Trading public parking space

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***Abstract*—Our paper investigates normative abstractions for the way drivers pursue parking space and respond to pricing policies about public and private parking facilities. The drivers are viewed as strategic agents who make rational decisions while attempting to minimize the cost of the acquired parking spots. We propose auction-based systems for realizing centralized parking allocation schemes, whereby drivers bid for public parking space and a central authority coordinates the parking assignments and payments. These are compared against the conventional uncoordinated parking search practice under fixed parking service cost, formulated as a resource selection game instance. In line with intuition, the auctioning system increases the revenue of the public parking operator exploiting the drivers' differentiated interest in parking. Less intuitively, the auction-based mechanism does not necessarily induce higher cost for the drivers: by avoiding the uncoordinated search and thus, eliminating the cruising cost, it turns out to be a preferable option for both the operator and the drivers under various combinations of parking demand and pricing policies.**

***Keywords*—Parking games, auctions, aggregate cost, revenue**

I. INTRODUCTION

The high demand for parking space in city centers has always been a challenge in the process of city planning. The city authorities draw on both public and private parking facilities and more recently deploy parking assistance systems (*e.g.*, [1] [2]), to respond to the parking needs of the car volumes that daily visit popular in-city destinations. Under the conventional parking search practice, drivers choose between the cheap but scarce on-street parking spots and the more expensive option of private parking space. In fact, drivers selfishly pursue to minimize the cost of access to parking facilities. However, the intuitive decision to head for the cheaper or free-of-cost on-street parking space, combined with the scarcity of public parking capacity in urban curbside of typical center areas, give rise to *tragedy of commons* effects and highlight the game-theoretic dynamics behind the parking spot selection problem.

In earlier work [3], we have formulated and studied the game that arises from the conventional parking search behavior under a fixed parking cost model. The drivers in search for parking space are viewed as rational strategic agents that choose either to compete for the cheaper but scarce on-street parking spots or head for the more expensive private parking lots. In the first case, they run the risk of failing to get a spot and having to *a posteriori* take the more expensive alternative, this time suffering the additional *cruising cost* in terms of time and fuel consumption. Drawing on realistic charging schemes, we have derived the equilibrium strategies of the drivers and assessed their (in)efficiency via game-theoretic measures such as the social cost and the Price of Anarchy. We summarize these results in Section II of this paper.

In this paper, we ask whether and how much can centralized parking assistance systems combined with more aggressive pricing schemes improve the outcome for both the on-street parking space operator *and* the drivers. More specifically, in Section III we propose different *auction* mechanisms for the assignment of on-street parking space. In fact, auction mechanisms have been used under various concepts in different disciplines. In network science, research efforts on node transactions devise auction-based schemes to address the challenge of resource (energy, bandwidth and storage space) sharing among multiple networking users [4]. Our paper approaches the process of parking space selection in urban environments as a network resource allocation problem. Indeed, the auctioning of parking spots is a promising key-idea that has only recently started to gain interest [5]. The number of available auctioned spots is announced to the drivers, who submit their bids for them, expressing what they are willing to pay for a parking spot in that particular occasion with complete information for the overall parking demand. As central mechanisms, auctions determine who gets a parking spot and at what cost, saving the additional expenses of cruising in the non-assisted, uncoordinated parking search, while unleashing the conventional buying rules in public parking operation. Indeed, the analytical results in Section IV show that, as expected, auctions always raise the revenue of the public parking operator since they adapt payments to what drivers are willing to pay for on-street parking space. Nevertheless, this does not come necessarily at the expense of drivers, who save the cruising cost and find the auctioning system less expensive on average, under various combinations of parking demand and pricing policies.

II. THE PARKING SPOT SELECTION GAME

In the parking spot selection game, the set of players consists of drivers who circulate within a city area in search for parking space. The players have to decide whether to drive towards the scarce low-cost on-street public parking spots or the more expensive private parking lot(s). All parking spots that lie in the same public or private area are assumed to be of the same value for the players. Thus, the decisions are made on the two *sets* of parking spots rather than individual set items. The two sets jointly suffice to serve all parking requests.

