Contention issues in congestion games

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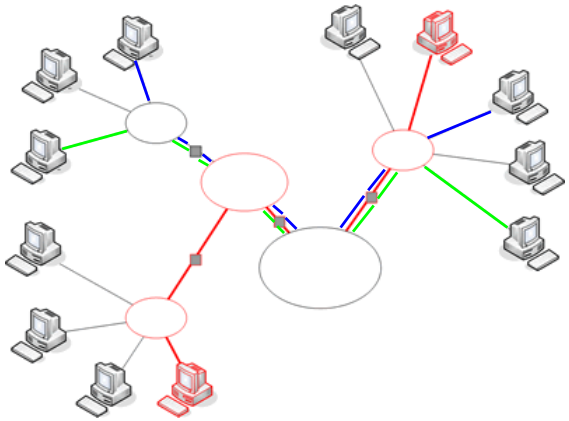
Motivation

- Games in which players can time their participation
  with the hope that fewer players will compete for the same resources.
  - TCP congestion control policy is such a strategy

A first step to the study of the important class of
congestion games with time-dependent strategies.

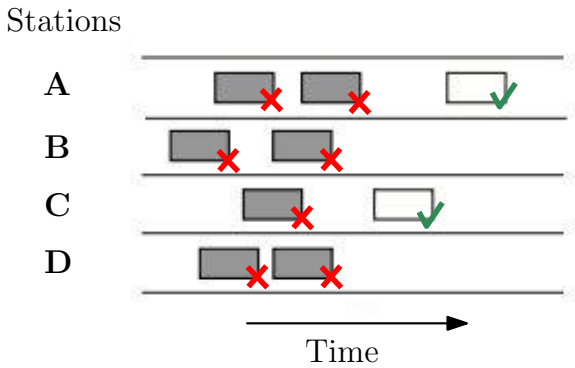
Congestion and Contention

Congestion
as in Internet routing



resource sharing ⇒ higher cost
Strategy: Set of resources

Contention
as in Ethernet / wireless protocols



resource sharing ⇒ nobody succeeds
Strategy: Timing

In between:

The cost depends on **both** the set of selected resources **and** timing (eg. TCP).

⇓

Strategy: Set of resources + Timing

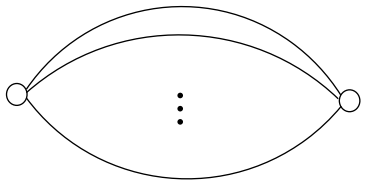
Our game-theoretic abstraction

Congestion game with time dimension

- Strategy: which path to use and when (probability $p_{e,t}$)
- Payoff: depends on the number of users using the same links at the same time

Assumptions:

- underlying network: A set of parallel links with affine latencies.



