}+1+p(q-1)}P_{P,u\to v} > 0$. It is evident that $\frac{dP_{P,u\to v}}{dp} > 0$ is satisfied for $p < p_{0,u\to v}$. Consequently, $p^{\hat{B}}_{0,u\to v} < p_{0,u\to v}$.

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