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The collective decision making on parking space selection can be formulated as an instance of the strategic *resource selection games*, whereby N players (*i.e.*, drivers) compete against each other for a finite number of S on-street public parking spots. More specifically, drivers who decide to compete for the cheaper on-street public parking space undergo the risk of not being among the S winner-drivers to get a public spot. In this case, they have to eventually resort to private parking space, only after wasting extra time and fuel (plus patience supply) on the failed attempt. The expected cost of competing for public parking space, w_{pub}, is therefore a function of the number of competing drivers k, and is given by

$$w_{pub}(k) = min(1, S/k)c_{pub,s} + (1 - min(1, S/k))c_{pub,f} \quad (1)$$

where $c_{pub,s}$ is the fixed cost of successfully competing for public parking space, whereas $c_{pub,f} = \gamma \cdot c_{pub,s}, \gamma > 1$, is the cost of competing, failing, and eventually paying for private parking space.

On the other hand, the cost of private parking spots is fixed

$$w_{priv}(k) = c_{priv} = \beta \cdot c_{pub,s} \quad (2)$$

where $1 < \beta < \gamma$, namely the excess cost $\delta \cdot c_{pub,s}$, with $\delta = \gamma - \beta > 0$, reflects the cruising cost in terms of wasted time and fuel till eventually heading to the private parking space.

If N_{pub} denotes the number of drivers that decide to compete for public parking space, then the aggregate drivers' cost paid by the entire population is given by

$$C(N_{pub}) = \begin{cases} c_{pub,s}\left[N\beta - N_{pub}(\beta - 1)\right], \text{if } N_{pub} \leq S \\ c_{pub,s}\left[N_{pub}\delta - S(\gamma - 1) + \beta N\right], \text{if } S < N_{pub} \leq N \end{cases}$$

whereas the revenue for the public parking space operator is given by:

$$R(N_{pub}) = \begin{cases} N_{pub}c_{pub,s}, \text{if } N_{pub} \leq S \\ Sc_{pub,s}, \text{if } S < N_{pub} \leq N \end{cases} \quad (3)$$

In [3], we have analyzed the parking spot selection game assuming both complete and probabilistic knowledge of the parking demand, *i.e.*, the number of drivers seeking for parking space, as well as complete uncertainty about it. The main finding for the strategic parking spot selection game is that, for parking demand exceeding the supply ($N > S$), the number of competing drivers in the equilibrium state $N_{pub,eq} = \min(N, N_0)$, with $N_0 = \frac{S(\gamma-1)}{\delta}$, exceeds the optimal number S that would compete for and succeed in getting an on-street parking spot in the ideal scenario[1]. In other words, an expected number of $N_{pub,eq} - S$ ends up wasting time, fuel, and psychological resources on needless cruising without eventually saving the more expensive private parking fee. On the contrary, when $N \leq S$, all drivers head to the area of public parking.

The resulting aggregate drivers' cost C_g when $N_{pub,eq}$ drivers compete (*i.e.*, under the equilibrium states of the game) amounts to

$$C_g \equiv C(N) = c_{pub,s}\left[N\gamma - \min(N, S)(\gamma - 1)\right], \text{ if } N \leq N_0 \text{ and}$$
$$C_g \equiv C(N_0) = c_{pub,s}\beta N, \text{ if } N > N_0 \quad (4)$$

which, for $N > S$, exceeds the optimal cost value $C_{g,opt} \equiv C(S) = c_{pub,s}\left[S + \beta(N - S)\right]$, the ratio $C_g/C_{g,opt}$ expressing the "price of anarchy" of the game and quantifying the penalty of lack of coordination across the drivers. On the other hand, the revenue R_g for the public parking space operator becomes

$$R_g \equiv R(N) = \min(N, S)c_{pub,s}, \text{ if } N \leq N_0 \text{ and}$$
$$R_g \equiv R(N_0) = Sc_{pub,s}, \text{ if } N > N_0 \quad (5)$$

III. The auction-based parking allocation

Parking assistance schemes can help overcome the inefficiencies that result from the uncoordinated selfish behavior of drivers. These systems rely on wireless communication systems for delegating the parking space assignment task to a central server, which: a) gets informed about the status of on-street public parking spots; b) collects the requests and bids of drivers for parking space; and c) determines who is assigned a public parking spot and at what cost, and notifies the drivers. In this paper, in particular, we propose and analyze an *auction-based* system for the management of the public parking space drawing on the theory of *multi-unit auctions with single-unit demand* [6].

