

Algorithms for Space-Time Equalization of Wireless Channels

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Abstract. In this thesis we investigate receiver techniques for maximum likelihood (ML) joint channel/data estimation in flat fading multiple-input multiple-output (MIMO) channels. The performance of iterative least squares (LS) for channel estimation combined with sphere decoding (SD) for data detection is examined for block fading channels, demonstrating the data efficiency provided by the semi-blind approach. The case of continuous fading channels is addressed with the aid of recursive least squares (RLS). The observed relative robustness of the ML solution to channel variations is exploited in deriving a block QR-based RLS-SD scheme. For the multi-user MIMO scenario, the gains from exploiting temporal/spatial interference color are assessed. We also derive the optimal training sequence for ML channel estimation in the presence of co-channel interference (CCI). In the second part of the thesis we propose two new adaptive equalizers for direct sequence code division multiple access (DS-CDMA) systems operating over time-varying and frequency selective channels. The equalizers consist of a number of serially connected stages and detect users in an ordered manner, applying a decision feedback equalizer (DFE) at each stage. Both the equalizer filters and the order in which the users are extracted are updated in a RLS manner, efficiently realized through time- and order-update recursions.

1 Introduction

During the last two decades, there has been an explosion in the services offered by wireless telecommunication networks, which is boosted from the relevant growth of the technologies of Informatics and Telecommunications. At the same time, there are new challenges for the development of the next generation telecommunication systems. Two of the most basic technologies for the evolution of the new services in the wireless networks are the multiple-input multiple-output (MIMO) and the code division multiple access (CDMA) systems. In this PhD thesis, we worked on the design and analysis of space-time signal processing algorithms for this kind of systems.

Specifically, we investigate iterative and recursive least squares (LS) algorithms for maximum-likelihood (ML) joint channel/data estimation, that are

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both data efficient and computationally attractive. The proposed schemes use the sphere decoding (SD) algorithm for data detection, and short training sequences for an initial channel estimation. We studied three new algorithms [10] for block- and continuous-fading frequency flat MIMO channels. Moreover, in the case of DS-CDMA systems, we propose two new adaptive equalizers of the successive interference cancellation (SIC) type operating over time-varying and frequency selective channels. Their development relies on the formulation of a DS-CDMA system as one with multiple inputs and multiple outputs and the adoption of existing adaptive solutions of the BLAST-type for MIMO channel equalization [1, 12, 7].

2 Semi-blind maximum-likelihood joint channel/data estimation for correlated channels in multiuser MIMO networks

2.1 Signal and System Model

Consider a MIMO communications system, with M_T transmit and M_R receive antennas, where $M_R \geq M_T$, and frequency flat fading channels. The received signal vector at time n is given by

$$\mathbf{x}(n) = \mathbf{H}_0(n)\mathbf{s}_0(n) + \mathbf{v}(n) \quad (1)$$

where $\mathbf{H}_0(n) \in \mathbb{C}^{M_R \times M_T}$ is the channel matrix, assumed of full column rank, $\mathbf{s}_0(n) \in \Omega^{M_T \times 1}$ denotes the input signal vector taking values from a finite alphabet (FA) Ω with cardinality $Q = |\Omega|$, and $\mathbf{v}(n) \in \mathbb{C}^{M_R \times 1}$ is composed of colored interference (CCI) and additive, temporally and spatially white, zero mean Gaussian noise.

2.2 Single-User Case

Maximum Likelihood Estimation. In the absence of multiuser interference, $\mathbf{v}(n)$ in (1) is only composed of white Gaussian noise. Thus, the problem of ML estimation can be formulated as

$$\min_{\mathbf{s}_0(n) \in \Omega^{M_T \times 1}, \mathbf{H}_0(n) \in \mathbb{C}^{M_R \times M_T}} \|\mathbf{x}(n) - \mathbf{H}_0(n)\mathbf{s}_0(n)\|^2 \quad (2)$$

It is clear that, given the input data $\mathbf{s}_0(n)$, the solution for the channel $\mathbf{H}_0(n)$ is given by its least squares (LS) estimate. For a known channel, the ML-optimal input vector is to be searched among all Q^{M_T} candidate M_T -tuples from $\Omega^{M_T \times 1}$. Sphere decoding (SD) [2] is known to be a computationally efficient alternative to exhaustive enumeration [4]. The basic idea is to reduce the number of candidates by searching only within a hypersphere centered at $\mathbf{x}(n)$ using a QR decomposition (QRD) of the channel matrix.

