Connectivity Issues for Optical Wireless Networks

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Abstract. Optical Wireless Communications (OWC) networks are becoming more and more popular for delivering broadband traffic since they are introducing significant advantages against the other alternative technologies. Operating wavelengths range from ultraviolet (UV) to the infrared (IR) portion of the electromagnetic spectrum and present significant attenuation from channel impairments. As a result, the transmission range is significantly reduced when a single hop is used.

Therefore, multi-hop operation, which is a common technique in wireless radiofrequency (RF) communication systems, is adopted in order to increase the effective distance between transmitter and receiver. To improve their reliability connectivity issues need to be investigated. Connectivity has been investigated in RF ad hoc networks (either one or two dimension) in contrast with OWC networks. The present dissertation aims to examine this research area by connecting the minimum transmission range ensuring connectivity with a plethora of parameters such as the adopted modulation and/or coding format, the transmitted power, the supported data rate and the error probability. Analytical expressions are extracted and the derived results are depicted using appropriate figures. The outcomes constitute a valuable tool to design such networks in practice.

Keywords: k-connectivity, node isolation probability, multi-hop networks, UV-C transmission, NLOS propagation, weather effects, underwater optical wireless networks.

1 Dissertation Summary

Optical wireless has been launched as an attractive candidate technology to provide broadband communications. The way optical wireless transceivers operate is more or less the same as fiber optics ones; however, since laser signals are now transferred through the atmosphere, the path loss between the transmitter and the receiver is getting raised due to a plethora of pernicious factors that appear. In this dissertation three types of optical wireless networks have been examined: non-line-of-sight (NLOS) optical transmission in the ultraviolet UV-C spectral region, free space optics (FSO)
in the infrared spectral region and underwater optical wireless transmission in the visible spectral region. More details on these networks are given in the following.

Transmission in the ultraviolet UV-C spectral region (particularly from 200nm to 280nm), also known as the solar-blind band, exhibits some unique characteristics. Firstly, most of the solar radiation is getting absorbed by the ozone in the upper atmosphere, leading to almost negligible background noise at the Earth’s surface. Secondly, the UV-C light generated from terrestrial sources is strongly scattered due to the presence of suspended particles in the atmosphere. The insignificant background solar radiation and the strong atmospheric scattering enable the activation of NLOS communication links with large field-of-view (FoV) receivers which allow a large amount of scattered light collection. Recent advances in hardware have led to the emergence of low cost semiconductor laser diodes and miniaturized LEDs at UV-C frequencies making this new technology a quite promising solution for short-range communications [1].

On the other hand, FSO transfers broadband services via line of sight (LOS) links and the common candidates operating wavelengths are 780nm, 950nm, and 1550nm. Weather, propagation distance, scattering, absorption, turbulence, pointing error effects, laser wavelength, and data rates are some of the deterministic and random elements that contribute to the overall performance of an optical wireless link. Even if all the deteriorating factors play a significant role, designers and implementers should particularly take meteorological phenomena into consideration when they intend to deploy a robust FSO network in practice. Fog, snow, and rain cause the scattering of laser signals in the atmosphere. Scattering makes a portion of the light beam traveling from a source deflect away from the intended receiver. Another atmospheric effect under clear weather conditions is the turbulence induced by random changes in the atmospheric refractive index. As a result, random phase and irradiance fluctuations (scintillation) of the optical signals are observed at the receiver [2]. Furthermore, the FSO links also depend on the pointing error performance. Pointing errors occur due to mechanical misalignment or errors in tracking systems. Among all phenomena, fog brings about the greatest repercussions since it is constituted of small water droplets having dimensions near the size of infrared wavelengths. Snow and rain also influence the FSO performance, though their impact is significantly less than that of fog. Note that these weather phenomena rarely occur concurrently.

Finally, underwater optical communications were based on simple point to point links delivering composite applications such as monitoring the ocean environment, mapping of the sea floor, and sensing purposes [3]. These systems are usually operating in the blue/green spectrum region where absorption is minimum compared to other wavelengths. The aquatic medium contains almost 80 different elements, dissolved or suspended in pure water, with different concentrations, as well as phytoplanktons, zoo planktons and many marine organisms and plants. These components may redirect the transmitting light or transform it into heat due to the two fundamental physical processes, namely absorption and scattering. Obviously, the light transmittance is sensitive to high wavelengths since these two physical processes have a highly spectral dependence.