In particular, N drivers (buyers) bid in a single auction for no more than one of S spare on-street public parking spots (non-divisible, physically identical goods). Drivers (bidders) are assumed to be symmetric: their valuations of parking spots are i.i.d RVs continuously distributed in the same interval $[v_{min}, v_{max}]$ and $F_V()$, $f_V()$ are their cumulative distribution and probability density functions, respectively. An appropriate choice for this interval is $[c_{pub,s}, c_{priv}]$. In other words, the operator of the public parking resources will typically impose a threshold on the selling price, *i.e.*, a reserve price, that will be no less than the on-street public parking spot price under fixed cost. Drivers, in turn, will account for this lower bound in their bidding decisions, while they will not be willing to pay more than what the private parking operator charges. Although each driver is aware of the distribution of his competitors' valuations, upon bidding, he can only know the realization of his own RV (*i.e.*, his bid). Bidders are also assumed to be risk-neutral, *i.e.*, they seek to maximize their expected profit from bidding, and free of budget constraints [6].

In general, if $\mathcal{N} = \{1, ..., N\}$ with N > 1 is the set of drivers who seek parking space, a selling auction mechanism consists of three components: the set of *bids* $\mathcal{B}_i$ (increasing functions of valuations) for each driver $i \in \mathcal{N}$; an *allocation rule* $\pi : \mathcal{B}_1 \times ... \times \mathcal{B}_N \rightarrow \mathcal{D}(\mathcal{N})$, where $\mathcal{D}$ is the set of probability distributions over $\mathcal{N}$ determining who are awarded parking spots, and a *payment rule* $p : \mathcal{B}_1 \times ... \times \mathcal{B}_N \rightarrow \mathbb{R}^N$ for the selling price of each allocated spot. Out of the variety of options, hereafter we consider the three most thoroughly analyzed implementations, the *uniform-price*, *discriminatory-price* and *Vickrey* auctions. All three auction formats are *standard* in that they assign the parking spots to the users that submit the highest bids. Under single-unit demand and

[1] For given N, any value for the expected number of competing players $0 \leq N_{pub} \leq N$ can be realized through an appropriate choice of the symmetric mixed-action strategy. Indeed, the parking spot selection game has, in addition to the pure equilibria, a unique symmetric mixed-action Nash equilibrium $p^{NE} = (p^{NE}_{pub}, p^{NE}_{priv})$, with $p^{NE}_{pub} = \min\left(1, \frac{N_0}{N}\right)$ and $p^{NE}_{pub} + p^{NE}_{priv} = 1$.

symmetric risk-neutral bidders, all three auctions are also *efficient* in the sense that they assign the parking spots to the users that value them most[2]. In other words, they induce equilibrium states, whereby the top-bids are submitted by the drivers that value the parking spots most. On the other hand, whereas all three auctioning mechanisms follow the same allocation rule, they differ in the payment rule they apply.

- Under the *uniform-price auction* (*upa*) and the *Vickrey auction* (*va*), all parking spots are sold at the same price, the "market-clearing price", which is equal to the first losing bid, *i.e.*, the $(S+1)^{th}$ highest over all drivers' bids.
- Under the *discriminatory-price auction* (*dpa*), the winning drivers pay an amount equal to their individual bids.

In the sequel, we first define the equilibrium bidding strategies when the drivers are aware of the number of competitors; for instance, because the parking assistance system provides them with this information. We then discuss their effectiveness from the bidders' and operator's perspective, given that the auctioned parking spots do not suffice to fulfil the entire parking demand. Otherwise, it is trivial to show that the centralized auction's and the distributed practice's outcomes (*i.e.*, parking spot allocation and winners' payments) coincide.

1) Uniform-price and Vickrey auction: Both the single-unit demand uniform-price and Vickrey auction mechanisms come under the broader category of incentive-compatible (truthful) mechanisms in that the equilibrium strategy, $\beta(v)$ for the drivers is to bid their real valuations v,

$$\beta_{upa}(v) = \beta_{va}(v) = v \tag{6}$$

For $N > S$, the conditional expectation of the driver's payment for a given valuation v is

$$\begin{aligned} p_{upa}(v) = p_{va}(v) &= Pr(V_{(N-S,N-1)} < v) \\ &\quad \cdot E\{V_{(N-S,N-1)} | V_{(N-S,N-1)} < v\} \\ &= \int_{v_{min}}^{v} y f_{V_{(N-S,N-1)}}(y) dy \end{aligned} \tag{7}$$

where $E\{\cdot\}$ is the expectation operator and $V_{(k,n)}$ is the k^{th} order statistic of the n competing valuations (*i.e.*, the k^{th} smallest out of n samples drawn from RVs $V_1, ..., V_n$) with probability density function $f_{V_{(k,n)}}(y) = \{B(k, n-k+1)\}^{-1}\{F_V(y)\}^{k-1}\{1-F_V(y)\}^{n-k} f_V(y)$, where $B(\cdot,\cdot)$ stands for the complete Beta function [7].

