System-Independent Threshold and BER Estimation in Optical Communications Using the Extended Generalized Gamma Distribution

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Abstract

Two important tasks with respect to the optimized configuration of an optical communications system are those of the performance evaluation and the receiver decision threshold estimation. In this paper, a new training-based BER and threshold estimation technique is proposed relaxing the assumption of Gaussian distributed received signals. The proposed method is similar in philosophy to the Gaussian Approximation one, and is system-independent and simulation-based. This means that the probability density function (pdf) of the sampled electrical current is estimated based on training data provided via simulations without any assumptions on the specific configuration of the communications system under consideration. The novelty of the paper is that for the first time a combination of a generalized form of the gamma distribution together with the noncentral chi-square distribution have been used for the modelling of the pdfs of the spaces and the marks respectively.

I. Introduction

The performance evaluation of optical fiber communication systems is of high interest since it can be used to find the optimum combination of system components. These include the type of fibers and their ordering in the final fiber, the pulse shapes, the optical and electrical filters of the receiver and the proper concatenation of the optical amplifiers. An other important issue which is related to the system performance is the optimum selection of the decision threshold.

Lightwave communications have the advantage of achieving very low bit error rates (BER). As a result, the BER is practically impossible to estimate straightforwardly with Monte Carlo (MC)

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simulations[1]. Thus, several efforts have been made for efficient bit error probability estimation, using either analytical or semi-analytical methods. An other important parameter which needs to be estimated for amplitude modulation signalling formats is the threshold value at the receiver end. The accuracy in the estimation of the threshold value is crucial to the actual performance of the receiver.

Apart from the efficiency and accuracy that a BER and threshold estimator should exhibit, another important attribute is the flexibility to incorporate new types of devices easily. With respect to analytical methods, usually a number of simplifying assumptions have to be made, such as ideal non-return-to-zero (NRZ) pulse formats, non-dispersive fibers and ideal, or explicitly specified optical and electrical filters [1], [2], [3]. Recently, more general and accurate approaches have also been presented but they need additional mathematical effort for adaptation to the configuration of the system under consideration and they are usually computationally complex and (e.g., [4], [5]).

Although analytical methods are of great importance, due to their lack of generality and/or intractable mathematically derived solutions which they imply, in practice we usually resort to semi-analytical, training based techniques with the best candidate being the Gaussian approximation (GA) method [6], [7]. The power of the above method is its simplicity and that it can be used independently of the specific configuration of the communication system.

In this paper, a new system-independent threshold and BER estimation technique is proposed. The estimation of the unknown values is realized in two steps in a similar manner to the GA method. In the first step, a number of received samples is generated via simulations. In the second step, the generated samples are used in order to estimate the probability density function (pdf) of the marks and the spaces leading straightforwardly to the computation of the threshold value and the corresponding BER. In contrast to the GA, which assumes that the pdf of the power of the corrupted by noise received pulses is Gaussian, a more general scheme is adopted where the real pdf of the spaces is approximated by the extended generalized gamma distribution (EGGD) and the pdf of the marks is approximated by the noncentral chi-square distribution.

The remainder of the paper is organized as follows: In section II the optical communication

[1]The performance evaluation of a system operating in the error free region (BER= 10^{9}) implies the estimation of about 10^{11} transmitted pulses.

system under consideration is described and in section III a general description of the simulation-based BER and decision threshold estimation methods is given. Sections IV and V deal with the proposed techniques for the pdf estimation of the spaces and the marks distribution correspondingly. Finally, the performance of the proposed methods is shown and compared with that of the GA method in section VI.

II. Optical Fiber receiver Model

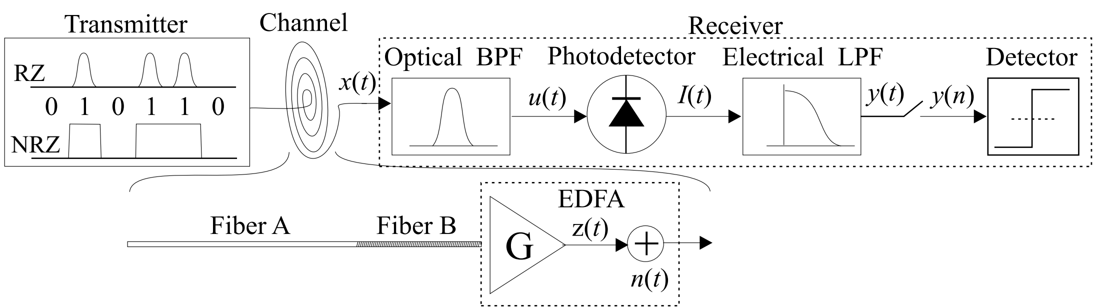

Fig. 1. General diagram of an optically preamplified direct detection communication system.

