

Consistency of Chordal RCC-8 Networks

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What is Qualitative Spatial Reasoning?

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- Qualitative spatial reasoning is based on qualitative abstractions of spatial aspects of the common-sense background knowledge, on which our human perspective on the physical reality is based.

Reasons for Qualitative Spatial Reasoning

- Two main reasons why non-precise, qualitative spatial information may be useful:
 - 1 Only partial information may be available (e.g. we may know that one region is *disconnected* from another without knowing the precise geometry of the regions)
 - 2 General constraints holding among geographical objects are often most naturally stated in qualitative terms (e.g. we may wish to state that one region is *part of* another region)

Applications of Qualitative Spatial Reasoning

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- Qualitative spatial reasoning is an important subproblem in many *applications*, such as:
 - Robotic navigation
 - High level vision
 - Geographical information systems (GIS)
 - Reasoning and querying with semantic geospatial query languages (stSPARQL, GeoSPARQL)

Region Connection Calculus

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- The Region Connection Calculus (RCC) is a first-order language for representation of and reasoning about topological relationships between extended spatial regions
- RCC abstractly describes regions, that are non-empty regular subsets of some topological space which do not have to be internally connected
- Relationships between spatial regions can be defined based on the $C(a, b)$ *connected* relation, which is true if the topological closures of the regions a and b share a common point

The RCC-8 Calculus

- RCC-8 is a *constraint* language formed by the combination of the following eight jointly exhaustive and pairwise disjoint *base relations*:
 - disconnected (DC)
 - externally connected (EC)
 - equal (EQ)
 - partially overlapping (PO)
 - tangential proper part (TPP)
 - tangential proper part inverse (TPPi)
 - non-tangential proper part (NTPP)
 - non-tangential proper part inverse (NTPPi)

The Eight Basic Relations of the RCC-8 Calculus

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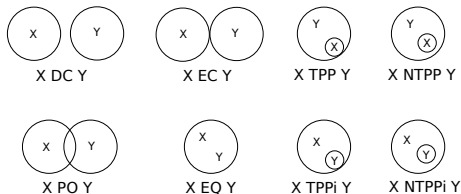


Figure: Two dimensional examples for the eight base relations of RCC-8

From these basic relations, combinations can be built. For example, proper part (PP) is the union of TPP and NTPP.

The RCC-8 Composition Table

o	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	*	DC,EC PO,TPP NTPP	DC,EC PO,TPP NTPP	DC,EC PO,TPP NTPP	DC,EC PO,TPP NTPP	DC	DC	DC
EC	DC,EC PO,TPPi NTPPi	DC,EC PO,TPP TPPi,EQ	DC,EC PO,TPP NTPP	EC,PO TPP NTPP	PO TPP NTPP	DC,EC	DC	EC
PO	DC,EC PO,TPPi NTPPi	DC,EC PO,TPPi NTPPi	*	PO TPP NTPP	PO TPP NTPP	DC,EC PO,TPPi NTPPi	DC,EC PO,TPPi NTPPi	PO
TPP	DC	DC,EC	DC,EC PO,TPP NTPP	TPP NTPP	NTPP	DC,EC PO,TPP TPPi,EQ	DC,EC PO,TPPi NTPPi	TPP
NTPP	DC	DC	DC,EC PO,TPP NTPP	NTPP	NTPP	DC,EC PO,TPP NTPP	*	NTPP
TPPi	DC,EC PO,TPPi NTPPi	EC,PO TPPi NTPPi	PO TPPi NTPPi	PO,EQ TPP TPPi	PO TPP NTPP	TPPi NTPPi	NTPPi	TPPi
NTPPi	DC,EC PO TPPi NTPPi	PO TPPi NTPPi	PO TPPi NTPPi	PO TPPi NTPPi	PO,TPP NTPP NTPPi TPPi,EQ	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

RCC-8 Example

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- Two houses are connected via a road. Each house is located on an own property. The first house possibly touches the boundary of the property; the second one surely does not. Is the network consistent?

```
house1 DC house2
house1 {TPP, NTPP} property1
house1 {DC, EC} property2
house1 EC road
house2 { DC, EC } property1
house2 NTPP property2
house2 EC road
property1 { DC, EC } property2
```

- Using a *path consistency* algorithm, we can refine the following relation:

```
house2 { DC } property1
```

RCC-8 Example

- Two houses are connected via a road. Each house is located on an own property. The first house possibly touches the boundary of the property; the second one surely does not. Is the network consistent?

```
house1 DC house2
house1 {TPP, NTPP} property1
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house2 NTPP property2
house2 EC road
property1 { DC, EC } property2
```