Therefore, the unconditional (*ex ante*) expectation of the driver's payment is given by

$$\begin{aligned} p_{upa} = p_{va} &= \int_{v_{min}}^{v_{max}} p_{upa}(v) f_V(v) dv \\ &= \frac{S}{N} E\{V_{(N-S,N)}\} \end{aligned} \tag{8}$$

while the *expected* revenue of the public parking operator becomes

$$\begin{aligned} R_a \equiv E\{R_{upa}\} = E\{R_{va}\} &= N p_{va} \\ &= S E\{V_{(N-S,N)}\} \end{aligned} \tag{9}$$

and is collected from the drivers with the top S bids.

On the other hand, drivers with the $N-S$ lowest bids resort to private parking facilities, all paying the fixed cost $v_{max} = c_{priv}$. Thus, the *expected* aggregate drivers' cost turns out to be

$$C_a \equiv E\{C_{upa}\} = E\{C_{va}\} = S E\{V_{(N-S,N)}\} + (N-S) v_{max} \tag{10}$$

For $N \leq S$, it is trivial to show that,

$$\begin{aligned} p_{upa} = p_{va} &= v_{min} \\ R_a \equiv E\{R_{upa}\} = E\{R_{va}\} &= N v_{min} \quad (11) \\ C_a \equiv E\{C_{upa}\} = E\{C_{va}\} &= N v_{min} \quad (12) \end{aligned}$$

2) Discriminatory-price auction: The discriminatory-price auction mechanism is the multi-unit counterpart of the single-unit *first-price* auctions. Vickrey, already in [8], showed that the expected revenue for all multi-unit auctions with single-unit demand featuring the same allocation rule is the same, a demonstration of the *revenue equivalence principle*. Therefore,

$$\begin{gathered} p_{dpa}(v) = p_{upa}(v) = p_{va}(v), \\ R_a = E\{R_{dpa}\} \text{ and } C_a = E\{C_{dpa}\} \end{gathered} \tag{13}$$

For $N > S$, the equilibrium bidding strategy equals

$$\begin{aligned} \beta_{dpa}(v) &= E\{V_{(N-S,N-1)} | V_{(N-S,N-1)} < v\} \\ &= \frac{1}{F_{V_{(N-S,N-1)}}(v)} \int_{v_{min}}^{v} y \cdot f_{V_{(N-S,N-1)}}(y) dy \end{aligned} \tag{14}$$

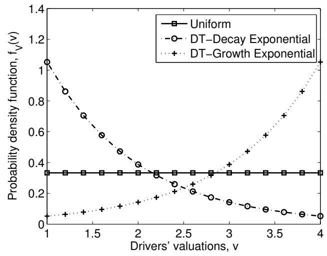
Otherwise,

$$\beta_{dpa}(v) = v_{min} \tag{15}$$

IV. NUMERICAL RESULTS

In Sections II and III we have outlined the game formulations of the two main practices in managing on-street public parking space and derived the equilibrium behaviors they induce. Under conventional uncoordinated search for on-street public parking, drivers have the chance to pay a lower parking fee when they succeed in getting a public parking spot. However, they run the risk of paying a normalized per-hour cruising cost $\delta c_{pub,s}$, on top of the private more expensive parking fee, when they fail to seize a public parking spot and, eventually, drive to a private parking lot. On the other hand, the auctioning of public parking places exploits the diverse drivers' personalities and level of interest for parking and allows for higher payments for public parking space, while saving the "price of anarchy" paid in the absence of coordination as under the aforementioned game formulation. In this section, we explore how different pricing schemes and the drivers' personalities and interest in parking (as captured in their valuation distributions) affect a) the revenue achievable by the public parking service operator; and b) the resulting per-driver expected cost of the parking service, under the two radically different paradigms of parking space management.

[2]In general, reserve prices introduce a positive probability that the auctioned object remains unsold impacting on the efficiency of the mechanism. Herein, however, this event is excluded, since drivers' bids range in $[c_{pub,s}, c_{priv}]$.



Distribution of drivers' valuations	$f_V(v)$
Uniform	$\frac{1}{v_{max}-v_{min}}$
Doubly-truncated *decay* exponential	$\frac{e^{-v}}{e^{-v_{min}}-e^{-v_{max}}}$
Doubly-truncated *growth* exponential	$\frac{e^{v}}{e^{v_{max}}-e^{v_{min}}}$

Fig. 1. Probability density functions for drivers' valuations of public parking spots, $c_{pub,s} = 1$, $\beta = 4$.