Fig. 1 shows the diagram of a lightwave communication system under consideration in this paper. The transmitter converts the information message to an On Off Keying (OOK) modulated sequence of either return-to-zero (RZ) or non-return-to-zero (NRZ) optical pulses. The optical channel consists of one or more fibers having specific dispersion characteristics connected in sequence. When the fiber nonlinear distortions are considered to be negligible, which is a fairly accurate in the cases of relatively low transmission powers, the fiber can be well modelled as a filter with frequency response

$$H(\omega) = \exp\left(-\frac{\alpha L_f}{2}\right)\exp\left(j\frac{\beta_2\omega^2 L_f}{2}\right)\exp\left(-j\frac{\beta_3\omega^3 L_f}{6}\right) \tag{1}$$

where, L_f is the fiber length α is the fiber loss and $\beta_2,\ \beta_3$ are the first-order and the second-order dispersion coefficients respectively. The fiber is followed by an erbium doped fiber optical amplifier (EDFA) of gain G which introduces amplified spontaneous emission (ASE) noise, which is the main source of noise in the receiver. It has a nearly white spectral density over the bandwidth of interest, and is well modelled with a complex white Gaussian noise $n(t)$ having a two sided spectral density $N_0/2$. The parameter $N_0 = n_{sp}(G-1)hv$ [8] where h is Planck's constant, v is the frequency of interest and n_{sp} is the spontaneous emission parameter which takes various imperfections into account.

At the receiver, the signal is optically filtered in order to reject the ASE noise frequencies outside the signal spectrum and/or to "extract" the signal under consideration in Wavelength Division Multiplexing (WDM) systems. The photodetector, which is mathematically described as a square-law device $(I(t) = |u(t)|^2)$, converts the optical signal to its electrical counterpart. Finally, a lowpass electrical filter further filters out the ASE noise before the detection of the received samples.

Marcuse [1] and Humblet et. al [2] were the first to derive analytical closed form expressions for the bit error probability of lightwave systems with optical amplifiers, under the assumptions of ideal optical bandpass filter and ideal integrate-and-dump electrical filter. In addition, the amplitude of all the marks (symbol "1") and all the spaces (symbol "0") were assumed to be a constant value and strictly zero respectively. In this ideal configuration, it was found that the received marks obey a noncentral chi-square (NCX2) distribution [9]

$$p_1(y) = \frac{1}{2\sigma^2}\left(\frac{y}{s^2}\right)^{\frac{n-2}{4}} e^{-(s^2+y)/2\sigma^2} I_{n/2-1}\left(\sqrt{y}\frac{s}{\sigma^2}\right),\ y \geq 0 \tag{2}$$

and the spaces obey a central chi-square distribution

$$p_0(y) = \frac{1}{\sigma^n 2^{n/2}\Gamma(\frac{1}{2}n)} y^{n/2-1} e^{-y/2\sigma^2},\ y \geq 0 \tag{3}$$

where $2\sigma^2$ equals to the power spectral density, N_0, of the ASE noise, I_α denotes the αth-order modified Bessel function of the first kind and $\frac{n}{2} = B_o/B_e$ is the number of modes per polarization state in the received optical spectrum, with B_o and B_e being the optical bandwidth and the electrical bandwidth at the receiver, respectively. Furthermore, s^2 indicates the energy of the received signal and $\Gamma(\cdot)$ denotes the gamma function.

Although the Gaussian distribution is not a good approximation of the chi-square distribution, the GA could be justified, due to the central limit theorem, for large values of n. However, this is not the case in practice. Despite this, the minimum probability of error (PE) with respect to different thresholds, estimated by the GA method, is unexpectedly close to the real minimum PE of the system [2]. On the other hand the GA fails in the estimation of the optimum threshold. Due to this inaccuracy of the GA, the ideal chi-square model is still preferred in many cases [10].