- Using a *path consistency* algorithm, we can refine the following relation:

```
house2 { DC } property1
```

The RSAT Reasoning Problem

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- RSAT in the RCC-8 framework, is the reasoning problem of deciding consistency of a set of spatial formula Θ , i.e., whether there is a spatial configuration where the relations between the regions can be described by Θ .
- RSAT is NP-Complete!
- However, tractable subsets S of RCC-8 exist for which the consistency problem can be decided in **polynomial** time

The RSAT Reasoning Problem

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- RSAT is **NP-Complete!**
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- RSAT is NP-Complete!
- However, tractable subsets S of RCC-8 exist for which the consistency problem can be decided in **polynomial** time

Maximal Tractable Subsets

- First, we define as \mathcal{NP}_8 the set of relations that by themselves result in NP-completeness when combined with the set of base relations:

$$\mathcal{NP}_8 = \{ R \mid (\{PO\} \not\subseteq R \text{ and } (\{NTPP\} \subseteq R \text{ or } \{TPP\} \subseteq R) \\ \text{and } (\{NTPPi\} \subseteq R \text{ or } \{TPPi\} \subseteq R)) \\ \cup \{ \{EC, NTPP, EQ\}, \{DC, EC, NTPP, EQ\}, \\ \{EC, NTPPi, EQ\}, \{DC, EC, NTPPi, EQ\} \}$$

- The following subsets are maximal tractable subsets that contain all base relations:

$$\mathcal{H}_8 = (RCC-8 \setminus \mathcal{NP}_8) \setminus \{ R \mid (\{EQ, NTPP\} \subseteq R \text{ and } \{TPP\} \not\subseteq R) \\ \text{or } (\{EQ, NTPPi\} \subseteq R \text{ and } \{TPP\} \not\subseteq R) \}$$

$$\mathcal{C}_8 = (RCC-8 \setminus \mathcal{NP}_8) \setminus \{ R \mid (\{EC\} \subset R \text{ and } \{PO\} \not\subseteq R) \text{ and } \\ R \cap \{TPP, NTPP, TPI, NTPPi, EQ\} \neq \emptyset \}$$

$$\mathcal{Q}_8 = (RCC-8 \setminus \mathcal{NP}_8) \setminus \{ R \mid (\{EQ\} \subset R \text{ and } \{PO\} \not\subseteq R) \text{ and } \\ R \cap \{TPP, NTPP, TPI, NTPPi\} \neq \emptyset \}$$

Maximal Tractable Subsets

- First, we define as \mathcal{NP}_8 the set of relations that by themselves result in NP-completeness when combined with the set of base relations:

$$\mathcal{NP}_8 = \begin{aligned} & \{ R \mid (\{PO\} \not\subseteq R \text{ and } (\{NTPP\} \subseteq R \text{ or } \{TPP\} \subseteq R) \\ & \quad \text{and } (\{NTPPi\} \subseteq R \text{ or } \{TPPi\} \subseteq R)) \} \\ & \cup \{ \{EC, NTPP, EQ\}, \{DC, EC, NTPP, EQ\}, \\ & \quad \{EC, NTPPi, EQ\}, \{DC, EC, NTPPi, EQ\} \} \end{aligned}$$

- The following subsets are maximal tractable subsets that contain all base relations:

$$\mathcal{H}_8 = (\text{RCC-8} \setminus \mathcal{NP}_8) \setminus \{ R \mid (\{EQ, NTPP\} \subseteq R \text{ and } \{TPP\} \not\subseteq R) \\ \text{or } (\{EQ, NTPPi\} \subseteq R \text{ and } \{TPP\} \not\subseteq R) \}$$

$$\mathcal{C}_8 = (\text{RCC-8} \setminus \mathcal{NP}_8) \setminus \{ R \mid (\{EC\} \subset R \text{ and } \{PO\} \not\subseteq R) \text{ and } \\ R \cap \{TPP, NTPP, TTPi, NTPPi, EQ\} \neq \emptyset \}$$

$$\mathcal{Q}_8 = (\text{RCC-8} \setminus \mathcal{NP}_8) \setminus \{ R \mid (\{EQ\} \subset R \text{ and } \{PO\} \not\subseteq R) \text{ and } \\ R \cap \{TPP, NTPP, TTPi, NTPPi\} \neq \emptyset \}$$

Path Consistency

- Approximates consistency and realises *forward checking* in a backtracking algorithm
- Checks the consistency of triples of relations and eliminates relations that are impossible though iteratively performing the operation

$$R_{ij} \leftarrow R_{ij} \cap R_{ik} \diamond R_{kj}$$

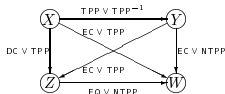
until a fixed point \bar{R} is reached.