For the pricing policy, we adopt values used in the municipal parking system in the city of Athens [9]. In particular, $c_{pub,s} \leq 2$ €, and $\beta \leq 7$, for 60-minute period. The cruising cost parameter δ is allowed to range in $(0, 10]$. On the other hand, we consider three alternatives for the distribution of the drivers' valuations, $f_V(v)$. In all three of them, V lies within an interval $[v_{min}, v_{max}] = [c_{pub,s}, \beta c_{pub,s}]$, yet the mass of the distribution is spread differently over this interval (see Fig. 1):

Doubly-truncated decay exponential valuations: This instance of valuation function corresponds to scenarios, whereby drivers are not willing to pay high for a parking spot. It could model driving in the center during leisure hours, where the acquisition of a parking spot is less urgent. The moments of the $(N-S)^{th}$ order statistic can be computed numerically through the recurrence relations derived by Joshi in [10].

Doubly-truncated growth exponential valuations: The mass in this valuation distribution is concentrated towards the rightmost values of its support. Compared with the doubly-truncated decay exponential distribution, this one can model driving in the city center during busy hours for business purposes.

Uniform valuations: This is the intermediate scenario, where the valuation of parking spots for individual drivers may lie anywhere in $[v_{min}, v_{max}]$ equiprobably. In this case, the expected value of the $(N-S)^{th}$ order statistic can be also computed through the mean value of the generalized Beta distribution $f(v; N-S, S+1)$, for $v \in [v_{min}, v_{max}]$, that is,

$$E\{X_{N-S,N}\} = v_{min} + \frac{N-S}{N+1}(v_{max} - v_{min}) \tag{16}$$

We consider medium to high parking demand levels (up to 160 drivers) and limited public parking supply ($S = 20$ spots) during the time window over which the parking requests are issued.

Figures 2a and 2b plot the aggregate drivers' cost as a function of the parking demand intensity (*i.e.*, number of drivers, N), under the distributed parking spot selection game and centralized parking auctioning system, respectively. In line with intuition, the aggregate drivers' cost increases with the parking demand under both parking allocation approaches. Under the distributed game (see Fig. 2a), the aggregate drivers' cost grows as the penalty cost for cruising between the public and private parking facilities (*i.e.*, δ) increases. Under the auctioning system (see Fig. 2b), the valuation distribution induces the following ordering of the aggregate drivers' costs (also formally proven in the Appendix)

$$C_a^g \geq C_a^u \geq C_a^d \tag{17}$$

where the superscripts g, u and d indicate quantities derived under growth exponential, uniform and decay exponential valuations, respectively.

On the parking operator's side, the revenue from auctioning the public parking spots exceeds that under the fixed-cost distributed parking service provision (see Fig. 2c). This is expected since the same number of drivers park in public space under both practices and these drivers pay *at least* $c_{pub,s}$ in the first case, while they pay *exactly* $c_{pub,s}$ in the latter case. The operator exploits the differentiated drivers' interest in the lower-cost public parking space and adapts the payments to what they are willing to pay for it. Thus, the revenue under the three valuation distributions is strictly ordered, with the growth exponential valuations inducing the highest revenue values and the decay exponential valuations the lowest values (ref. Appendix).

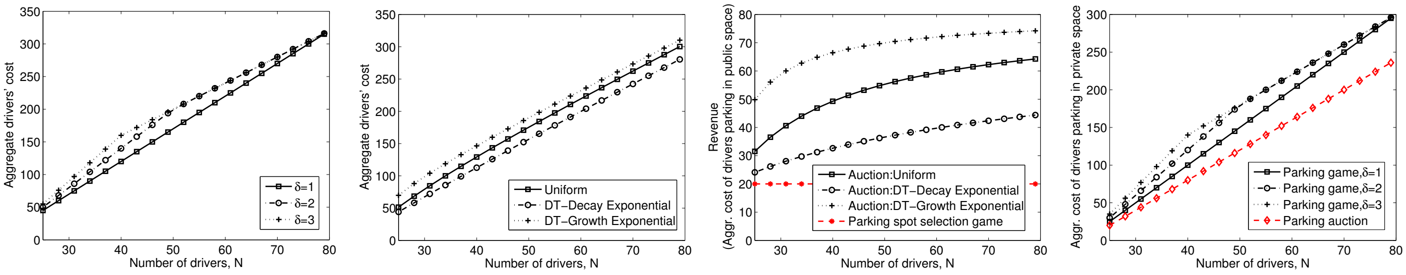
On the drivers' side, the picture is mixed as some drivers pay more and some pay less for public parking space under the auctioning system. Specifically, on one hand, the aggregate cost of the drivers parking in public space (*i.e.*, operator's revenue) under the auctioning system exceeds that under the parking spot selection game, as Figure 2c illustrates. On the other hand, the aggregate cost of the drivers parking in private space under the auctioning system is lower than that under the parking spot selection game, as shown in Figure 2d. This is due to the fact that under the auctioning system all bidders that are not awarded public parking spots enjoy the benefits from the coordination of drivers' parking spot selection, avoid the "price of anarchy" and end up paying the same fixed cost c_{priv}, irrespective of their valuations.