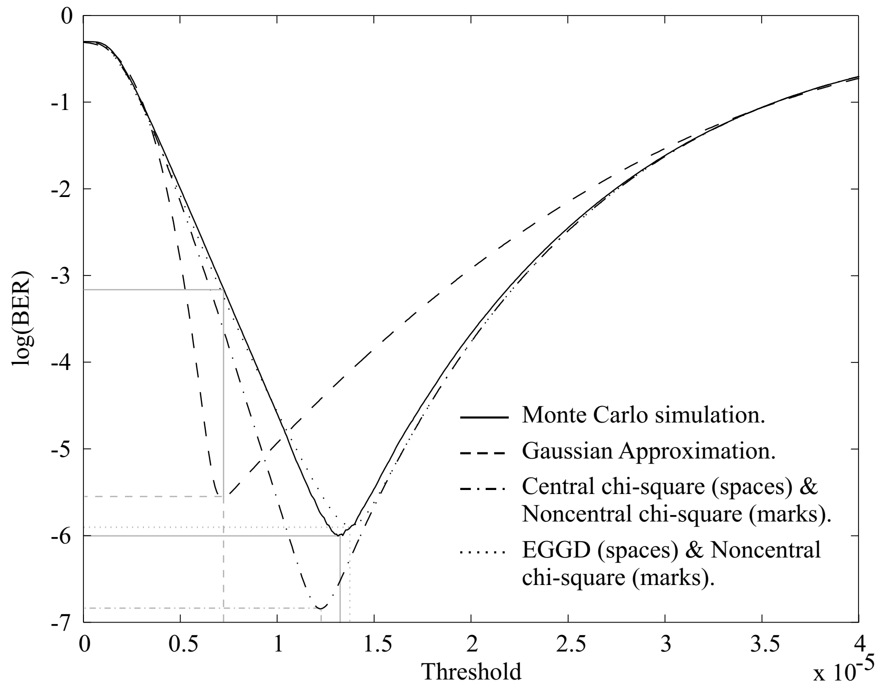

Fig. 2. Probability of error with respect to different threshold values.

However, in real conditions, neither the GA nor the chi-square approximation (CSA) is accurate due to the non-ideal filtering processes which take place at the receiver. Fig. 2 shows the above inaccuracy for a receiver consisting of a Fabri-Pèrot optical filter (FPF) with 3dB bandwidth $B_o = 1.8R$ and a fifth-order electrical filter of bandwidth $B_e = 0.8R$ having a Bessel transfer function. R denotes the transmission rate which in the above example equals to 10 Gbps. The actual performance of the communication system in different threshold values is shown with the solid curve and it is obtained via Monte Carlo (MC) simulations. The dashed curve shows the estimated bit-error-rate (BER) when both the marks and the spaces are modelled with Gaussian distributions. The dashed-dotted curve corresponds to the CSA with central and noncentral chi-square (NCX2) distribution for the modelling of the spaces and the marks respectively. The horizontal lines indicate the minimum BER obtained by the different methods while the vertical lines point the threshold value in which the minimum BER is achieved. Clearly, although the GA estimates the minimum BER closely (4-5 fold increase in BER), this is realized “accidentally” in the sense that if the best GA threshold would be adopted the actual performance of the system would be seriously deteriorated (solid horizontal line).

In this paper, the extended generalized gamma distribution (EGGD) for the spaces and the NCX2 distribution for the marks modelling is adopted since they can be more flexible in the approximation of the actual distributions.

III. Training-based Threshold and BER calculation principle

Under a training-based approach, threshold and BER estimates can be obtained in three successive steps.

In the first step, the optical communication system, including the stages of the transmitter, the receiver and the effects of the transmission medium, is simulated for an a-priori known sequence of transmitted pulses. The received electrical current is baudrate sampled and the resulting samples are grouped in two sets S_0 and S_1 according to the corresponding transmitted pulse (space or mark). The above training sample grouping would be adequate in an idealized system configuration [1]. However, in real lightwave communication systems neither the strictly zero spaces nor the constant marks assumption is valid, due to the dispersive effects taking place during the pulses' transmission and the inter-symbol interference (ISI) caused by the limited filter bandwidth of the optical and electrical receiving filters. As a result, the noiseless received samples are classified in 2^L states[2], depending on the different combinations of L successive pulses with L being the number of pulses which interfere with each other. Thus, if the system under consideration introduces significant ISI, each one of the sets S_0, S_1 need to be further partitioned into 2^{L-1} subsets $S_{0,i}$ and $S_{1,i}$, $i = 1,\ 2, \ldots, 2^{L-1}$ since the data of these subsets are distributed differently to each other.

In the second step, the pdfs of the data sets are estimated using either parametric estimation, such as the GA method, where the data are assumed to be distributed in accordance with specific models, or non-parametric techniques like histogram or Monte Carlo simulation methods. In this paper we focus on parametric estimation since it is generally faster with respect to the required training samples. After the second step, 2^L pdfs, $f_{0,i}$ and $f_{1,i}$, $i = 1,\ 2, \ldots, 2^{L-1}$ have been fitted to the data of the subsets. Note that the pdfs in their parameterized form are not necessarily the same for all the subsets.

[2]Half of them correspond to transmitted space and the rest corresponds to marks

In the final step, as optimum threshold value, δ, estimate is chosen as the one which minimizes the BER and the resulting BER corresponds to this optimum threshold:

$$\begin{aligned} B\hat{E}R &= \underbrace{min}_{\delta}\left\{\frac{1}{2^L}\left(\sum_{i=1}^{2^{L-1}}\int_{\delta}^{\infty} f_{0,i} + \sum_{i=1}^{2^{L-1}}\int_{-\infty}^{\delta} f_{1,i}\right)\right\} && (4)\\ &= \underbrace{min}_{\delta}\left\{\frac{1}{2^L}\left(\sum_{i=1}^{2^{L-1}}(1-F_{0,i}) + \sum_{i=1}^{2^{L-1}} F_{1,i}\right)\right\}, && (5)\end{aligned}$$

where, $F_{0,i}$ and $F_{1,i}$ denotes the cumulative distribution function (cdf) of the samples related to the ith state of the spaces and the marks respectively.