If $R_{ij} = \emptyset$ for a pair (i, j) then R is inconsistent, otherwise \bar{R} is *path-consistent*.

- Computing \bar{R} is done in $O(n^3)$

Is Path Consistency Sufficient?

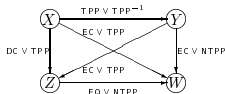
- Path consistency does **not imply** consistency
- The following set of spatial constraints is path-consistent but not consistent



- Still, path consistency is **sufficient** for deciding consistency, if only relations in any tractable subset that contains all base relations, like \hat{H}_8 , C_8 , Q_8 , are used

Is Path Consistency Sufficient?

- Path consistency does not imply consistency
- The following set of spatial constraints is path-consistent but not consistent



- Still, path consistency is **sufficient** for deciding consistency, if only relations in any tractable subset that contains all base relations, like $\hat{\mathcal{H}}_8$, \mathcal{C}_8 , \mathcal{Q}_8 , are used

About PyRCC8..

- PyRCC8¹ is an efficient qualitative spatial reasoner written in pure Python. It employs PyPy², a fast, compliant implementation of the Python 2 language



- PyRCC8 offers a path consistency algorithm for solving tractable RCC-8 networks and a backtracking-based algorithm for general networks

¹<http://pypi.python.org/pypi/PyRCC8>

²<http://pypy.org/>

Path Consistency Algorithm

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Path-Consistency(C)

Input: A constraint network C

Output: True or False

```
1: Q ← {(i, j) | 1 ≤ i < j ≤ n} // Initialize the queue
2: while Q is not empty do
3:   select and delete an (i, j) from Q
4:   for k ← 1 to n, k ≠ i and k ≠ j do
5:     t ← Cik ∩ (Cij ∘ Cjk)
6:     if t ≠ Cik then
7:       if t = ∅ then
8:         return False
9:       Cik ← t
10:      Cki ← t̄
11:      Q ← Q ∪ {(i, k)}
12:     t ← Ckj ∩ (Cki ∘ Cij)
13:     if t ≠ Ckj then
14:       if t = ∅ then
15:         return False
16:       Ckj ← t
17:       Cjk ← t̄
18:       Q ← Q ∪ {(k, j)}
19: return True
```

Implementations

- Based on Simple Queue
 - Implementation with non-weighted arcs
- Based on Priority Queue (process most restrictive arc first)
 - Implementation with exactly weighted arcs
 - Implementation with approximately weighted arcs, using the approach by Van Beek and Manchak [1]

Queue Structure

- Simple Queue (combines set and deque):
 - Membership checking: $O(1)$
 - Push: $O(1)$
 - Pop: $O(1)$
- Priority Queue (combines dictionary and heapq):
 - Membership checking: $O(1)$
 - Push: $O(\log(n))$
 - Pop: $O(\log(n))$

Comparing PC Implementations of Different Reasoners

- We compare the PC implementation of PyRCC8 to the PC implementations of the following qualitative spatial reasoners:
 - Renz's solver³
 - GQR⁴
- Different size n of instances from $A(n, d = 9.5, l = 4.0)$ were used

³<http://users.rsise.anu.edu.au/%7Ejrenz/software/rcc8-csp-solving.tar.gz>

⁴<http://sfbtr8.informatik.uni-freiburg.de/R4LogoSpace/Tools/gqr.html>

Comparison Diagram

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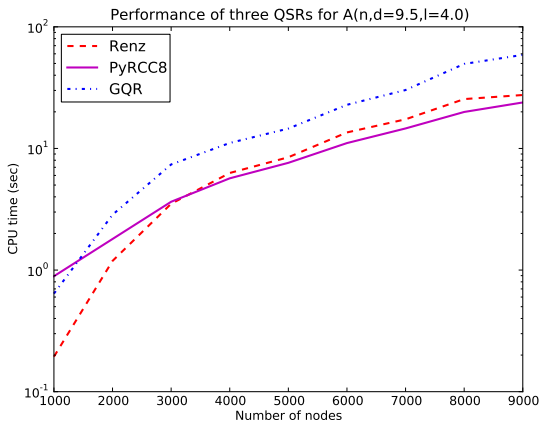
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Using the Admingeo Dataset to Compare PC Implementations of Different Reasoners