Overall, when the excess cost (in terms of fuel and time wasted on cruising) due to the lack of coordination in the distributed parking game, exceeds the excess cost from bidding *over* the fixed minimum cost $c_{pub,s}$ (collected by the operator), *both* the drivers and the operator are doing better under the auctioning system. Otherwise, the distributed parking spot selection represents a less expensive practice for the drivers. In the remainder of this section, we compare the per-driver cost under the two parking space management practices and explore the conditions on the number of drivers and the cruising, public and private costs under which the aforementioned win-win situation emerges in the auctioning practice.

Let Δ denote the difference between the per-driver cost under the conventional distributed parking spot selection game, C_g/N, and its counterpart under the centralized auction-based allocation, C_a/N, that is,

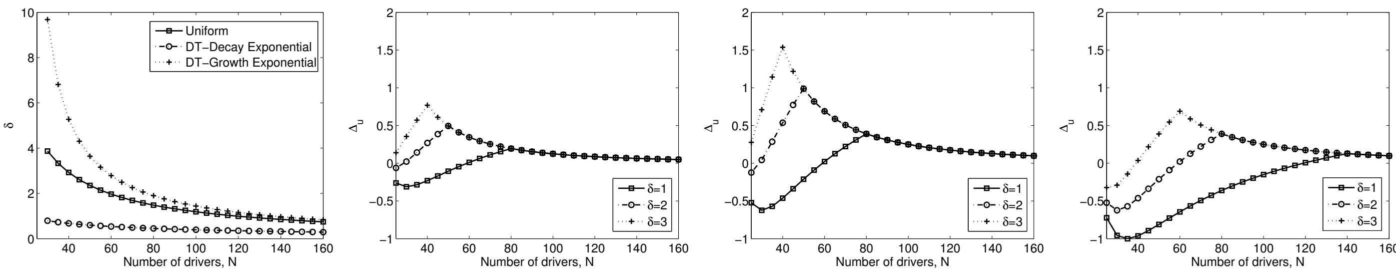
$$\Delta = \frac{1}{N}(C_g - C_a) \tag{18}$$

For $\Delta > 0$ ($\Delta < 0$), this difference expresses the *excess* cost that drivers pay in the parking spot selection game (auctioning system) compared to the auctioning system (parking spot selection game). Drivers are indifferent over the two approaches for $\Delta = 0$.



a. b. c. d.

Fig. 2. Aggregate drivers' costs and revenues as functions of the number of drivers, $c_{pub,s} = 1, \beta = 4$: (a) Aggregate drivers' cost under the parking spot selection game (for $\delta \in \{1, 2, 3\}$); (b) Aggregate drivers' cost under the auctioning system (for the three valuation distributions); (c) Operator's revenue under the auctioning system and the parking spot selection game; (d) Aggregate cost of the drivers parking in private parking space under the auctioning system and the parking spot selection game.



a. $\beta = 7, c_{pub,s} = 1$ b. $\beta = 4, c_{pub,s} = 1$ c. $\beta = 4, c_{pub,s} = 2$ d. $\beta = 7, c_{pub,s} = 1$

Fig. 3. (a): Values of cruising cost that zero difference Δ for different number of drivers; (b), (c), (d): Difference Δ_u as a function of the number of drivers, under various pricing schemes.

