



The antennas preassignment problem

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Accepted 3 March 2004

Abstract

We deal here, with a real-world problem, called ANTENNAS PREASSIGNMENT problem. We first show how it can be modeled as a particular kind of circular cutting problem. Next, we develop a simulated annealing heuristic solving it with good performance. Our method is based upon the definition of an energy function reflecting the objective to be optimized; small values of this function correspond to local optima of good quality for our problem.

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1. Introduction

In this paper we deal with the following problem. We wish to build antennas in order that they cover a given land-area. We assume that the antennas coverage is a circle of a certain radius and that the area to be covered is rectangular. We also assume that the technical characteristics of all antennas are identical except of their surface-coverage abilities that can differ the ones from the others. Since land-area is still under construction and/or internal transformation, we cannot have, at the initial stage, a complete knowledge of the places where antennas must be stalled, but one has an a priori knowledge of the types of antennas needed. On the other hand, coverage of the area dealt is relatively urgent in such a way that one cannot wait that all constructions and transformations planned are accomplished before ordering antennas (given that ordering and fabrication delays of such components is rather long). Hence, one needs that a certain quantity of antennas is available once area is ready for coverage, even if the total quantity of them is not sufficient to cover the whole area. A way to tackle fuzziness occurred is the following:

We are given a certain number of types of circular-coverage antennas and a rectangular area to be covered by them; the objective is to place antennas in such a way that

1. coverage areas of antennas do not mutually intersect and
2. the total coverage of rectangular area is maximized.

This is what we call ANTENNAS PREASSIGNMENT problem. Obviously, its solution does not constitute a feasible solution for the initial problem; however, it represents a feasible estimation of the lower bound on the number of antennas needed to be ordered so that coverage works are not delayed (even if additional components will have to be ordered during works).

ANTENNAS PREASSIGNMENT turns out to be a member of a well-known family of combinatorial problems, called *circular cutting problems*. Our problem can be formally expressed in the following way:

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Instance. A rectangle R of length L and width W (this is the area to be covered) and ℓ types of cycles, type $i \in \{1, \dots, \ell\}$, been characterized by its surface s_i (that models, in a proper way, the surface-coverage ability of an antenna of type i ; observe that once s_i is given, radius r_i is also fully determined). We suppose that the number n_i of antennas of type i , $i = 1, \dots, \ell$, is at least $\lfloor L \times W/s_i \rfloor$, i.e., that the number of antennas of any type i is sufficient that R is covered without surface-coverage overlapping exclusively by antennas of type i .

Objective. A placement of circles in R in such a way that no two of them have non-empty intersection and that the maximum of the surface of R is covered. We assume that the coverage of R is the sum of the surfaces of the circles placed by the solution.

Obviously, the problem at hand is **NP-hard**, since it is an immediate generalization of **KNAPSACK** [3]. We propose in this paper an algorithm based upon simulated annealing (see [1,2,6] for details) that approximately solves **ANTENNAS PREASSIGNMENT**. We first model feasible solutions of the problem in terms of an energy function to be minimized. We then propose an algorithm that searches for local optima energy function. These local optima correspond to solutions for **ANTENNAS PREASSIGNMENT**.

Let us note that this paper is simultaneously new, as it models and solves a real-world problem, and a kind of continuation and extension of [5], where a simulated annealing method has been proposed for an analogous version of circular cutting problem. The energy model and the perturbation-mechanism in [5] were less exhaustive and fine than here. Despite, results obtained there are very interesting. This, together with the fact that we have tackled almost the same problem working on an industrial contract, has motivated us in solving it by extending and refining the work in [5].

2. The simulated annealing model

Denote by S_0 the surface of R , by S_r the set $\{(x, y) : x \geq L \text{ and } 0 \leq y \leq W\}$, and by \bar{S} the set $\mathbb{R}^2 \setminus (S_0 \cup S_r)$; finally, denote by S_i the surface of piece i .