- We compare the PC implementation of PyRCC8 to the PC implementations of the following qualitative spatial reasoners:
 - Renz's solver
 - GQR
 - Pellet Spatial⁵
- The admingeo⁶ dataset (11761 regions / 77910 relations) was used which was properly translated to fit the input format of the different PC implementations

⁵<http://clarkparsia.com/pellet/spatial/>

⁶<http://data.ordnancesurvey.co.uk/ontology/admingeo/> 

Evaluation with a Large Dataset

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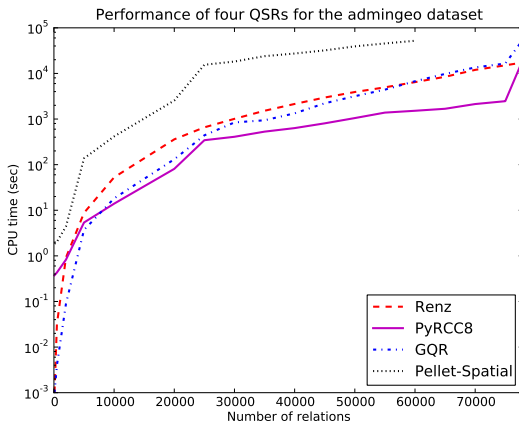
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- To explore the search space for the general case of RCC-8 networks in order to solve an instance Θ of RSAT, some sort of backtracking must be used.
- We implemented two backtracking algorithms:
 - 1 A strictly recursive one
 - 2 An equivalent iterative one which resembles recursion

Recursive Consistency Algorithm

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Consistency(C)

Input: A constraint network C

Output: A refined constraint network C' if C is satisfiable or None

- 1: **if** not **Path-Consistency**(C) **then**
- 2: **return** **None**
- 3: **if** no constraint can be split **then**
- 4: **return** **C**
- 5: **else**
- 6: choose an unprocessed constraint $x_i R x_j$ and split R into $S_1, \dots, S_k \in S: S_1 \cup \dots \cup S_k = R$
- 7: Values $\leftarrow \{S_l \mid 1 \leq l \leq k\}$
- 8: **for** V in Values **do**
- 9: replace $x_i R x_j$ with $x_i V x_j$ in C
- 10: result = **Consistency**(C)
- 11: **if** result \neq **None** **then**
- 12: **return** result
- 13: **return** **None**

Iterative Consistency Algorithm

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Consistency(C)

Input: A constraint network C

Output: A refined constraint network C' if C is satisfiable or None

```
1: Stack ← {} // Initialize stack
2: if not Path-Consistency(C) then
3:   return None
4: while 1 do
5:   if no constraint can be split then
6:     return C
7:   else
8:     choose an unprocessed constraint  $x_i R x_j$  and split  $R$  into  $S_1, \dots, S_k \in S: S_1 \cup \dots \cup S_k = R$ 
9:     Values ←  $\{S_l \mid 1 \leq l \leq k\}$ 
10:    while 1 do
11:      if not Values then
12:        while Stack do
13:          Values = Stack.pop()
14:          if Values then
15:            break
16:          else
17:            return None
18:        V = Values.pop()
19:        replace  $x_i R x_j$  with  $x_i V x_j$  in C
20:        if Path-Consistency(C) then
21:          break
22:        Stack.push(Values)
23: raise RuntimeError, Can't happen
```

Heuristics

- Split set specific
 - Base relations set: Average branching factor of 4.0
 - Horn relations set ($\hat{\mathcal{H}}_8$): Average branching factor of 1.4375
- Constraint specific
 - Static/Dynamic: constraint processing is done *statically* before or *dynamically* during the search
 - Local/Global: constraint evaluation based on *local* heuristic weight or *global* heuristic criterion
- Value specific
 - Choice of a sub-relation based on its constrainedness

Comparing PyRCC8 to Other Reasoners

- We compare PyRCC8 to the following qualitative spatial reasoners:
 - Renz's solver⁷
 - GQR⁸
- Different size n of instances from $A(n, d = 9.5, l = 4.0)$ were used