Block Fading. Assuming block fading and dropping time indices, (1) can be re-written as

$$\mathbf{X} = \mathbf{H}_0 \mathbf{S}_0 + \mathbf{V} \quad (3)$$

where \mathbf{X} denotes the $M_R \times N$ output matrix, \mathbf{S}_0 is the $M_T \times N$ input matrix, and $\mathbf{V} \in \mathbb{C}^{M_R \times N}$ is the noise matrix. N denotes the length of the data block, over which the channel matrix is assumed constant. Let $\mathbf{H}_0^{(0)}$ denote the estimate of \mathbf{H}_0 that may have resulted from a (short) training period. This can be improved, and as a consequence the data estimates as well, via an iterative procedure consisting of alternately optimizing the data estimate based on the current channel estimate and vice versa. Table 1 summarizes the general ALS ML scheme, where $\mathbf{H}_0^{(i)}$ and $\mathbf{S}_0^{(i)}$ are the channel and data estimates in the i -th

Table 1. ALS for joint ML channel estimation/data detection.

Given:	$\mathbf{X}, \mathbf{H}_0^{(0)}$
Step 0	$i = 0$ Repeat until convergence
Step 1	$i = i + 1$
Step 2	$\mathbf{S}_0^{(i)} = \arg \min_{\mathbf{S}_0 \in \Omega^{M_T \times N}} \ \mathbf{X} - \mathbf{H}_0^{(i-1)} \mathbf{S}_0\ ^2$
Step 3	$\mathbf{H}_0^{(i)} = \mathbf{X} \mathbf{S}_0^{(i)\dagger}$

iteration. Two well-known examples are *iterative least squares with projection (ILSP)* and *iterative least squares with enumeration (ILSE)* [16]. ILSP is a simple approach, where the ML solution is only approximated, by projecting onto the FA each of the entries of the soft LS data estimate, $\mathbf{H}_0^{(i-1)\dagger} \mathbf{X}$. In ILSE, full enumeration is performed, thus obtaining the exact ML solution but at a very high computational cost. An exact ML ALS scheme of lower expected complexity would result if SD were utilized instead to detect each of the columns of $\mathbf{S}_0^{(i)}$ in Step 2 above. This algorithm will henceforth be referred to as *iterative least squares with SD (ILS-SD)*.

Continuous Fading. For continuous fading channels an online version of the ILS-SD algorithm is employed using the recursive least squares (RLS) algorithm. Considerable computational savings would result if the Q, R factors were tracked instead of the channel matrix itself [8]. An additional reduction in the computational burden of the receiver can be achieved by performing the Q, R update once in every T samples instead of on a sample by sample basis. Between two consecutive updates, SD is based on the available QRD as if the channel remained constant in the meantime. This ‘sub-sampled’ channel tracking scheme is suggested by the observed robustness of the ML solution to mild changes in the channel [10] and leads to significant computational savings with little or no performance loss. The proposed algorithm, will be referred to hereafter as RLS-SD.

2.3 Multi-User Case

Interference Color. Here, we consider the case where the interference component, $\{\mathbf{v}(n)\}$, may be correlated both in time and space. Therefore the received signal process $\{\mathbf{x}(n)\}$ is also temporally correlated. To exploit this fact, we employ more than one consecutive received samples to *jointly* detect the corresponding input vectors [14]. Stacking N consecutive received samples together we can write:

$$\underbrace{\begin{bmatrix} \mathbf{x}(n) \\ \mathbf{x}(n-1) \\ \vdots \\ \mathbf{x}(n-N+1) \end{bmatrix}}_{\bar{\mathbf{x}}} = \underbrace{\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_0 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}_0 \end{bmatrix}}_{\bar{\mathbf{H}}_0} \underbrace{\begin{bmatrix} s_0(n) \\ s_0(n-1) \\ \vdots \\ s_0(n-N+1) \end{bmatrix}}_{\bar{s}_0} + \underbrace{\begin{bmatrix} \mathbf{v}(n) \\ \mathbf{v}(n-1) \\ \vdots \\ \mathbf{v}(n-N+1) \end{bmatrix}}_{\bar{\mathbf{v}}}$$