Case $N > N_0$: By equations (4) and (18) we have that

$$\begin{aligned}\Delta &= \frac{1}{N}\left(N\beta c_{pub,s} - C_a\right) \quad (19)\\ &= \frac{1}{N}\left[N\beta c_{pub,s} - [R_a + (N-S)\beta c_{pub,s}]\right]\\ &= \frac{S}{N}\left(\beta c_{pub,s} - E\{V_{N-S,N}\}\right) > 0\end{aligned}$$

since the per-spot expected payment $E\{V_{N-S,N}\}$ is strictly smaller than the cost of private parking space. Therefore, for $N > N_0$, with $N_0 = \frac{S(\gamma-1)}{\delta}$, drivers are always better off with the auctioning system. Consequently, if the demand is high enough, win-win situations always emerge under the auctioning practice.

Case $N \leq N_0$: Unlike the first case, when $N \leq N_0$ (*i.e.*, $\delta \leq \frac{S(\beta-1)}{N-S}$) the picture is not clear from the drivers' perspective. By equations (4) and (18) we have that

$$\Delta = \frac{1}{N}\left[c_{pub,s}[N\gamma - S(\gamma-1)] - C_a\right] \quad (20)$$

Therefore, the two parking assignment options can be equivalent or either of them prevail. In particular, the two parking assignment options become equivalent when

$$\delta = \frac{1}{N-S}\left[\frac{C_a}{c_{pub,s}} + S(\beta-1) - N\beta\right] \quad (21)$$

The cruising cost that achieves equivalence is plotted in Figure 3a as a function of the parking demand. By equation (21) and as shown in Figure 3a, the equivalence is possible by decreasing (increasing) the cruising cost as the parking demand increases (decreases). In addition, the higher the drivers' valuations are, the higher revenue the operator gains, the higher the aggregate drivers' cost the auctioning system induces and, finally, the more the cruising between the area of public and private parking should cost to counterbalance the higher payments of drivers under the auctioning system. This causal relation between valuations and cruising cost parameter is clearly seen in Figure 3a.

In order to proceed further and identify conditions under which the win-win situations emerge (*i.e.*, $\Delta > 0$), we need a specific valuation distribution to consider. In the sequel, we study the difference $\Delta_u = \frac{1}{N}(C_g - C_{a,u})$ between the induced *per-driver* expected cost under the parking spot selection game and that under the auctioning system with uniformly distributed drivers' valuations. By equations (4), (10) and (16), with $[v_{min}, v_{max}] = [c_{pub,s}, \beta c_{pub,s}]$, the difference Δ_u is

$$\Delta_u = \begin{cases} c_{pub,s}\frac{(N-S)}{N}\left[\delta - (\beta-1)\frac{S}{N+1}\right], \text{if } N \leq N_0 \\ c_{pub,s}(\beta-1)\frac{S(S+1)}{N(N+1)}, \text{if } N > N_0 \end{cases} \quad (22)$$

As Figures 3b, c, d and equation (22) suggest, the shape of Δ_u function is primarily determined by the relation between the number of drivers N and the number $N_0 = \frac{S(\gamma-1)}{\delta}$. The turning point at $N = N_0$ is shifted to the left as (a) the public parking capacity, S, decreases; or (b) the cruising cost, δ, increases; or (c) the cost of private parking space, β, drops.

Impact of number of drivers: For given public parking capacity and charging parameters and if $N > N_0$, we have already shown that drivers always prefer the auctioning system (*i.e.*, $\Delta_u > 0$). However, as Figures 3b, c and d also illustrate, Δ_u approaches zero as the demand increases which suggests that the auctioning system will have marginal advantage, irrespective of the applied charging scheme. Indeed, the difference Δ_u is strictly decreasing with N since,

$$\frac{\vartheta\Delta_u}{\vartheta N} = -c_{pub,s}(\beta-1)\frac{S(S+1)(2N+1)}{[N(N+1)]^2} < 0 \quad (23)$$

On the contrary, under lower parking demand ($N \leq N_0$), no scheme dominates. Drivers end up paying less on average under the auctioning scheme only if $\frac{S(\beta-1)}{\delta} - 1 < N \leq N_0$.

Impact of cruising, private and public parking costs: For given parking demand and supply, Δ_u increases with the cruising cost, δ, as shown in Figures 3b, c, d and captured in equation (24),

$$\frac{\vartheta \Delta_u}{\vartheta \delta} = \begin{cases} c_{pub,s} \frac{(N-S)}{N} > 0, \text{if } \delta \leq \frac{S(\beta-1)}{N-S} \\ 0, \text{if } \delta > \frac{S(\beta-1)}{N-S} \end{cases} \tag{24}$$