The deteriorating factors that are introduced by the propagation channel reduce both the maximum obtainable data rate and the transmission range and an efficacious
solution to mitigate these impairments is to employ relay-assisted techniques. In a multi-hop transmission, the total transmission path is divided into smaller distances between relays or hops which suffer from less loss. At each relay, the received optical field is processed and forwarded to the next one. In that way, an effective serial relayed network can provide services at far distances. Transmission through relays is quite a common practice in wireline and RF wireless communication systems. A plethora of such relevant studies has appeared recently; see, e.g., the newly published books of Uysal [4] or Dohler and Li [5].

Multi-hop networks can properly operate if connectivity between their nodes is satisfied. A fully connected network contains a path from any node to another. When there is no path between at least one source–destination pair, the network is disconnected. Obviously, connectivity plays a critical role for wireless networking. Connectivity was mainly studied for RF wireless networks where the propagation suffers from various severe, random in nature impairments such as path loss, multi-path fading, and shadowing [6], [7]. Another metric of network connectivity is the node isolation probability which can be defined as the probability that a random node cannot communicate with any other nodes. Some of the studies on this topic for one-dimensional relayed networks are as follows. In [8], the probability of having a wireless network composed of at most C clusters is extracted. In [9], an ad hoc network consisting of nodes and base stations is considered, and the probability of node to base station connectivity is derived. Analytical expressions for the probability that a wireless network is connected are presented in [10], as well. Finally, Miorandi and Altman [11] obtained exact results for the coverage probability, the node isolation probability, and the connectivity distance for various node placement statistics.

In the context of optical wireless networking there appears an absence of similar works in the open technical literature mainly due to the fact that the interest on this specific field is nowadays in season. That was the key motivation for this dissertation and the outcomes were the first papers in the areas of ultraviolet communications ([12], [13]), free space optics ([14]) and underwater optical communications ([15]). The numerical results of this dissertation are of significant value for telecom researchers working toward a flexible optical wireless network deployment in practice.

2 Results and Discussion

2.1 k-connectivity issues

The multi-hop optical wireless network network can be represented as an undirected graph $G$ with a set of vertices $V$ and a set of edges $E$ [16]. The set of vertices has cardinality $n$ and represents the set of nodes, while the set of the edges corresponds to the OWC links between the nodes. The node degree, $d(u)$, is defined as the number of links of a node (i.e., the number of neighbor nodes within its range). An isolated node has a null node degree. The minimum node degree, $d_{\text{min}}$ of $G$ is defined as the minimum value among the node degrees. Since the nodes are placed in fixed positions, the existence of isolated nodes is undesirable. In terms of communication networks, the probability that each node in a multi-hop OWC network has a minimum
node degree \( d_{\text{min}} \geq k \) depends on the node density, \( \rho \), as well as the transmission range, \( R \) and is given by [17]

\[
P(d_{\text{min}} > k) = \left(1 - \sum_{i=0}^{k-1} \frac{(\pi R^2)^i}{i!} e^{-\pi R^2} \right)^n
\]

(1)

A path is defined as a sequence of successive edges on \( G \). The existence of a path between two nodes denotes that they are connected. \( G \) is connected when a path exists between all pairs of nodes. Similarly, we say that \( G \) is \( k \)-connected (\( k \geq 1 \)) when \( k \) mutually independent paths exist between all pairs of nodes [16]. The conditions (1) ensure that every node has at least one neighboring node within its range. However, the event \( d_{\text{min}} > 0 \) is not a sufficient condition for ensuring the connectivity of the network. It can be proven that, if \( n \gg 1 \), then

\[
P(G \text{ is } k\text{-connected}) = P(d_{\text{min}} \geq k)
\]

(2)

for \( P(d_{\text{min}} \geq k) \) almost 1 [16].

Fig. 1. Graph connectivity: (a) non connected, (b) 1-connected, (c) 2-connected and (c) 3-connected.
2.2 k-connectivity issues for ultraviolet UV-C two-dimensional multi-hop networks

In [12] we consider a multi-hop network configuration consisting of several NLOS UV-C communication sensors. A typical UV-C link between a transmitter (Tx) and a receiver (Rx) is shown in Fig. 2. Both the Tx and Rx face vertically upwards; i.e., they have 90° apex angle. In this scenario, the Tx transmits a signal vertically upwards having a beam divergence angle $\theta_T$. The cone produced by the Tx beam intersects the Rx FOV cone of $\theta_R$ degrees. The separation between Tx and Rx is $r$, while the distances from the common volume $V$ to the Tx and Rx are $r_1$ and $r_2$, respectively. A communication link is established when the optical power is backscattered by particles inside the volume produced by the intersection of the two cones and reaches the Rx node.