link e, k users

cost for each user: $\ell_e(k) = a_e k + b_e$

- strategies: non-adaptive, *symmetric*

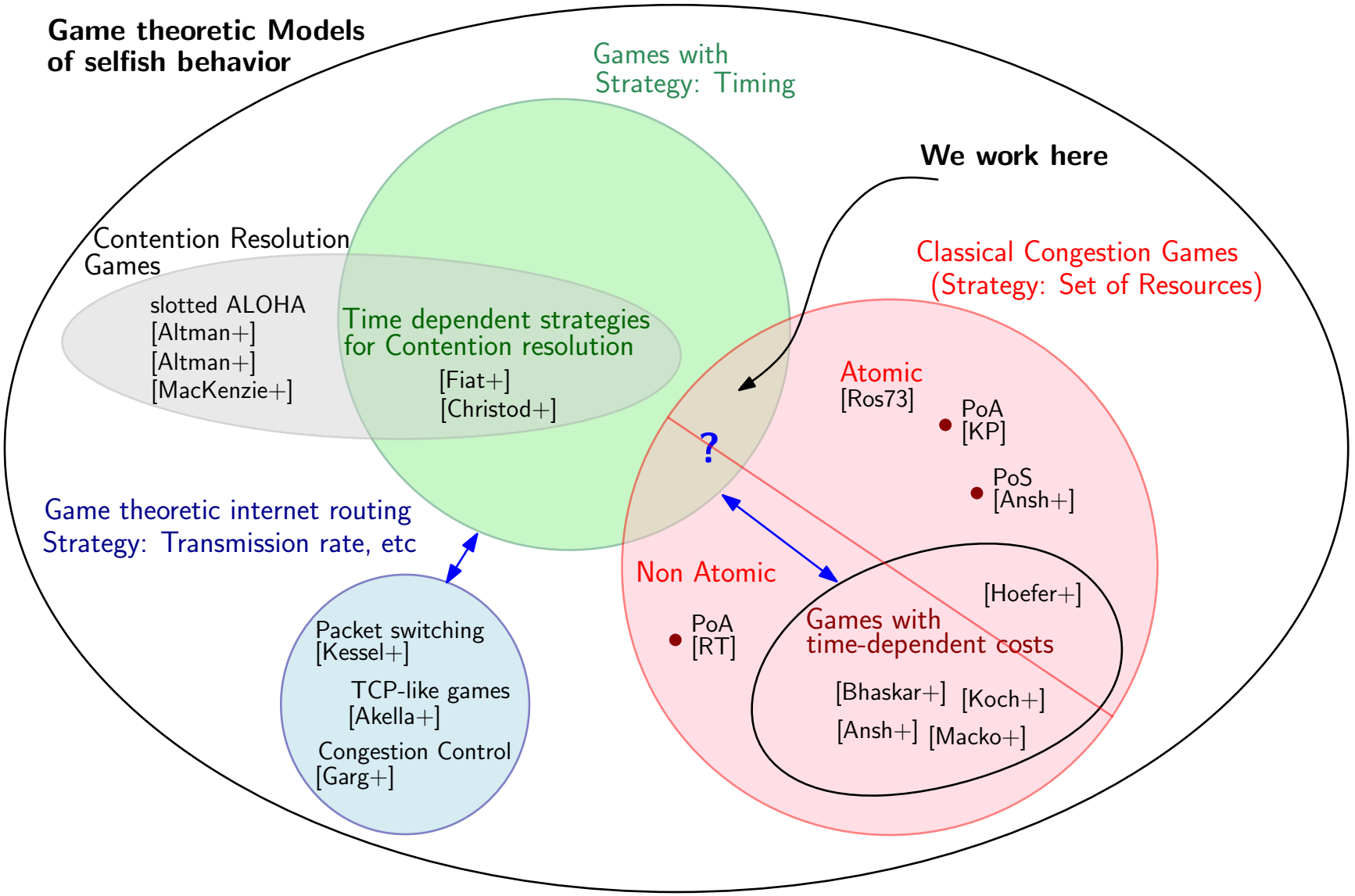
Outline

1. Related Work

2. Our Work
   - latency models & derived games
   - structural properties of conveyor belt games
   - study of symmetric Nash equilibria in boat model
   - study of symmetric Nash equilibria in conveyor belt model