The dependance of Δ_u from the private parking cost, β, can be analyzed from equation (25). Namely, Δ_u increases with β under high parking demand (*i.e.*, $N > N_0$ or equivalently $\beta < 1 + \frac{\delta(N-S)}{S}$), thus motivating more drivers to compete for the scarce on-street parking space and increasing the "price of anarchy" of the uncoordinated parking search. However, under low-to-medium parking demand (*i.e.*, $N \leq N_0$ or equivalently $\beta \geq 1 + \frac{\delta(N-S)}{S}$), any increase in β raises the payments in the auctioning system and hence, reduces the advantage of saving the cruising cost. This trend is also shown in Figures 3b, d.

$$\frac{\vartheta \Delta_u}{\vartheta \beta} = \begin{cases} c_{pub,s} \frac{-S(N-S)}{N(N+1)} < 0, \text{if } \beta \geq 1 + \frac{\delta(N-S)}{S} \\ c_{pub,s} \frac{S(S+1)}{N(N+1)} > 0, \text{if } \beta < 1 + \frac{\delta(N-S)}{S} \end{cases} \tag{25}$$

Finally, by equation (26) we infer that Δ_u increases as the public parking gets more expensive (cheaper), while the distance between public and private parking is significant (close). Namely, drivers benefit from the auctioning system if the cruising cost outweighs the cost of bidding over a higher reserve price. This effect is also shown in Figures 3b, c.

$$\frac{\vartheta \Delta_u}{\vartheta c_{pub,s}} = \begin{cases} (\beta-1)\frac{S(S+1)}{N(N+1)} > 0, \text{if } \delta > \frac{S(\beta-1)}{N-S} \\ \frac{(N-S)}{N}\left[\delta - (\beta-1)\frac{S}{N+1}\right] \geq 0, \text{if } \frac{S(\beta-1)}{N+1} \leq \delta \leq \frac{S(\beta-1)}{N-S} \\ \frac{(N-S)}{N}\left[\delta - (\beta-1)\frac{S}{N+1}\right] < 0, \text{otherwise} \end{cases} \tag{26}$$

V. Conclusions

In this paper, we propose auction-based mechanisms for allocation of public parking space and analyze their effectiveness in terms of the induced drivers' cost and achievable revenue by the public parking operator. These mechanisms are compared against the conventional uncoordinated parking space search with fixed parking service cost. While the operator profits from auctioning the public parking resources, exploiting the diverse drivers' personalities and interest in parking (as captured by their valuation distributions), the comparative study reveals that drivers also benefit due to the savings of the "price of anarchy". A detailed analytical study determines the conditions under which such win-win situations emerge. It turns out that this is always the case under high parking demand.

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Appendix

We want to prove that under the auction-based mechanism, there is a strict ordering of the drivers' expected payments (*i.e.*, operator's revenue) with respect to the three valuation functions, that leads to the following ordering

$$C_a^g \geq C_a^u \geq C_a^d \tag{27}$$

Equivalently, we want to derive a similar relationship for the $(N-S)^{th}$ order statistics of the three valuation functions.

The proof proceeds in three steps. Firstly, we note that there are first-order stochastic dominance relationships between the three cumulative distribution functions, that is

$$F_V^g(v) \prec F_V^u(v) \prec F_V^d(v) \tag{28}$$

as can be readily seen in the following Figure.

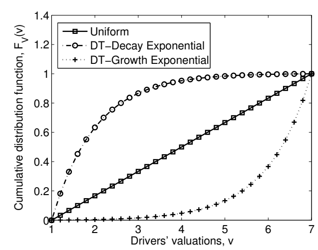

Fig. 4. Stochastic ordering of the three valuation functions $F_V(v)$, ($v_{min} = 1$, $v_{max} = 7$).

Then, we need to recall that the cumulative distribution function of the $(N-S)^{th}$ order statistic is written [7]

$$F_{(N-S,N)}(v) = \int_0^{F(v)} \frac{N!}{S!(N-S-1)!} t^{N-S-1}(1-t)^S dt \tag{29}$$

Therefore, the first-order dominance relationships in the drivers' valuations as given in (28) is inherited by their $(N-S)^{th}$ order statistics. As a result it holds that,

$$F^g_{V_{(N-S,N)}}(v) \prec F^u_{V_{(N-S,N)}}(v) \prec F^d_{V_{(N-S,N)}}(v) \tag{30}$$

Finally, the ordering in (27) emerges directly when relating the expected values of the valuations to their cumulative distribution functions through a general relation concerning non-negative RVs [11],

$$E\{X_{(N-S,N)}\} = \int_0^{\infty} \left(1 - F_{(N-S,N)}(x)\right) dx \tag{31}$$