or

$$\bar{\mathbf{x}} = \bar{\mathbf{H}}_0 \bar{s}_0 + \bar{\mathbf{v}} \quad (4)$$

All the interferers' channels are assumed to obey the well-known Kronecker model [13, 18], with the same receive fading correlation matrix. Assume, moreover, an *interference-limited environment*, where CCI overwhelms background noise [14]. One can then show that the interference correlation matrix is given by [17]

$$\mathcal{R}_{\bar{\mathbf{v}}} = E(\bar{\mathbf{v}}\bar{\mathbf{v}}^H) = \mathcal{R}_t^* \otimes \mathcal{R}_s \quad (5)$$

where \mathcal{R}_t^* and \mathcal{R}_s are the *temporal* and the *spatial* interference colors, respectively.

Maximum-Likelihood Estimation. Under the assumption of Gaussianity for $\{\mathbf{v}(n)\}$ [15], the ML joint channel estimation / data detection problem for (4) can be formulated as

$$\min_{\bar{s}_0, \bar{\mathbf{H}}_0} \left[\mathcal{R}_{\bar{\mathbf{v}}}^{-1/2} (\bar{\mathbf{x}} - \bar{\mathbf{H}}_0 \bar{s}_0) \right]^H \left[\mathcal{R}_{\bar{\mathbf{v}}}^{-1/2} (\bar{\mathbf{x}} - \bar{\mathbf{H}}_0 \bar{s}_0) \right]$$

where $\mathcal{R}_{\bar{\mathbf{v}}}^{-1/2}$ is a Hermitian square root of $\mathcal{R}_{\bar{\mathbf{v}}}^{-1}$. Utilizing the relation $\mathcal{R}_{\bar{\mathbf{v}}}^{-1/2} = \mathcal{R}_t^{-*/2} \otimes \mathcal{R}_s^{-1/2}$, resulting from (5), the ML problem for \mathbf{H}_0 and \mathbf{S}_0 is now formulated as

$$\min_{\mathbf{S}_0, \mathbf{H}_0} \left\| \mathcal{R}_s^{-1/2} \mathbf{X} \mathcal{R}_t^{-1/2} - \mathcal{R}_s^{-1/2} \mathbf{H}_0 \mathbf{S}_0 \mathcal{R}_t^{-1/2} \right\|^2 \quad (6)$$

where $\bar{\mathbf{x}} = \text{vec}(\mathbf{X})$ and $\bar{s}_0 = \text{vec}(\mathbf{S}_0)$. Hence, the solution is given by the Gauss-Markov estimator (GME) [5]:

$$\hat{\mathbf{H}}_0 = \mathbf{X} \mathcal{R}_t^{-1} \mathbf{S}_0^H \left(\mathbf{S}_0 \mathcal{R}_t^{-1} \mathbf{S}_0^H \right)^{-1} \quad (7)$$

from which we can observe that the channel estimate involves only the temporal correlation of the interference. Note also, that this is an *unbiased* estimate of \mathbf{H}_0 , that is, $E(\hat{\mathbf{H}}_0) = \mathbf{H}_0$ and the corresponding covariance matrix is given by [5]:

$$\mathbf{C}_{\hat{\mathbf{H}}_0} = E \left[(\hat{\mathbf{H}}_0 - \mathbf{H}_0) (\hat{\mathbf{H}}_0 - \mathbf{H}_0)^H \right] = (\tilde{\mathbf{S}}_0 \tilde{\mathbf{S}}_0^H)^{-1} = (\mathbf{S}_0 \mathcal{R}_t^{-1} \mathbf{S}_0^H)^{-1}. \quad (8)$$

Optimal Training for Channel Estimation. To save bandwidth, one would like to devote as few as possible symbols to training the channel estimator. Thus, given a fixed training sequence length, N_t , we want to compute the $M_T \times N_t$ training matrix \mathbf{S}_0 that minimizes $\text{tr}(\mathbf{C}_{\hat{\mathbf{H}}_0})$, subject to a constraint on the total energy consumed for training. Formally:

$$\min_{\mathbf{S}_0} \text{tr} \left[(\mathbf{S}_0 \mathcal{R}_t^{-1} \mathbf{S}_0^H)^{-1} \right] \quad (9)$$

$$\text{s.t. } \text{tr}(\mathbf{S}_0 \mathbf{S}_0^H) \leq E_T \quad (10)$$

The solution to this problem is provided in the following:

Theorem 1 *The class of training matrices optimizing the criterion (9), (10) is given by*

$$\mathbf{S}_0^{\text{opt}} = \mathbf{U} \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{M_T} \end{bmatrix} \mathbf{G}_{M_T}^H \quad (11)$$

where \mathbf{U} can be any unitary $M_T \times M_T$ matrix and

$$\sigma_i = \sqrt{\frac{\sqrt{\lambda_i}}{\sum_{j=1}^{M_T} \sqrt{\lambda_j}} E_T}, \quad i = 1, 2, \dots, M_T, \quad (12)$$

with $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_{M_T}$ being the M_T smallest eigenvalues of \mathcal{R}_t and \mathbf{G}_{M_T} the $N_t \times M_T$ matrix of corresponding (orthonormal) eigenvectors, in that order.

Iterative Joint Channel / Data Estimation. In practice, due to the highly increased complexity that the detection of a longer sequence entails, detection is performed in pairs of vectors ($N = 2$). The ML data detection problem will then be written as in (6):

$$\min_{\mathbf{s}_0(n-1), \mathbf{s}_0(n)} \left\| \mathcal{R}_s^{-1/2} [\mathbf{x}(n) \ \mathbf{x}(n-1)] \mathcal{R}_t^{-1/2} - (\mathcal{R}_s^{-1/2} \mathbf{H}_0) [\mathbf{s}_0(n) \ \mathbf{s}_0(n-1)] \mathcal{R}_t^{-1/2} \right\|^2 \quad (13)$$

or

$$\min_{\bar{\mathbf{s}}_0 \in \Omega^{2M_T \times 1}} \left\| \left(\mathcal{R}_t^{-*/2} \otimes \mathcal{R}_s^{-1/2} \right) \bar{\mathbf{x}} - \left(\mathcal{R}_t^{-*/2} \otimes \mathcal{R}_s^{-1/2} \mathbf{H}_0 \right) \bar{\mathbf{s}}_0 \right\|^2 \quad (14)$$

In the training period, \mathcal{R}_t is of size $N_t \times N_t$, with N_t being the training sequence length as above. However, in the detection phase, described by (14), the temporal correlation matrix is 2×2 . One can simply compute these two matrices separately, with the aid of sample averaging.

Then, the ML channel estimation problem can be written as

$$\min_{\mathbf{H}_0} \left\| \mathcal{R}_s^{-1/2} \mathbf{X} \left(\mathbf{I}_{N/2} \otimes \mathcal{R}_t^{-1/2} \right) - \mathcal{R}_s^{-1/2} \mathbf{H}_0 \mathbf{S}_0 \left(\mathbf{I}_{N/2} \otimes \mathcal{R}_t^{-1/2} \right) \right\|^2 \quad (15)$$

with a 2×2 matrix $\mathcal{R}_t^{-1/2}$. Solving for \mathbf{H}_0 , we obtain an estimate as in (7) where \mathcal{R}_t^{-1} should be replaced by $\mathbf{I}_{N/2} \otimes \mathcal{R}_t^{-1}$. The proposed iterative procedure will be henceforth referred to as *ILS-SD-R*.

2.4 Simulation Results

The effectiveness of ILS-SD, as compared to SD detection based on the trained channel estimate can be seen in Fig. 1(a) for uncorrelated Rayleigh block fading channels. The performance of ILSP is also shown. As expected, both trained SD

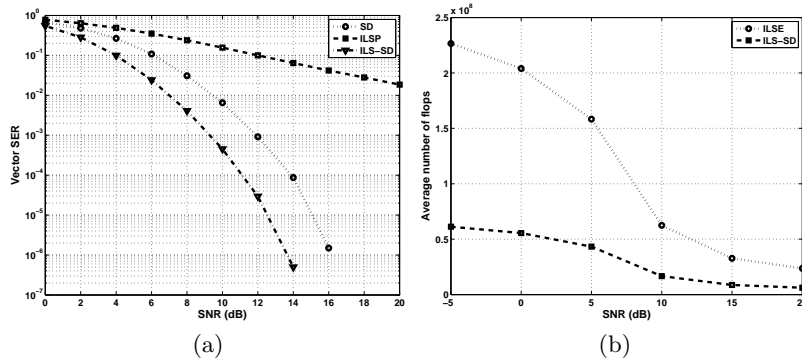


Fig. 1. (a) Comparing ILS-SD with SD based on training only and ILSP. Uncorrelated Rayleigh flat fading channels with $M_T = M_R = 4$ and QPSK input. (b) Computational requirements for convergence of the ILS-SD algorithm as compared to ILSE.