The general idea of our method is the following:

- we first place circles in \mathbb{R}^2 without overlappings;
- next, suitably defining an energy function that describes feasible solutions of **ANTENNAS PREASSIGNMENT** and their values:
 - we first force circles to be concentrated in the strip $S_0 \cup S_r$, even if this concentration produces overlappings in both S_0 and S_r ;
 - finally, we eliminate overlappings from S_0 by moving, in a proper way, circles out from S_0 .

2.1. The energy function

The energy function is based upon the following requirements:

1. situations where: two circles intersect mutually, or a circle intersects R , or, finally, a circle lies in the exterior of R , have to be avoided in a feasible solution;
2. the greater the number of circles feasibly placed in R , the higher the usage (covering) of the surface of R .

We will describe in what follows, how to construct an energy function E , global or local optima of which correspond to good **ANTENNAS PREASSIGNMENT** solutions. Set $m = \sum_{i=1}^{\ell} n_i$, and denote by $A(i)$ the area covered by circle i .

In order to introduce a maximum number of circles in the rectangle, we “strain” them to be concentrated to the bottom-left corner of R . By introducing a large number of circles in R we fulfill requirement 2 but this greedy policy may introduce intersections between circles not only inside R , but also outside, therefore violation of requirement 1 may be happened.

Because of this contradiction between requirements 1 and 2, a trade-off between them is performed by using penalty parameters. We use four such parameters:

- p_1 penalizing non-empty intersections between circles inside R ,
- p_2 penalizing intersections of the circles with \bar{S} ,
- p_3 penalizing intersections of the circles with S_r , and
- p_4 penalizing the distance from the point $(0, 0)$ (bottom-left corner of R).

We split the expression for the energy into two parts: $E^{(1)}$ and $E^{(2)}$. The first one will control whether our configuration respects requirement 1. For any undesired position of a circle, the measure of “how much requirement 1 is violated” is expressed by an area, which will take part in the expression of energy presented below.

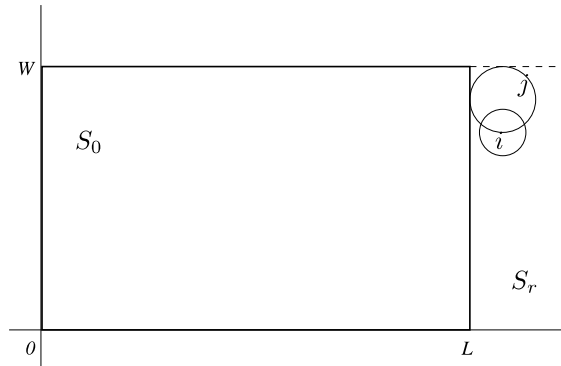


Fig. 1. Case 1.

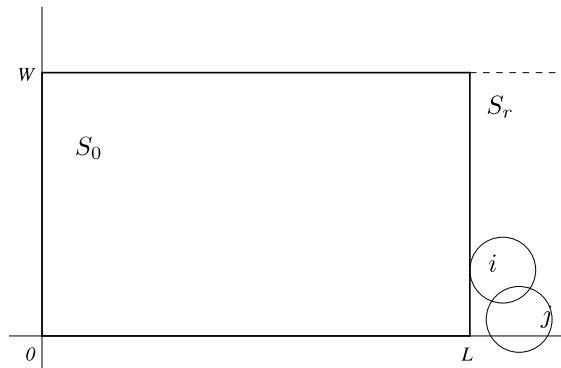


Fig. 2. Case 2.

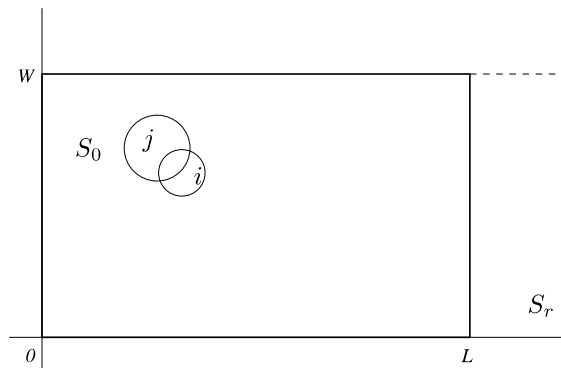


Fig. 3. Case 3.