IV. Spaces approximation using the EGGD

The extended generalized gamma distribution[3] is a flexible four-parameter family of distributions, mainly used in reliability theory, with pdf [11]

$$f(y) = \frac{b}{\Gamma(\rho)} a^{-b\rho}(y-c)^{b\rho-1} e^{-\left(\frac{y-c}{a}\right)^b},\ y > c. \tag{6}$$

The distribution has two shape parameters (b and ρ), one scale parameter (a) and one location parameter (c) and includes as special cases the exponential, Weibull, gamma and log-normal distributions.

The idea of using the EGGD for the approximation of the pdfs of the spaces comes from the fact that the chi-square distribution, which is the actual distribution in the ideal case, belongs to a subfamily of the gamma distribution and it can be modelled accurately by the EGGD $(a = 2\sigma^2,\ b = 1,\ \rho = \frac{n}{2},\ c = 0)$. In the case that the lightwave communication system under consideration operates differently to the idealized setting which leads to chi-square distributions, we expect that the four free parameters of the EGGD will allow it to adapt itself to the shape of the actual distributions satisfactorily.

The parameter estimation of the EGGD from a set of training data samples (obtained by simulations) can be realized with several methods where compromises have to be made with respect to accuracy, stability and computational effort. Two techniques are proposed here, the

[3]It is also referred to as generalized four-parameter gamma distribution.

method of moments (MOM), where the distribution is fitted to the data with the aid of the estimates of the up to the first three moments, and an iterative likelihood maximization scheme.

According to the MOM method, tentative estimates for the parameters $a(c),\ b(c)$ and $\rho(c)$ are obtained for a series of trial values of the location parameter $c = c_k,\ k = 1 \ldots K$ solving the system of equations [12]:

$$\frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{\psi^{''}(\tilde{\rho}(c))}{[\psi^{'}(\tilde{\rho}(c))]^{\frac{3}{2}}} \tag{7}$$

$$\tilde{b}(c) = \frac{\mu_2\psi^{''}(\tilde{\rho}(c))}{(\mu_3\psi^{'}(\tilde{\rho}(c)))} \tag{8}$$

$$\tilde{a}(c) = \exp[\mu_1^{'} - \tilde{b}^{-1}(c)\psi(\tilde{\rho}(c))] \tag{9}$$

where $\mu_r^{'},\ \mu_r$ are the rth raw moment and the rth central moment of the vector $\ln(\mathbf{x} - c)$ and $\mathbf{x} = \{x_i,\ i = 1, \ldots, N\}$ is the vector of training data. Moreover,

$$\Psi'(x) = \frac{\partial \ln \Gamma(x)}{\partial x}, \quad \Psi''(x) = \frac{\partial^2 \ln \Gamma(x)}{\partial x^2}$$

are the digamma and trigamma functions respectively. More specifically, for the solution of the above system of equations, the population moments are replaced by sample moments and the order parameter $\tilde{\rho}(c)$ is obtained from eq. 7 by iterative means. Then, estimates for the rest of the parameters are straightforwardly obtained by solving the rest of the equations successively. The final set of estimated parameters based on the MOM is chosen to be the $\hat{a} = \tilde{a}(\hat{c})$, $\hat{b} = \tilde{b}(\hat{c})$, $\hat{\rho} = \tilde{\rho}(\hat{c})$ where $\hat{c}$ is the location value which maximizes the log-likelihood function $L_{max}(c) = L[\tilde{a}(\hat{c}),\ \tilde{b}(\hat{c}),\ \tilde{\rho}(\hat{c}),\ c]$ of the EGGD given by:

$$\begin{aligned} L(a, b, \rho, c) &= N \ln b - N \ln \Gamma(\rho) - \frac{1}{a^b} \sum_{i=1}^{N} (x_i - c) \\ &- N b \ln a + (b\rho - 1) \sum_{i=1}^{N} \ln(x_i - c). \end{aligned} \tag{10}$$

The second techniques is based on the maximization of the log-likelihood of the EGGD. Many difficulties have been reported when the maximization is realized through the gradient of the likelihood, especially when the number of samples is limited as it is the case of lifetime data [11]. In fact, much of the difficulty arises because EGGD distributions with very different sets of parameter values look alike. However, in the case we investigate here, we did not face similar problems. Firstly, the length of the data set can be made arbitrarily long, due to the fact that they are obtained via simulations. Also, instead of using a gradient minimization scheme we obtain the parameter estimates directly using a simplex unconstrained nonlinear optimization method[4] [13]. The difficulties in obtaining maximum likelihood estimates can be further reduced by using a reparameterized form of the EGGD, [14]:

$$f(x) = \frac{\rho^{\rho-1/2}}{\sigma\Gamma(\rho)} \exp\left(\sqrt{\rho}\frac{x-\mu}{\sigma} - \rho e^{(y-\mu)/\sqrt{(\rho)}}\right) \tag{11}$$

where, $x = \ln(y)$, $\sigma = (b\sqrt{\rho})^{-1}$, $\mu = \ln a + \ln\rho/b$.