and ILS-SD perform much better than ILSP. ILS-SD is seen to greatly outperform trained-only SD. Moreover, we have seen [10] that ILS-SD converges faster than ILSP on the average, with the difference being more noticeable for low and medium values of the (per antenna) SNR. The computational savings in ILS-SD as compared to exhaustive enumeration (ILSE) are significant, as can be seen in the example of Fig. 1(b).

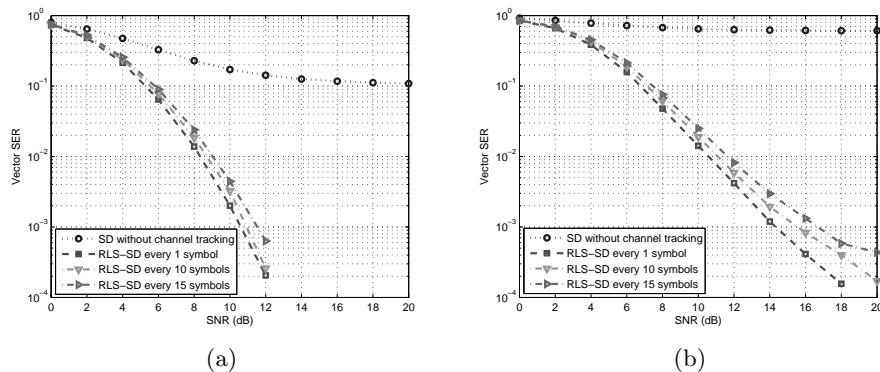


Fig. 2. Performance of RLS-SD. Results of simulation over uncorrelated Rayleigh flat fading channels using ten training symbols and for two mobile speeds: (a) 10 m/s and (b) 30 m/s. Probability of outage: 5%.

Some representative results from simulating RLS-SD for uncorrelated Rayleigh channels are shown in Fig. 2. For the sake of comparison, the results of employing SD with no channel tracking are also included. The loss in performance for RLS-SD when the update is done every $T > 1$ symbol periods is seen to be insignificant for sufficiently small values of T . It is worthwhile to notice that, similar results with that of Figs. 1(a), 2 have been also obtained [10] for correlated Rayleigh and Ricean channels.

The results for the multiuser case are demonstrated in Figs. 3, 4. The training-based SD scheme is also evaluated in Fig. 3. The results of ignoring the interference colors (temporal [9] and spatial/temporal) are also included. A considerable reduction in SER is seen to be achievable by employing optimal training in estimating the channel, especially for moderate to high SINR values compared with orthogonal (DFT) training. One can conclude that exploiting CCI color can result in significant performance gains. Moreover, as expected [14], it appears that the interference *spatial* correlation accounts for most of this benefit.

Using ILS-SD-R iterations results in Fig. 4. One can see that the gain from employing optimal training in ILS-SD-R initialization is now canceled by the iterative improvement procedure, especially for sufficiently long training sequences. Note, however, that, as seen in Fig. 4(b), optimal training can still result in faster convergence, at least in the moderate to low SINR regime.

3 Adaptive BLAST-type Decision-Feedback Equalization Schemes for Wideband DS-CDMA Systems

In the second part of the thesis we study adaptive equalization algorithms for DS-CDMA systems. We propose two new adaptive equalizers [11] for time-varying and frequency selective channels.