Next, we assume that a number of $n$ nodes are distributed at fixed positions on a service area $A$. Each node is independently placed on the service area according to a homogeneous Poisson point process. Assuming large values of $n$ and $A$, a constant node density, $\rho = n/A$, can be obtained. Under this assumption, the homogeneous Poisson point process can be obtained as the limiting case of the uniform distribution. Consider, now, the case where all the nodes have the same transmission range $R$, i.e., homogeneous range assignment. This means that every node covers a circular region with area $A' = \pi R^2$. Every source node forwards traffic towards one or more destination nodes provided that their cones are intersected. If this does not happen, the node is getting isolated. If the transmission range is short, the probability of having isolated nodes increases. In contrast, assuming a large range, interference problems may appear. Apparently, if a node can communicate with more than one neighbor, the network robustness significantly increases; hence a proper selection of the transmission range is a critical parameter for the network connectivity robustness.

![Fig. 2. UV-C NLOS link geometry.](image)
The determination of an appropriate path loss model is critical for the k-connectivity investigation. Chen et al. in [20] proposed an empirical channel path loss model based on a set of extensive measurements. In their study, the authors demonstrated a communication test-bed and collected path loss measurements, for various combinations of Tx and Rx apex angles. On the basis of these measurements, a simple power decay model was proposed in [21] and adopted in [12]. As a result, analytical expressions for k-connectivity have been obtained, assuming the most fundamental modulation formats, i.e., on-off keying (OOK) [27] and pulse position modulation (PPM) [27] and adopting noise modeling for daily (Gauss model) and nightly (Poisson model) operation. For example, eq. (3):

\[ P_{\text{OOKP}}(d_{\text{min}} > k) = \left( 1 - \sum_{i=0}^{k-1} \exp(-\rho n \left( \frac{n AP_T}{\sqrt{h c R_b \ln(2P_e)} \lambda} \right)^2 - \frac{i}{2} \right)^n \] (3)

presents the network k-connectivity for OOK assuming the Poisson noise model. In (3), \( P_t \) denotes the transmitted power, \( R_b \) is the supported data rate, \( P_e \) is the error probability, \( \lambda \) is the operating wavelength, \( h \) is the Planck’s constant, \( c \) is the speed of light, \( \eta \) is the quantum efficiency of the optical filter and photodetector, \( a \) is the path loss exponent and \( \xi \) is the path loss factor.

The aim of this study is to address the following question: Given a homogeneous range assignment \( R \), for a given modulation scheme, what is the minimum node density \( \rho \) required to achieve a k-connected network with probability close to 1?

2.3 Node isolation probability for serial ultraviolet UV-C multi-hop networks

In [13] we focus on the node isolation probability evaluation of a serial multi-hop UV-C network and derive analytical expressions assuming the OOK and PPM modulation formats. The network consists of \( n \) transceivers (nodes), deployed at fixed positions on a service interval with length \( l \), operating under NLOS conditions (Fig. 3). Every node is independently placed on the service interval according to a homogeneous one-dimensional PPP. Assuming large values of \( n \) and \( l \), a constant node density, \( \rho = n/l \), can be obtained. Under this assumption, the homogeneous PPP can be obtained as the limiting case of the uniform distribution [22]. Every node is equipped with a Tx and a Rx, with elevation angles of \( \beta_T \) and \( \beta_R \) degrees, respectively. The distance between a transceiver and its first neighbor is a random variable following a generalized Gamma distribution [23]. The Tx produces a cone, which has a beam divergence angle of \( \theta_T \) degrees, and intersects the Rx FOV cone of \( \theta_R \) degrees. A communication link is established when the optical power is backscattered by particles inside the common volume, generated by the intersection of the two cones, and reaches the Rx node.

We consider the case of homogeneous range assignment, i.e., all the nodes have the same transmission range \( R \). Every source node forwards traffic toward its first
neighbor node provided that their cones are intersected. If this does not happen, the node becomes isolated. If the transmission range is short, the probability of having isolated nodes increases. On the contrary, assuming a large value of $R$, the interference level for each node may be significantly increased, thus degrading the quality of the communication link.