The likelihood maximization parameter estimation method can be used in order to refine the estimates provided by the MOM if the latter estimates get adopted as the initialization parameter set of the simplex algorithm.

One of the advantages of the use of the EGGD in the standard or in the reparametrized form is that its cumulative distribution function (cdf)

$$F(y) = \frac{\gamma(\rho, \left(\frac{y-c}{a}\right)^b)}{\Gamma(\rho)} \tag{12}$$

which is required to calculate the probability of error, is given as a function of the incomplete and the complete gamma functions given by

$$\gamma(\rho, z) = \int_0^z t^{\rho-1}e^{-t}dt \text{ and } \Gamma(\rho) = \int_0^\infty t^{\rho-1}e^{-t}dt$$

respectively from which accurate numerical solutions can be obtained easily[5].

V. Marks approximation using the Noncentral Chi-square distribution

Although the distribution of the spaces can differ significantly from its “idealized” counterpart which is the central chi-square distribution, we noticed, based on our numerical experience, that in general this is not the case with respect to the distribution of the marks which, in the ideal case, are NCX2 distributed. In fact, the NCX2 distribution models the marks well even if the system configuration is not the ideal one.

[4]The Nelder-Mead Simplex Method is available in MATLAB.

[5]Both for the complete and the incomplete gamma functions as well as for the digamma and trigamma functions there are techniques for their numerical solution implemented in many commercially available software packages like MATLAB.

The three parameters of the noncentral chi-square distribution (eq. 2) are estimated here from training data via closed form equations based on the cumulant generation function leading to cumulants of the form [15].

$$k_r = 2^{r-1}(r-1)!(\sigma^2)^{r-1}(rs^2 + n\sigma^2),\ \ r = 1,\ 2, \ldots . \tag{13}$$

Estimates of at least up to the third order cumulants are needed and they are directly given by the row and central moments, i.e., $k_1 = \mu_1^{'}$, $k_2 = \mu_2$ and $k_3 = \mu_3$. More specifically, an estimate of the parameter s^2 is obtained solving the equation

$$k_3 \hat{s^2}^2 + (2k_3k_1 - 4k_2^2)\hat{s^2} + k_3k_1^2 - 2k_2^2k_1 = 0 \tag{14}$$

and $\hat{\sigma^2}$, $\hat{n}$ are given by equations

$$\hat{\sigma^2} = \frac{k_2}{2(\hat{s^2} + k_1)} \tag{15}$$

$$\hat{n} = \frac{k_1 - \hat{s^2}}{\hat{\sigma^2}}. \tag{16}$$

Since the cdf of the NCX2 distribution is not solvable analytically, here it is computed numerically using the Simpson's rule. However, accurate closed form approximations can be obtained with the aid of the saddlepoint approximation [16].

In the rare cases where the marks distribution had diverge from the NCX2 distribution it turned out the NCX2 had degenerate to a distribution similar to the Gaussian one. In such a case, the above method did not give reasonable estimates for the NCX2 distribution parameters, e.g., $\sigma,\ n,\ s\ >\ 0$. If that happened, the Gaussian approximation for the marks can be used instead.

The dotted curve in Fig. 2 corresponds to the combined approximation with the EGGD the spaces and the NCX2 for the marks.

VI. PERFORMANCE EVALUATION

In order to estimate the optimum threshold value and the BER, the distribution of each one of the states is estimated based on the repetitive transmission of an appropriate de Bruijn training sequence [17] which guarantees that all the states will be represented with the same number of samples.