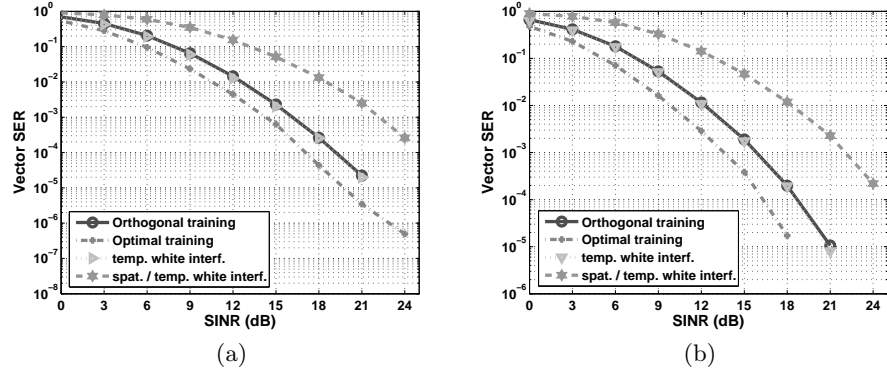


Fig. 3. Performance of SD scheme based on training only, with orthogonal and optimal training. Interference-limited environment (INR=20 dB) with (weakly) correlated Rayleigh channels. The effects of not taking the temporal and the spatial/temporal interference colors into account are also shown. (a) 8 training symbols (b) 12 training symbols.

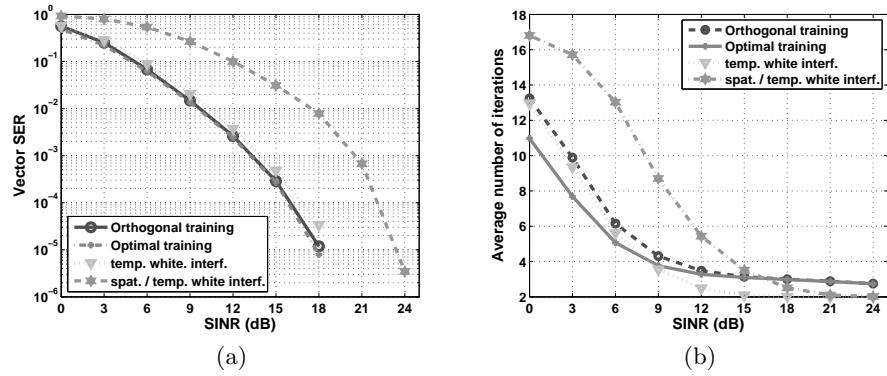


Fig. 4. Performance of the ILS-SD-R in the setup of Fig. 3. (a, b) 8 training symbols.

3.1 System Model

We consider the uplink of a symbol-synchronous DS-CDMA system with a spreading factor of P chips per symbol, K single-antenna users, and a single-antenna receiver. The users transmit independently symbol sequences which are spread through a short P -periodic spreading code $\mathbf{c}_i = [c_i(0) \ c_i(1) \ \cdots \ c_i(P-1)]^T$. The spreading codes are assumed to be known at the receiver. The transmission is through time-varying frequency selective channels, of length L with $L \leq P$.

Sampling at chip rate and collecting P successive measurements of the received signal x in a $P \times 1$ vector, a multiple-input multiple-output (MIMO) formulation with K inputs and P outputs [19] results for the DS-CDMA system. Similarly, collecting $P + L - 1$ successive samples of x instead of P , a new MIMO formulation with K inputs and $P + L - 1$ outputs results.

3.2 An Adaptive BLAST-type Equalization Scheme

An adaptive MIMO DFE detection scheme with variable detection order was proposed in [1] for flat time-varying channels. It was shown that the proposed technique performs similarly to V-BLAST algorithm with RLS channel tracking but at a reduced computational complexity. At each time instant, the receiver carries out the equalization in K serially connected stages. The users are detected in an ordered manner, applying a DFE at each stage. The stronger users are detected first, allowing easier detection for the weaker users [3]. The equalizer filters and the order of detection are updated at each stage by minimizing a set of LS cost functions for all candidate users. The user which attains the minimum cost is selected to be the next detected user.

An algorithm which exhibits the same BER performance as the above method but with reduced computationally complexity and favourable numerical behaviour was proposed in [12], based on the updating of the inverse Cholesky factor of the input autocorrelation matrix. An extension of this method to include frequency selective channels was developed in [7], where expanded input and weight vectors are used in order to eliminate both MAI and ISI.

Viewing a frequency selective DS-CDMA system as a MIMO system, as referenced in Section 3.1, the efficient square root LS algorithm of [7] can be straightforwardly applied for multiuser data detection. The resulting scheme will henceforth be referred to as the square root multiuser detection (SR-MUD) algorithm.

3.3 A RAKE-based Adaptive SIC Scheme

It is important to notice that, in the course of the SR-MUD algorithm, knowledge of users' code sequences is not required. An improved version of the SR-MUD algorithm can be developed, through incorporating knowledge of the code sequences by exploiting the RAKE receiver concept.