3. Open Problems

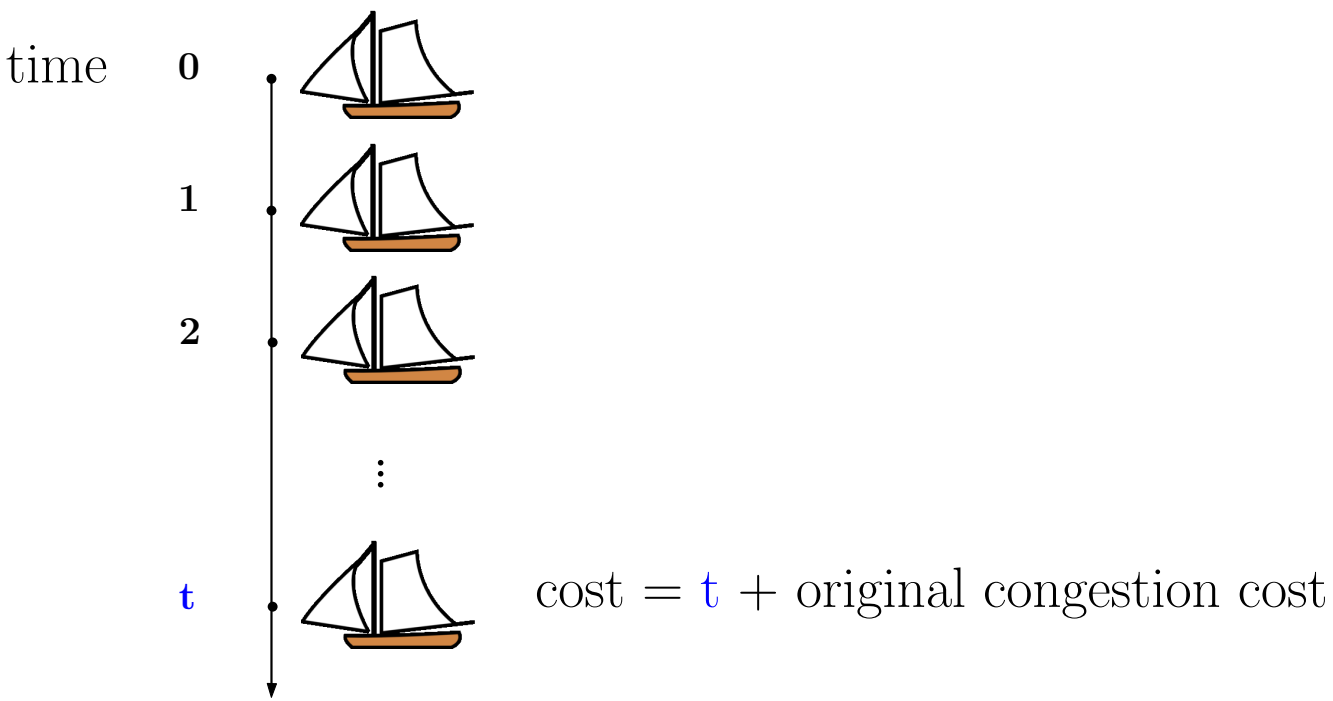
Related Work



Boat model

the latency of a player is influenced only by the players that start at the same time

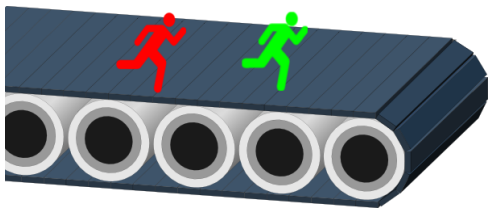
for each link:

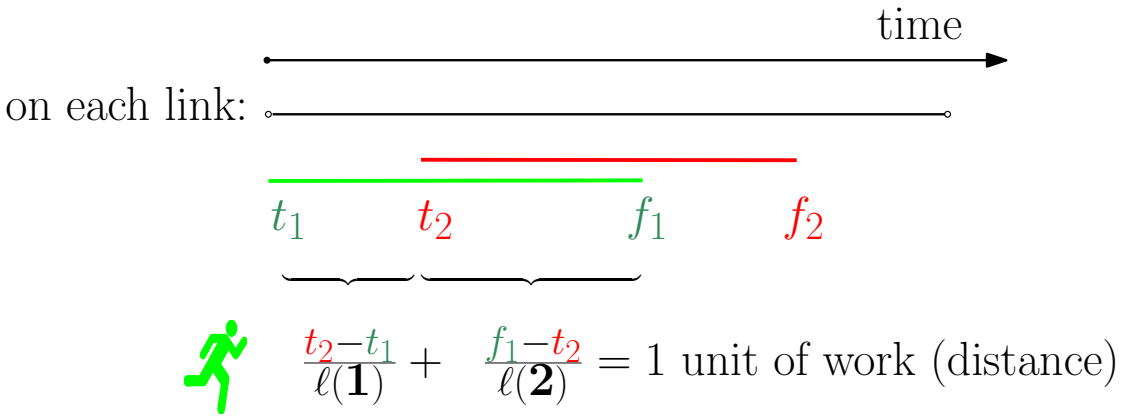


The speed of each boat depends only on the number of players on it.