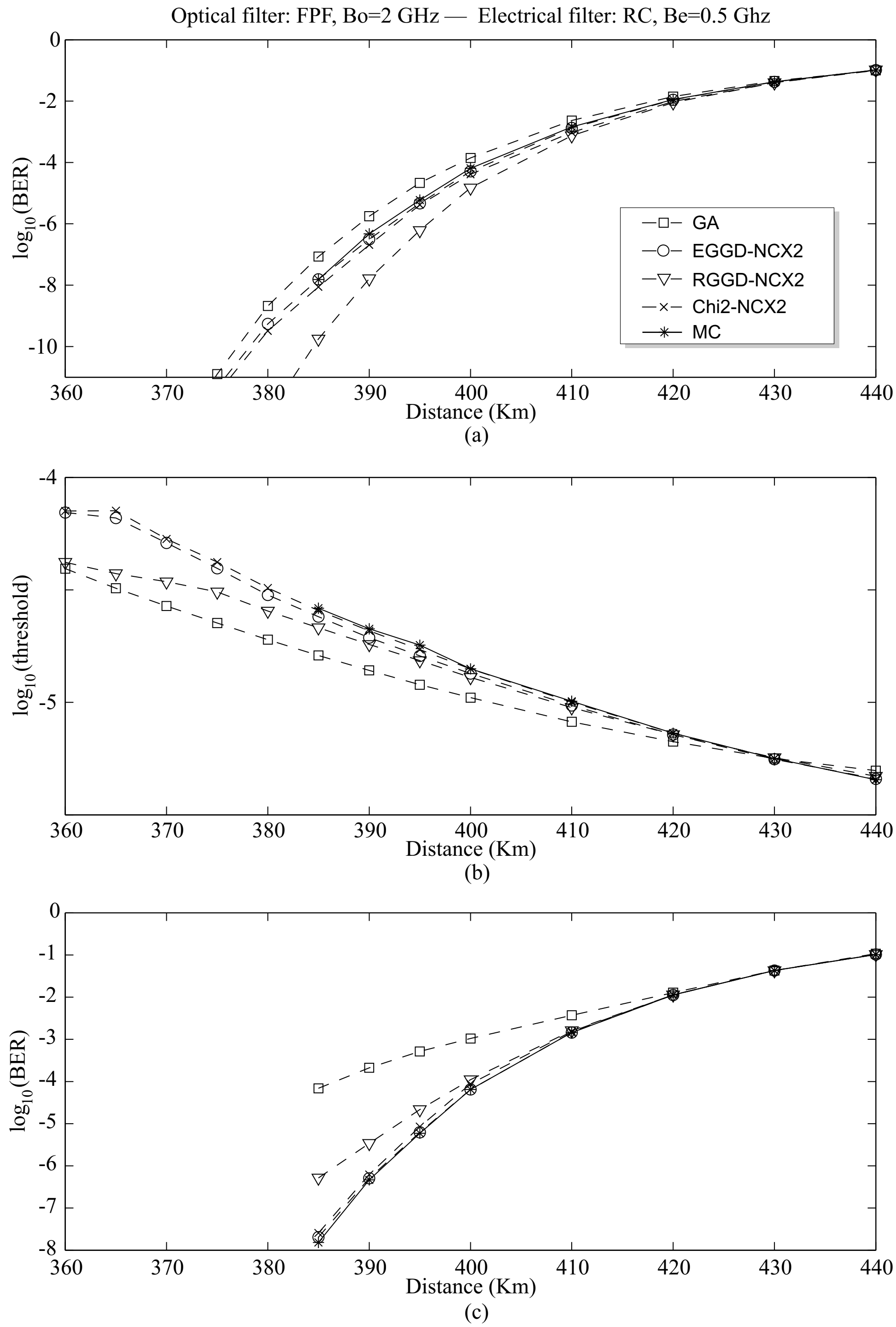

Fig. 3. (a) BER with respect to distance. (b) Threshold estimation with respect to distance. (c) BER achieved by adopting the estimated threshold.

System configuration: FPF optical filter with $2R$ 3-dB bandwidth and RC electrical filter with $0.5R$ 3-dB bandwidth