The structure of the new adaptive scheme is similar to SR-MUD. In this scheme the second MIMO formulation of the DS-CDMA system, presented in

Section 3.1, is used. However, a modified input signal is applied to the feedforward filter, utilizing the RAKE receiver idea. Specifically, the received signal is multiplied by a convolution matrix containing one-chip shifts of c_i and the output of this product consists the input of the feedforward filter. Hence, we take advantage of the known code sequences to lessen the effect of the other users. However, due to the non-orthogonality of the distorted code sequences, the feedback filter is necessary so as to eliminate the effect of the residual ISI and MAI. Moreover, based on the fact that the equalizer input vector can be expressed in an order-recursive manner an efficient order-update relation for the equalizer weights and the LS error energies can be obtained. Finally, through time- and order-update equations, we efficiently calculate the weights of the equalizers and determine the detection ordering. The proposed algorithm will henceforth be referred to as RAKE-RLS.

3.4 Simulation Results

The performance of the proposed adaptive schemes is compared with the RAKE receiver, the ASIC algorithm, and the linear receiver adapted via the exponentially weighted conventional RLS. The single user bound (SUB), is also shown as a benchmark. In our experiments, we simulate a near-far scenario, where the received amplitude of each user is determined such that $10 \log_{10}(A_i/A_{i+1})^2 = N$ dB, and the amplitude of the first user is set to 1.

The BER performance versus E_b/N_0 (dB) is depicted in Fig. 5 for $K = 7$, $L = 6$, $N = 2$ dB, and for different values of spreading factor P . The superiority of the proposed schemes is evident in the higher E_b/N_0 regime. Specifically, for small values of P ($P = 8$) and at high E_b/N_0 , SR-MUD outperforms RAKE-RLS, while for large values of P ($P = 128$) RAKE-RLS attains the best performance. The superiority of the proposed schemes have also been demonstrated [11] for different values of the channel length and the number of users.

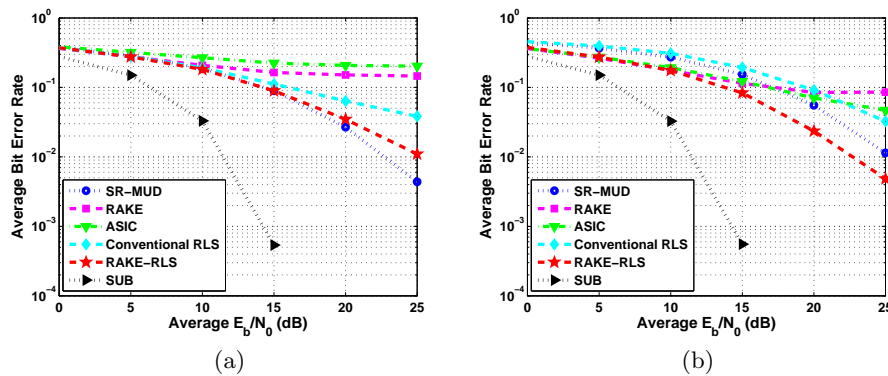


Fig. 5. BER vs. E_b/N_0 (dB) for $K = 7$, $L = 6$, $N = 2$ dB and spreading factor (a) $P = 8$ (b) $P = 128$.

4 Conclusions

Semi-blind schemes for ML joint channel estimation and data detection in MIMO flat fading channels were examined in this thesis. Both block-iterative and recursive algorithms were considered, to address block and continuous fading scenarios, respectively. The multiuser MIMO scenario, resulting in temporally/spatially colored CCI, was also addressed and the gains from exploiting CCI were assessed. The presented simulation results demonstrated the practical applicability of the investigated schemes in realistic environments. Moreover, two new adaptive equalization algorithms for time-varying and frequency selective channels in a DS-CDMA system were derived, based on the BLAST idea. The first algorithm results from a straightforward application of the idea of [1] to a MIMO-formulated DS-CDMA system, while the second one arises by incorporating the RAKE receiver concept to the first scheme. Both the equalizer filters and the optimum detection ordering are efficiently updated through time- and order-update equations. Improved BER performance is offered compared to existing adaptive DS-CDMA equalizers, in a near-far mobile environment and over a wide range of spreading factors, channel lengths and numbers of users.

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