Conveyor belt model

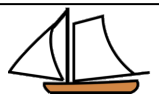
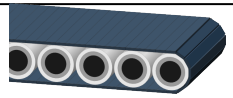
the latency of a player is affected by the players that share the system, even if they started earlier or later





The speed depends on the number of people on the belt:
During each time step, if k players use this link, each one completes work of $\frac{1}{\ell(k)}$.

Our results at a glance

	Boat 	**Conveyor belt** 
congestion game	✓	only for 2 players
existence of pure NE	✓	not always
exact network topology matters?	No	Yes
nature of symmetric NE	unique, probabilities drop linearly with time	
nature of optimal symmetric solution	structure that resembles the NE	
PoA, PoS	small (1.06)	

Are the conveyor belt games congestion games?

Only 2 player conveyor belt games are congestion games!

- For 2 players and arbitrary networks, there is a potential function.

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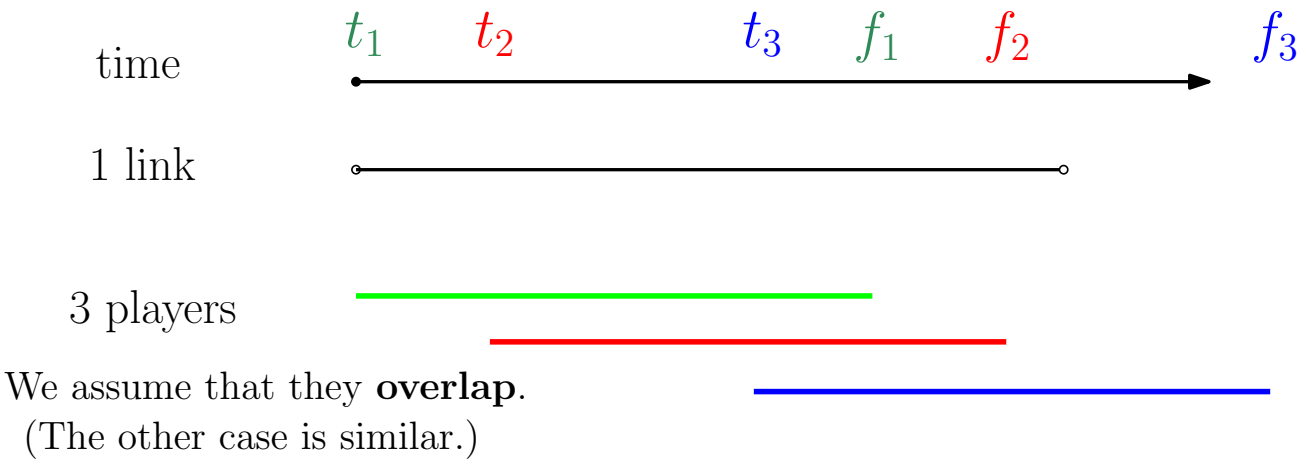
Only 2 player conveyor belt games are congestion games!

- For 2 players and arbitrary networks, there is a potential function.
- For 3 or more players (even on a single link): There are games that have no pure (asymmetric) equilibria, so they are not in general congestion games.

Conveyor belt games may not possess pure NE

No pure NE for 3 players on one link with $\ell(k) = 5k - 1$!



We assume that they **overlap**.
(The other case is similar.)

$$\underbrace{\frac{t_2 - t_1}{\ell(\mathbf{1})}}_{} + \underbrace{\frac{t_3 - t_2}{\ell(\mathbf{2})}}_{} + \underbrace{\frac{f_1 - t_3}{\ell(\mathbf{3})}}_{} = 1$$

& similar equations for the other 2 players

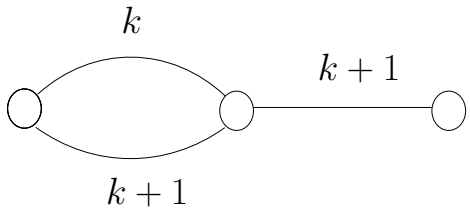
Best strategy for player 3: select $t_3 \geq f_1$ (no overlap - contradiction).