⁷ <http://users.rsise.anu.edu.au/~Ejrenz/software/rcc8-csp-solving.tar.gz>

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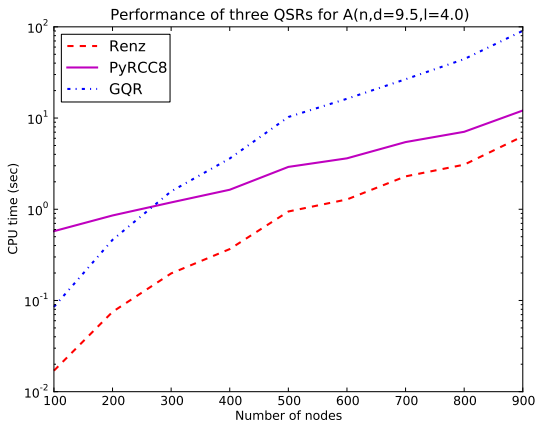
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- Up till now, all approaches in qualitative spatial reasoning enforce path consistency on a complete spatial network
- We propose enforcing path consistency on a *chordal* spatial network [2] as Chmeiss and Condotta have done for temporal networks [3], and we call this type of local consistency as ∇ -*path consistency* for clarity

∇ -Path Consistency

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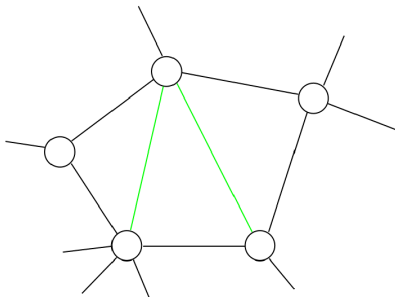
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- We propose enforcing path consistency on a *chordal* spatial network [2] as Chmeiss and Condotta have done for temporal networks [3], and we call this type of local consistency as ∇ -*path consistency* for clarity

Chordal Graph

- A graph is chordal if each of its cycles of four or more nodes has a *chord*, which is an edge joining two nodes that are not adjacent in the cycle
- An example of a chordal graph is shown below:



Triangulation

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- Triangulation of a given graph is done by eliminating the vertices one by one and connecting all vertices in the neighbourhood of each eliminated vertex with *fill edges*
- Determining a minimum triangulation is an NP-hard problem
- Use of several heuristics for sub-optimal solutions (e.g. minimum degree, minimum fill)
- Chordality checking can be done efficiently in $O(|V| + |E|)$ time, for a graph $G = (V, E)$ (e.g., with MCS, LexBFS)

Preliminaries

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- Let $G = (V, E)$ be an undirected chordal graph. There exists a tree T , called a *clique tree* of G , whose vertex set is the set of maximal cliques of G and whose edge set is the set of minimal separators of G .
- Let C be a constraint network from a given constraint satisfaction problem (CSP). We will use \mathcal{V}_C to refer to the set of variables of C . If \mathcal{V} is any set of variables, $C_{\mathcal{V}}$ will be the constraint network that results from C by keeping only the constraints which involve variables of \mathcal{V} .

Patchwork Property in RCC-8 Networks

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Definition

We will say that a CSP has the patchwork property if for any finite satisfiable constraint networks C and C' of the CSP such that $C_{V_C \cap V_{C'}} = C'_{V_C \cap V_{C'}}$, the constraint network $C \cup C'$ is satisfiable [4].

Proposition

The three CSPs for path consistent $\hat{\mathcal{H}}_8$, \mathcal{C}_8 , and \mathcal{Q}_8 networks, respectively, all have patchwork [4].

Proposition

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Proposition

Let C be an RCC-8 constraint network with relations from $\hat{\mathcal{H}}_8, \mathcal{C}_8$, and \mathcal{Q}_8 on its edges. Let G be the chordal graph that results from triangulating the associated constraint graph of C , and T a clique tree of G . Let C' denote the constraint network corresponding to G (C' is C plus some universal relations corresponding to fill edges). C is consistent if all the networks corresponding to the nodes of T are path consistent.

Example

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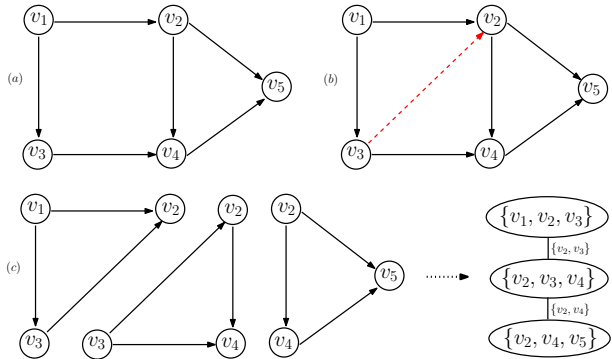
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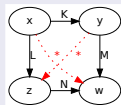
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Corollary

Corollary

Given the RCC-8 network shown below, suppose **K**, **L**, **M**, and **N** are RCC-8 relations from any tractable subset of RCC-8, and suppose the diagonals are the universal relation.