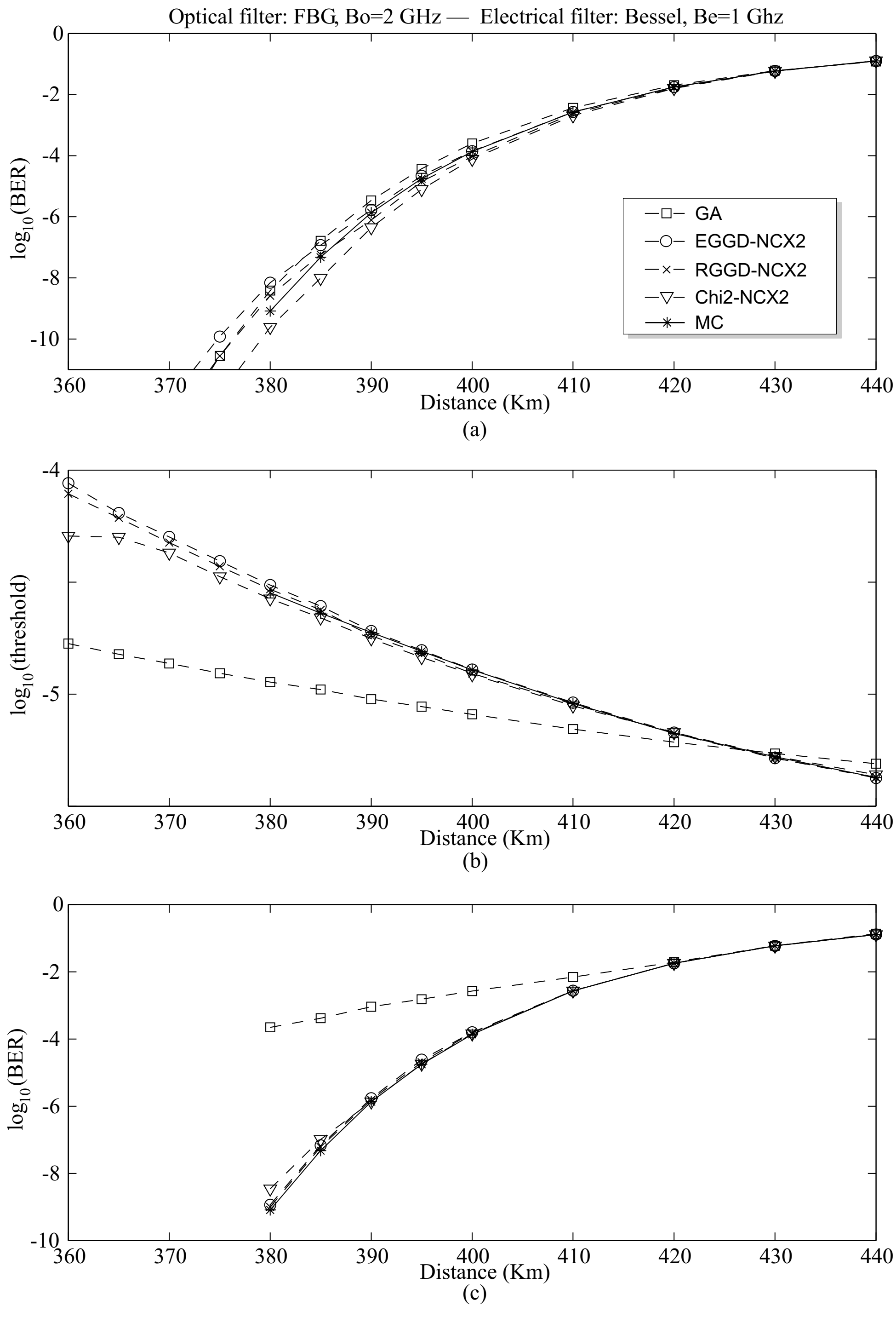

Fig. 4. (a) BER with respect to distance. (b) Threshold estimation with respect to distance. (c) BER achieved by adopting the estimated threshold.

System configuration: FBG optical filter with $2R$ 3-dB bandwidth and 5th order Bessel electrical filter with $1R$ 3-dB bandwidth