All the possible undesirable positions of circles and their contribution to the first part of the energy are:

1. if a circle j intersects i and if both i and j lie in S_r , $E_{ij}^{(1)} = p_3|A(i) \cup A(j)|$ (Fig. 1);
2. if a circle j intersects i and if i lies in S_r and j intersects \bar{S} , $E_{ij}^{(1)} = \max\{p_2, p_3\}|A(j)| + p_3(|A(i)| - |A(i) \cup A(j)|)$ (Fig. 2);
3. if a circle j intersects i and if both i and j lie in the interior of R , $E_{ij}^{(1)} = p_1|A(i) \cap A(j)|$ (Fig. 3);
4. if a circle j intersects i and if i lies in the interior of R and j intersects \bar{S} , $E_{ij}^{(1)} = (p_1 + p_2)|A(j)|$ (Fig. 4);
5. if a circle j intersects i and if both i and j also intersect \bar{S} , $E_{ij}^{(1)} = (p_1 + p_2)|A(i) \cup A(j)|$ (Fig. 5);

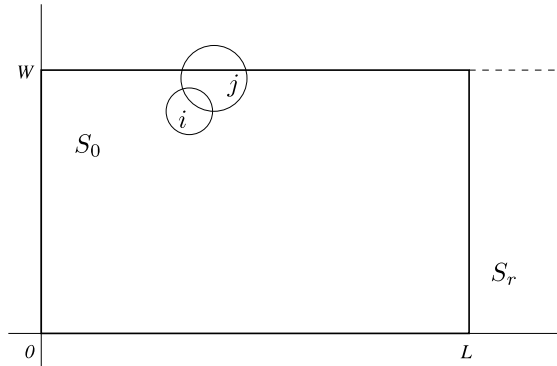


Fig. 4. Case 4.

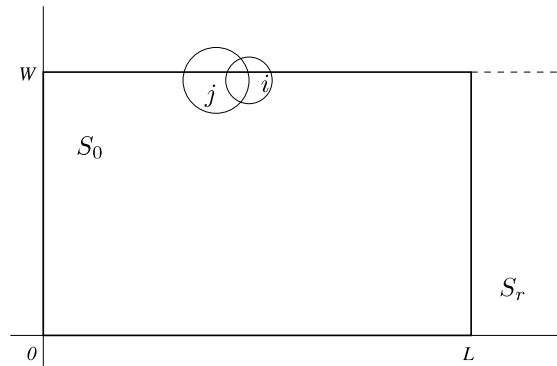


Fig. 5. Case 5.

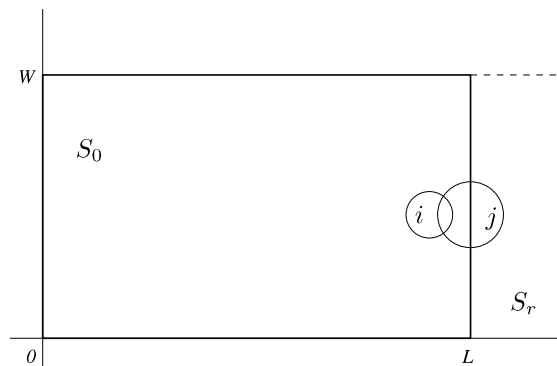


Fig. 6. Case 6.

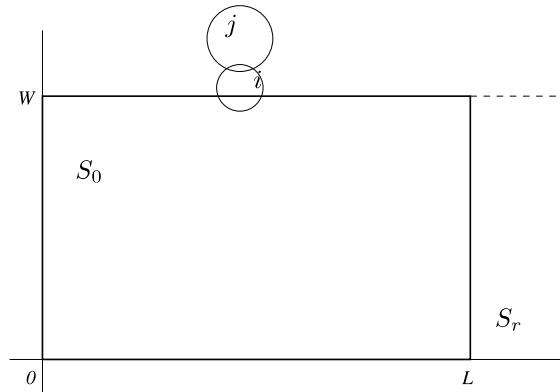


Fig. 7. Case 7.