The following hold:

- 1 If the network is inconsistent, then both diagonals cannot be instantiated.
- 2 Both of the diagonals lead to the same pruning of relations **K**, **L**, **M**, and **N**

- PyRCC8 ∇ is a chordal reasoner which was developed by extending PyRCC8
- Similarly to PyRCC8, PyRCC8 ∇ offers a ∇ -path consistency algorithm for solving tractable RCC-8 networks and a backtracking-based algorithm for general networks

∇ -Path Consistency Algorithm

∇ -Path-Consistency(C, G)

Input: A constraint network C and its chordal graph G

Output: True or False

```
1:  $Q \leftarrow \{(i, j) \mid (i, j) \in E\}$  // Initialize the queue
2: while  $Q$  is not empty do
3:   select and delete an  $(i, j)$  from  $Q$ 
4:   for each  $k$  such that  $(i, k), (k, j) \in E$  do
5:      $t \leftarrow C_{ik} \cap (C_{ij} \diamond C_{jk})$ 
6:     if  $t \neq C_{ik}$  then
7:       if  $t = \emptyset$  then
8:         return False
9:        $C_{ik} \leftarrow t$ 
10:       $C_{ki} \leftarrow \checkmark$ 
11:       $Q \leftarrow Q \cup \{(i, k)\}$ 
12:      $t \leftarrow C_{kj} \cap (C_{ki} \diamond C_{ij})$ 
13:     if  $t \neq C_{kj}$  then
14:       if  $t = \emptyset$  then
15:         return False
16:        $C_{kj} \leftarrow t$ 
17:        $C_{jk} \leftarrow \checkmark$ 
18:        $Q \leftarrow Q \cup \{(k, j)\}$ 
19: return True
```

Complexity Analysis

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- Let δ denote the maximum degree of a vertex of G
- For each arc (i, j) selected at line 3, we have at most δ vertices of G corresponding to index k such that v_i, v_j, v_k forms a triangle
- Additionally, there exist $|E|$ arcs in the network and one can remove at most $|\mathcal{B}|^9$ values from any relation that corresponds to an arc
- It results that the time complexity of ∇ -path consistency is $O(\delta \cdot |E| \cdot |\mathcal{B}|)$

⁹ \mathcal{B} refers to the set of base relations of RCC-8

Pruning Capacity

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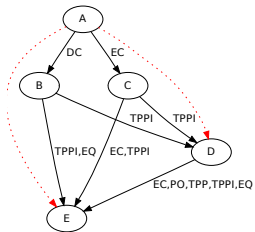
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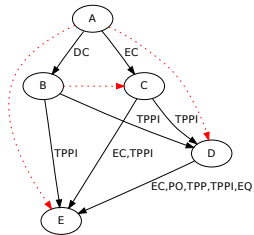
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(a) Chordal graph



(b) Complete graph

Recursive ∇ -Consistency Algorithm

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∇ -Consistency(C, G)

Input: A constraint network C and its chordal graph G

Output: A refined constraint network C' if C is satisfiable or None

```
1: if not  $\nabla$ -Path-Consistency(C, G) then
2:   return None
3: if no constraint can be split then
4:   return C
5: else
6:   choose unprocessed constraint  $x_i R x_j$ ; split R into  $S_1, \dots, S_k \in S: S_1 \cup \dots \cup S_k = R$ 
7:   Values  $\leftarrow \{S_l \mid 1 \leq l \leq k\}$ 
8:   for V in Values do
9:     replace  $x_i R x_j$  with  $x_i V x_j$  in C
10:    result =  $\nabla$ -Consistency(C, G)
11:    if result  $\neq$  None then
12:      return result
13: return None
```

Iterative ∇ -Consistency Algorithm

∇ -Consistency(C, G)