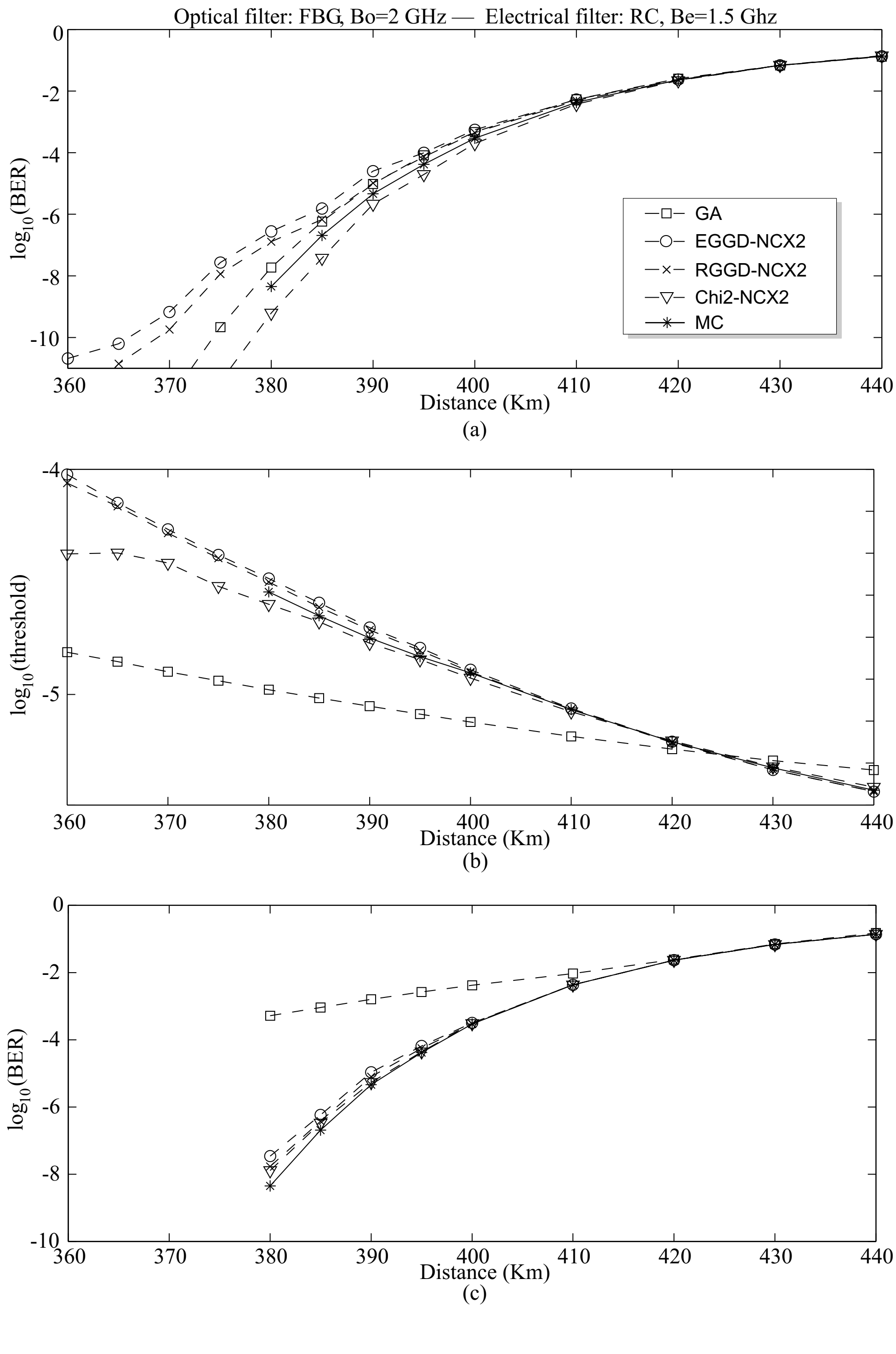

Fig. 5. (a) BER with respect to distance. (b) Threshold estimation with respect to distance. (c) BER achieved by adopting the estimated threshold.

System configuration: FBG optical filter with $2R$ 3-dB bandwidth and RC electrical filter with $1.5R$ 3-dB bandwidth

The simulated communication system consists of a fiber having dispersion parameter $D = -2$ and attenuation parameter $\alpha = 0.2dB$. Then the dispersion caused to the optical signal by the fiber above is partly compensated by an other fiber with $D = 17$ and $\alpha = 0.2dB$. In the simulation examples that follow, the optical signal is compensated only 70% in the sense that the length of the second fiber is 70% of the length required in order the signal to be fully compensated. The transmitted optical RZ pulses were Gaussian shaped with peak power $Po = 20$mW and full width at half maximum FWHM= 2^{-11}. Moreover, the transmission rate was $R = 10$ Gbps and the spontaneous emission parameter n_{sp} was set to 2.

The estimation techniques based on the EGGD and the reparametrized generalized gamma distribution (RGGD) have been tested under several combinations of optical and electrical filters. In the simulation examples that follows, only the MOM method for the fit of the EGGD have been used since the likelihood maximization method provided only slightly improved results. Moreover, the performance curves which are shown correspond to filter combinations that lead to the worst performance the proposed techniques. Fig. 3 corresponds to a Fabri-Pérot (FPF) optical filter with 3-dB bandwidth $Bo = 2R$ and a first-order RC low-pass electrical filter with 3-dB bandwidth $Be = 0.5R$. In Fig. 4 the optical filter has been modelled as a fiber Bragg grating (FBG) [18] with 3-dB bandwidth $Bo = 2R$ and the electrical filter is a fifth-order Bessel filter with 3-dB bandwidth $Be = 1R$. Finally, Fig. 5 shows the performance curves in the case of FBG optical filter with $Bo = 2R$ and RC electrical filter with $Be = 0.5R$.

For comparison, the estimates provided by the Gaussian and the ideal chi-square for the spaces and NCX2 for the marks(Chi2-NCX2) approximation methods are also shown. The actual performance of the communication system under the specific configuration is given by Monte Carlo (MC) simulations.

In all the simulation examples, the top graphs (Fig.3a, Fig. 4a) and Fig. 5a), correspond to the BER estimation. We see that the EGGD-NCX2 method provide better estimates over the GA method for probabilities of errors up to the error free region only in the simulation example of $Be = 0.5$ (Fig.3a) since in the case of $Be = 1$ (Fig. 4a) its estimate deviates from the actual one for BER less than 10^{-6}. The RGGD-NCX2 outperforms the GA method in both cases. In general, we have observed that the BER estimation accuracy of both proposed techniques deteriorates as the ratio of the optical filter bandwidth and the electrical filter bandwidth decreases. This is readily seen when the electrical filter bandwidth has been increased to $Be = 15$ (Fig. 5a).

However, it has to be noted that in many cases the electrical filter is significantly narrower than the optical filter leading to high Bo/Be ratios.

With respect to the threshold estimation (Fig.3b, Fig. 4b and 5b) the performance of the proposed methods is relatively insensitive to the fraction Bo/Be and estimates well the optimum threshold in all cases. Finally, in Fig.3c, 4c and Fig. 5c, we can observe how important is the accurate selection of the threshold parameter to the BER achieved by the receiver. We see that the thresholds estimated by the proposed methods provide performance very close to the optimum one.

VII. Conclusions

In this paper, a combination of the generalized gamma distribution with the noncentral chi-square distribution have been used for the estimation of the performance and the optimization of the decision threshold of a preamplified optical communications system. The proposed technique achieves much better performance than the Gaussian approximation method in the decision threshold estimation. As far as the probability of error estimation is concerned, the proposed method performs well up the error-free limit provided that the ratio of the optical and electrical bandwidths is higher than 2. However, with respect to the threshold estimation the proposed method gives accurate results for even lower bandwidth ratios.

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