$\Downarrow$

There are not finish times that satisfy this game's constraints.

In conveyor belt games the network topology matters

In conveyor belt games, a user's cost depends on the underlying network topology.



Consider 2 players.

- They finish at $f_1 = 7/2, f_2 = 9/2$.

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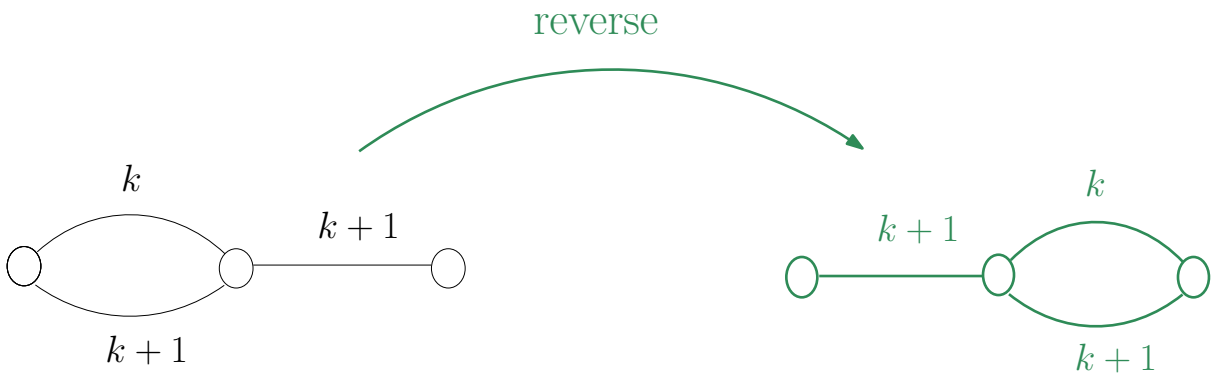


Consider 2 players.

- They finish at $f_1 = 7/2, f_2 = 9/2$.
- On the reversed they finish at $f_1' = 4 > f_1, f_2' = 5 > f_2$.

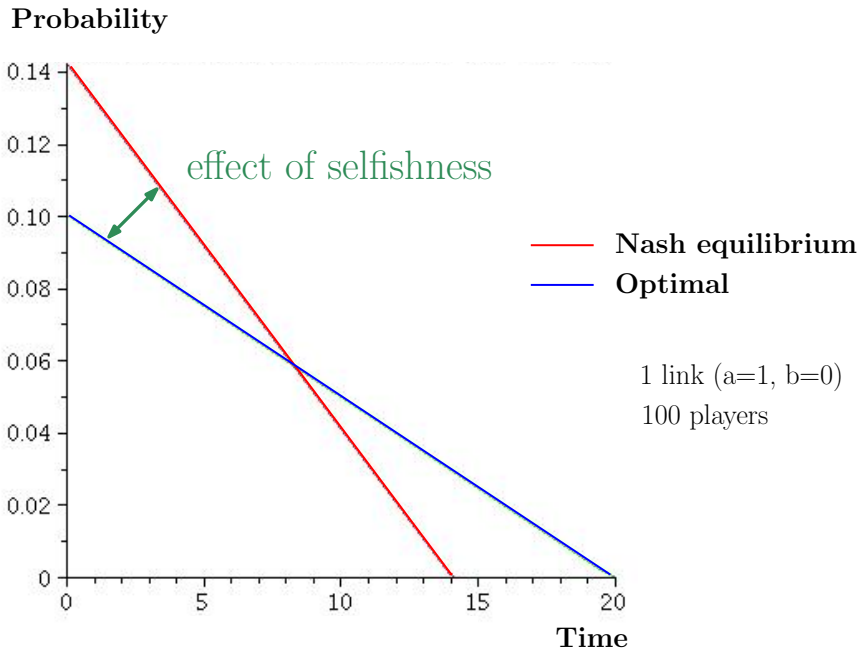
$\Downarrow$

The finish time of each player is not the same in these two networks!