Input: A constraint network C, A chordal graph G

Output: A refined constraint network C' if C is satisfiable or None

```
1: Stack  $\leftarrow$  {} // Initialize stack
2: if not  $\nabla$ -Path-Consistency(C, G) then
3:   return None
4: while 1 do
5:   if no constraint can be split then
6:     return C
7:   else
8:     choose unprocessed constraint  $x_i R x_j$ ; split R into  $S_1, \dots, S_k \in S: S_1 \cup \dots \cup S_k = R$ 
9:     Values  $\leftarrow$  { $S_l \mid 1 \leq l \leq k$ }
10:    while 1 do
11:      if not Values then
12:        while Stack do
13:          C, Values = Stack.pop()
14:          if Values then
15:            break
16:          else
17:            return None
18:        V = Values.pop()
19:        replace  $x_i R x_j$  with  $x_i V x_j$  in C
20:        if  $\nabla$ -Path-Consistency(C, G) then
21:          break
22:        Stack.push(C, Values)
23: raise RuntimeError, Can't happen
```


Comparing PyRCC8 ∇ to PyRCC8

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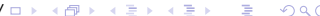
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- We compare PyRCC8 ∇ to PyRCC8, a complete graph dedicated reasoner, using the following data:
 - *Random instances* composed from the set of all RCC-8 relations
 - The *admingeo*¹⁰ dataset

¹⁰<http://data.ordnancesurvey.co.uk/ontology/admingeo/> 

Experimenting with Random Instances

- We generated instances from $A(100, d, l = 4.0)$, for d varying from 3 to 15 with a step of 0.5. For each series, 300 networks were generated using Renz's network generator¹¹
- We used the Horn relations set as our split set, and the dynamic/local constraint scheme with a weighted queue configuration, since it proved to be the best combination for both reasoners, confirming the results in [5]

¹¹<http://users.rsise.anu.edu.au/%7Ejrenz/software/rcc8-csp-solving.tar.gz> 

Comparison Diagram on CPU time

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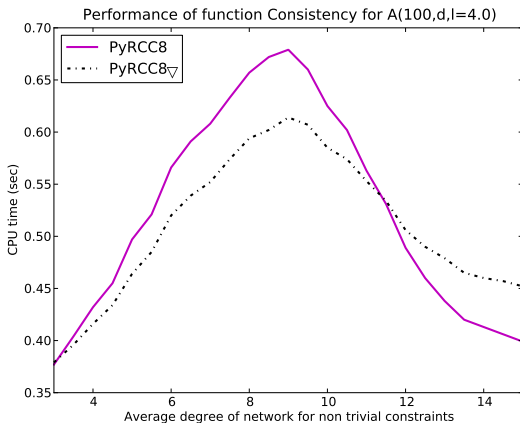
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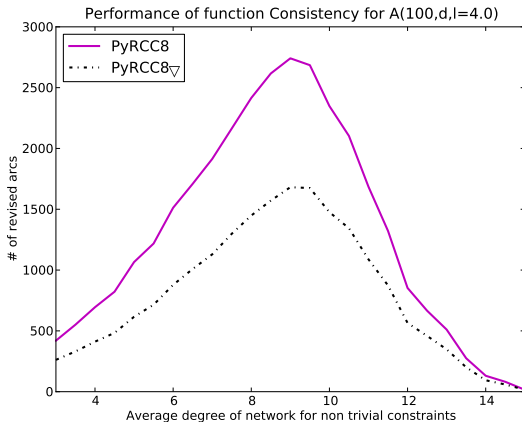
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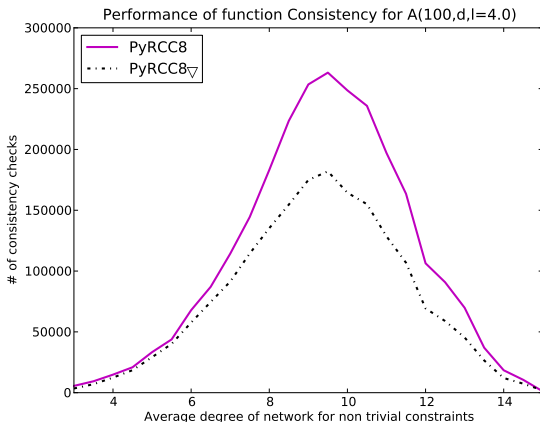
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	PyRCC8	PyRCC8▽	%
CPU time	0.524s	0.509s	2.80%
revised arcs	1300.681	801.204	38.40%
checked constraints	105751.173	74864.985	29.21%

Table: Comparison based on the average of different parameters

Experimenting with the Admingeo Dataset

- The *admingeo*¹² dataset consists of 11761 regions and 77910 base relations, thus being an extremely large and sparse network, making itself a good candidate for stress testing different path consistency implementations
- We used a simple queue configuration, since the weighted variants made no difference on this dataset other than using much more memory