![Diagram of Serial UV-C multi-hop network geometry.](image)

**Fig. 3.** Serial UV-C multi-hop network geometry.

A single scattering model is adopted, where each photon is assumed to be scattered at most once through its propagation from Tx to Rx. This model has been derived by Xu et al. in [19], as a fine approximation of the one introduced by Luettgen et al. in [18], which is presented in an integral form. Therefore, analytical expressions for the node isolation probability, $P_{iso}$, can be obtained in terms of the system parameters (i.e., transmitted power, supported data rate, and probability of error), as well as the geometrical configuration parameters (i.e., Tx and Rx elevation angles, Tx full beam divergence angle, and Rx FoV), and the node density. For example, eq. (4)

$$
P_{iso,OOK} = \exp\left(-2\rho \frac{\sin\beta}{k_s \beta_T} W_0 \left(\frac{\eta P_k k_s H(\mu) A_s \theta_T^2 \beta_T^2}{96 \sin \beta_T \sin^2 \beta_R (1 - \cos^2 \theta_R) \sqrt{\eta P_k k_s H(\mu)}}\right)\right)$$

(4)

presents the node isolation probability for OOK. In (4), $H(\mu)$ denotes the composite phase function, $k_s$ and $k_e$ are the extinction and scattering coefficients, respectively, $\eta$ is the quantum efficiency of the optical filter and photodetector, $A_s$ is the area of the receiving aperture and $N_0$ is the white noise power spectral density. Obviously, an appropriate tradeoff between the node density, $\rho$, and the transmission range, $R$, is required to ensure a minimum number of isolated nodes. The presented results not only investigate this trade off but also considering the impact of the transceiver geometrical configurations (elevation angles, divergence angles) and the interaction of several parameters such as the supported data rate, the transmitted power and the network length.
2.4 Weather effects on node isolation probability for serial FSO multi-hop networks

In [14] we consider a serial network architecture composed of \( n \) relays, i.e., \( n \) FSO transceiver nodes uniformly distributed in a service interval of length \( l \) according to a binomial point process (BPP) model (Fig. 4). The distance between a node and its \( k \)-th neighbor follows a generalized beta distribution given by eq. (1) in [14].

\[
P_{iso,fog} = \left[ 1 - 2w_0 \left( \frac{a_{fog}}{2\Theta R_{fog}} \sqrt{\frac{P_{thr}}{P_f}} \right) \right]^n
\]

(5)

gives the node isolation probability when fog is present. In (5) \( \Theta \) denotes the beam divergence and \( a_{fog} \) the attenuation coefficient due to fog. From this equation, we can conclude that an adequate node isolation probability for a given length, \( l \), depends on various network parameters and the number of nodes, \( n \).

2.5 k-connectivity issues for underwater optical wireless multi-hop networks

In [15] we consider a three-dimensional (3D) uOWC network consisting of nodes floating at different depths over a service aquatic volume. Each node is equipped with six sensors with FoV of 60° in order to cover all directions and allow transmission in three dimensions. A possible way to deploy such a network was proposed in [26]. Every sensor is placed at the bottom of the sea by an anchor and is connected with a buoy that can be inflated by a pump. Sensors are pushed towards the surface by the buoy. The depth adjustment can be controlled by arranging the wire length connecting the sensor to the anchor. By properly adjusting the depth, we can also construct a one-
dimensional uOWC network where nodes are arranged over a service length into the aquatic medium. The one dimensional arrangement can be considered as a special case of the general three dimensional node configuration.

The determination of the minimum achievable transmission range of each node is critical for the design and the connectivity investigation of the network. It is found, that this parameter is a function of network parameters (transmitted power, supported data rate, error probability), the operating wavelength and the chlorophyll concentration as shown in (6):

\[
R = \frac{2 \cos(\theta)}{c(\lambda)} \times W_0 \left[ \frac{c(\lambda)}{\cos(\theta)} \frac{1}{\eta A_{T} \eta A_{R} \cos(\theta)} \sqrt{\frac{2\pi(1-\cos(\theta_0))}{\eta A_{T} \eta A_{R} \cos(\theta)}} \left( -r_{dc} - \frac{r_{bg}}{2} \right) \right] (6)
\]

In (6), \(c(\lambda)\) denotes the extinction coefficient, \(\theta_0\) is the Tx beam divergence angle, \(\theta\) is the angle between the perpendicular to the Rx plane and the Tx-Rx trajectory, \(T\) is pulse duration, \(\eta\) is the detector counting efficiency while \(r_{dc}\) and \(r_{bg}\) are the sources of additive noise due to dark counts and background illumination, respectively.