The structure of symmetric mixed Nash Equilibria and optimal non-selfish solution in boat model

Both of them:

- are unique
- in each link the probabilities drop linearly with time

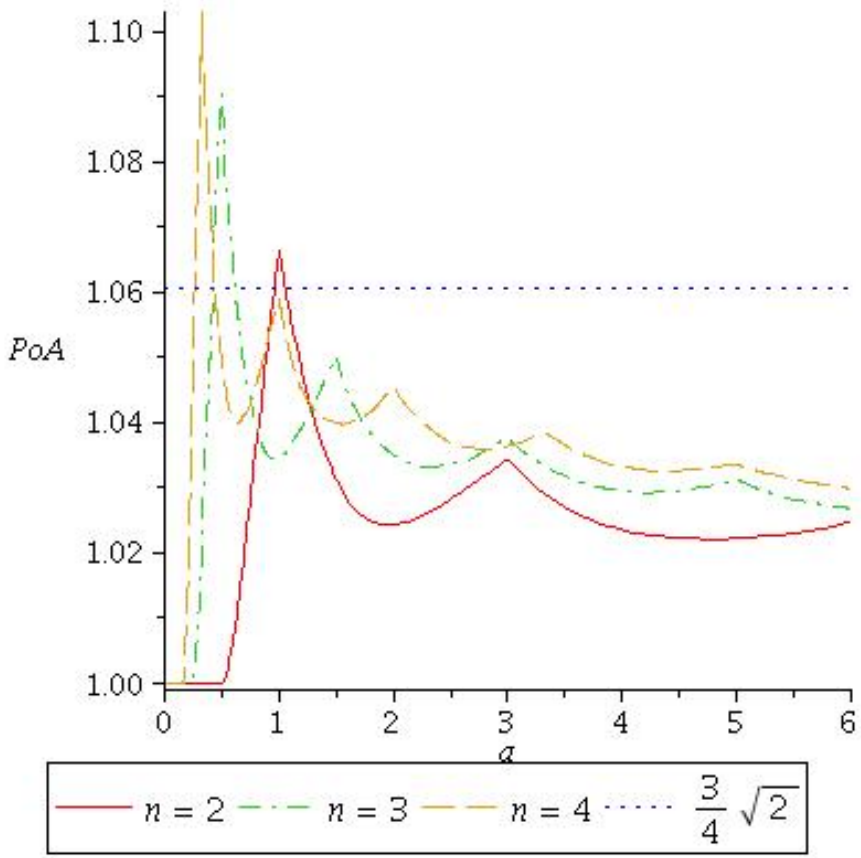


- We observe the bicriteria relation (also in [RT02]).
- Users are more greedy in NE than in OPT in the beginning of the game.

PoA/PoS of symmetric strategies in boat model

For a fixed network,

- the PoA tends to $3\sqrt{2}/4 \approx 1.06$ (assuming that number of players $\to \infty$)
- for a fixed number of players:



Structure of symmetric mixed Nash Equilibria and optimal non-selfish solution in conv. belt model (2 players)

Very similar to boat model, BUT:

the probabilities here are non-zero only at multiples of $\ell(1)$.

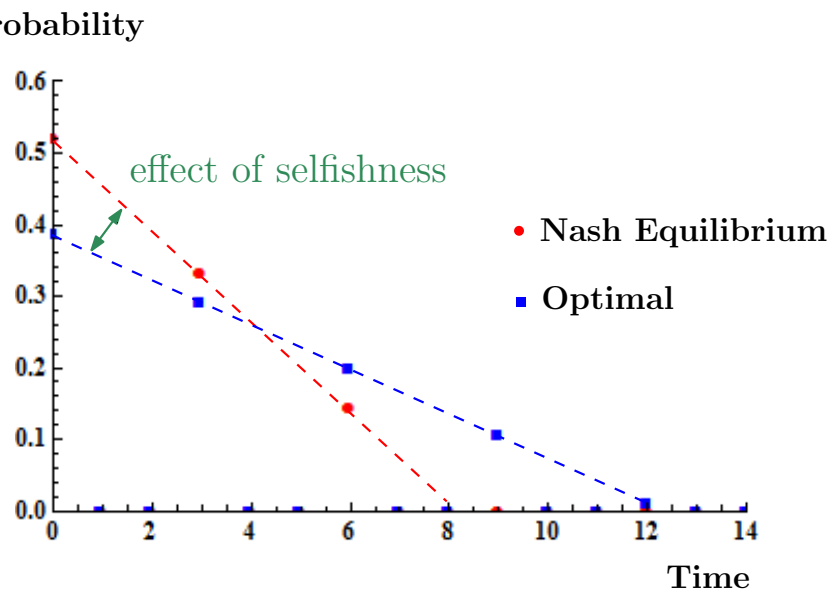
$\Rightarrow$ Either do not overlap or start together!