¹²<http://data.ordnancesurvey.co.uk/ontology/admingeo/> 

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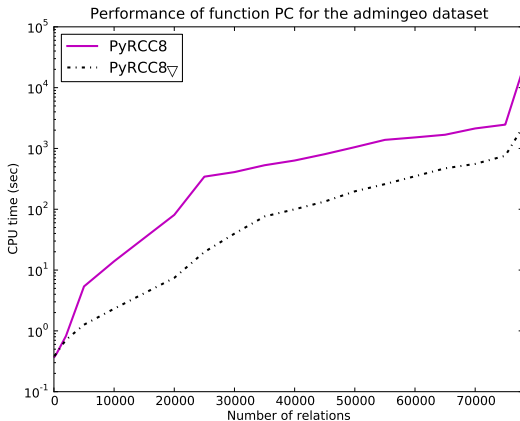
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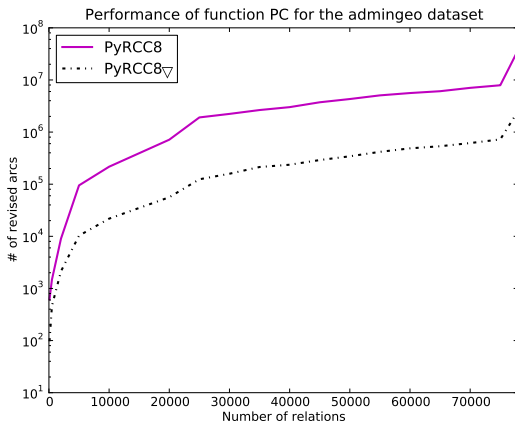
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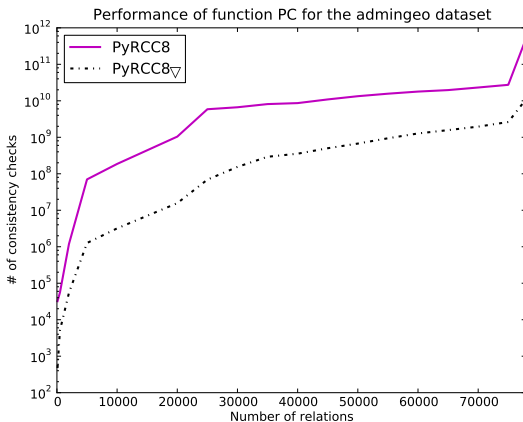
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	PyRCC8	PyRCC8▽	%
CPU time	1825.129s	289.203s	84.15%
revised arcs	4834133.78	373080.28	92.28%
checked constraints	$3.606e + 10$	$1.181e + 09$	96.72%

Table: Comparison based on the average of different parameters

Test Machine

- All experiments were carried out on a computer with an Intel Xeon 4 Core X3220 processor with a CPU frequency of 2.40 GHz, 8 GB RAM, and the Debian Lenny x86 64 OS
- Renz's solver and GQR were compiled with gcc/g++ 4.4.3
- PelletSpatial was run with OpenJDK 6 build 19, which implements Java SE 6
- PyRCC8 was run with PyPy 1.8, which implements Python 2.7.2
- Only one of the CPU cores was used for the experiments

Conclusions

- We made the case for a new generation of RCC-8 reasoners implemented in Python, and making use of advanced Python environments, such as PyPy, utilizing trace-based JIT compilation techniques
- We introduced ∇ -path consistency for RCC-8 networks
- We showed that ∇ -path consistency is sufficient to decide the consistency problem for the maximal tractable subsets $\hat{\mathcal{H}}_8$, \mathcal{C}_8 , and \mathcal{Q}_8 of RCC-8
- We implemented a chordal graph dedicated reasoner for RCC-8 networks
- We showed experimentally that ∇ -path consistency can offer a great advantage over full path consistency on sparse graphs

Main Points

- Explore self learning heuristics regarding variable and value selection
- Create module to generate spatial CSPs
- Transform PyRCC8 into a generic qualitative reasoner
- Use other methods of triangulation and compare the behavior of partial path consistency for these different methods
- Explore applications with distributed systems such as Pregel¹³
- Research on solving chordal spatial networks incrementally
- Perform experiments with other possible real datasets, such as GADM¹⁴

¹³<http://slideshare.net/shatteredNirvana/pregel-a-system-for-largescale-graph-processing>

¹⁴<http://gadm.geovocab.org/>

Acknowledge

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- I would also like to thank my colleagues, and Katia Papakonstantinou especially, for their help, interest, and advice

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Any Questions? 😊

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Any Questions? 😊