A set of analytical results is presented in order to study the connectivity behavior of an underwater multi-hop sensor network in the 300nm–700nm spectral region for two case studies. At first, a one-dimensional scenario is considered, where \(n = 100\) nodes are uniformly distributed at a given service length. Secondly, a three-dimensional scenario is considered, where \(n = 100\) nodes are uniformly distributed at a given service volume.

3 Conclusions

In the context of this dissertation we examined connectivity issues for three types of optical wireless communication networks.

At first, in order to extend the coverage region for a NLOS UV-C optical communication network, the network operation via multiple node-to-node hops is proposed and the system performance in terms of the k-connectivity property is investigated. The effect of several network parameters, such as the node density and the probability of error, were examined for uncoded OOK and PPM modulation schemes, assuming Poisson and Gaussian noise models. Various numerical results were illustrated, showing a useful outcome for telecom system designers for constructing a reliable UV-C network. Apparently, better results could be obtained by using more efficient modulation schemes, e.g., subcarrier intensity modulation, as well as incorporating coding, e.g., repetition or convolutional codes. Moreover, the use of different deployment geometries, e.g., use of directional beams, can obviously enhance the performance; however, the connectivity analysis in this case is getting much more complicated and is a subject of ongoing research.

Secondly, analytical expressions for the node isolation probability of a serial NLOS UV-C network were presented where transceivers are distributed statistically
An effective path loss model is used and transmission with both OOK and PPM modulation schemes was considered assuming Gaussian noise. Several illustrative examples were depicted to show the interaction between various parameters, including the node density, the data rate, the required amount of power to achieve a certain error probability floor, etc. Different geometrical transceiver configurations were examined in order to obtain the node density required to achieve $P_{\text{iso}} \approx 0$ as well. The adoption of other path loss models and the consideration of more effective modulation and/or coding schemes are some of the topics for further research.

Thirdly, we focused on the node isolation probability of a serial FSO network where transceivers are placed on a given path-link following an one-dimensional BPP. We used an effective path loss model and considered operation under the most critical weather phenomena, e.g., fog, haze, rain, and snow. Proper design scenarios were presented in order to reveal the interaction between the number of required nodes, the length of the service interval, and the weather condition parameters (visibility, rainfall/snowfall rate) so that to achieve $P_{\text{iso}} = 0$, as well. In the worst case scenario, i.e., having thick fog with a 50m visibility, a network operator needs 15 nodes to cover a service length of 1km [13]. The work can be improved in a number of ways, e.g., using different path loss models, considering other modulation formats, using forward error correction schemes, etc.

Finally, we considered a underwater optical wireless communication (uOWC) configuration consisting of uniformly distributed nodes communicating with each other using IM/DD with OOK modulation scheme. We, then, investigated the interaction between the node density and various parameters such as error probability, wavelength, transmitted power, data rate, etc, in order to achieve connectivity. As an example, assuming the one dimensional scenario with a service length of 1km and $P_{\text{e}} = 10^{-6}$ we find from that the required number of nodes in order to deploy a fully connected serial multi-hop uOWC network is 336 for $R_b = 10$Mbps and 184 for $R_b = 100$kbps [14]. Obviously, these values would be significantly reduced by adopting more efficient modulation and/or coding schemes. This work can be expanded in several ways. At first, the adopted path loss model is only an approximation of the received intensity at each node and does not capture the scattering component of the transmitted beam. Several channel models presented in the literature, e.g., ([28] or [29]) could be assumed and more realistic results would be obtained. Furthermore, alternative models to describe the absorption and scattering coefficients could be adopted. As an example, the distinction between “small” and “large” particles according to Haltrin’s model is determined by the index of refraction. However, this distinction could also be determined relative to the wavelength of illuminating resolution [30]. Moreover, the role of channel sharing/MAC/collisions should be clarified in case the non-isolated nodes do not operate successfully at a quoted bit rate.

4 References