1 link

latencies:

$\ell(1) = 3$

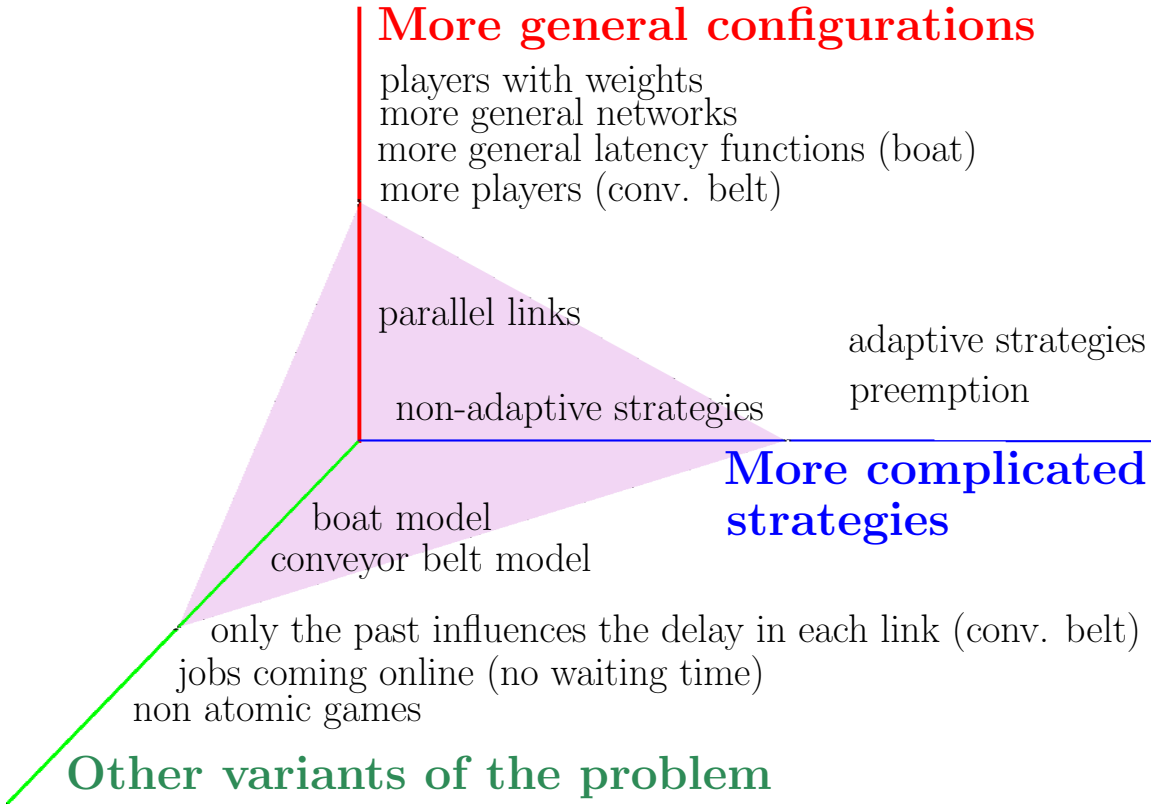
$\ell(2) = 19$



- Bicriteria relation.
- For a fixed network, PoA tends to $3\sqrt{2}/4 \approx 1.06$ (assuming latency $\to \infty$)

Open problems - ongoing work

Essentially any issue not covered in this talk is open!



Ongoing work:

- general networks
- adaptive strategies.

thank you for your attention