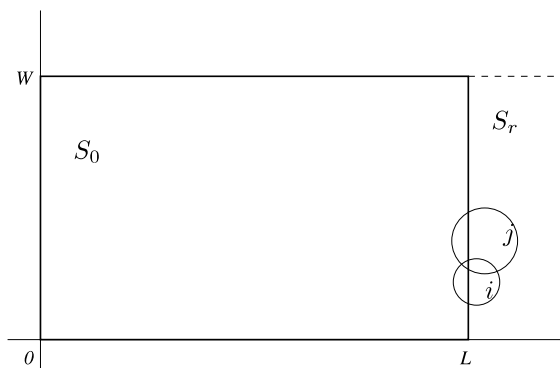


Fig. 8. Case 8.

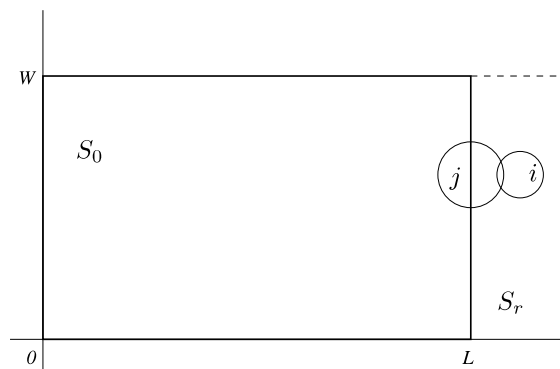


Fig. 9. Case 9.

6. if a circle j intersects i and if i lies in the interior of R and j intersects S_r , $E_{ij}^{(1)} = (p_1 + p_3)|A(j)|$ (Fig. 6);
7. if a circle j intersects i and if i intersects R and j lies in \bar{S} , $E_{ij}^{(1)} = p_2|A(i)|$ (Fig. 7);
8. if a circle j intersects i and if both i and j also intersect S_r , $E_{ij}^{(1)} = (p_1 + p_3)|A(i) \cup A(j)|$ (Fig. 8);
9. if a circle j intersects i and if j intersects R and i lies in S_r , $E_{ij}^{(1)} = p_3|A(i)|$ (Fig. 9);
10. if a circle j intersects i and if circle j intersects both \bar{S} and S_r and if i lies in \bar{S} or in S_r , then $E_{ij}^{(1)} = \max\{p_2, p_3\}|A(j)|$ (Fig. 10);

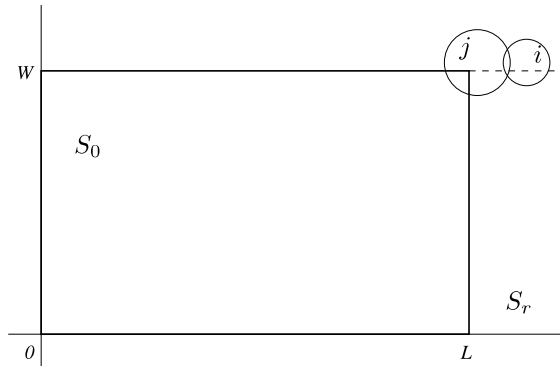


Fig. 10. Case 10.

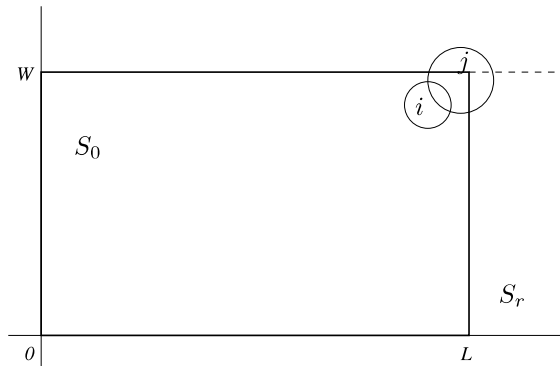


Fig. 11. Case 11.

- 11. if a circle j intersects i and if circle j intersects both \bar{S} and S_r and if i lies in the interior of R , then $E_{ij}^{(1)} = (p_1 + \max\{p_2, p_3\})|A(j)|$ (Fig. 11);
- 12. if a circle j lies in S_r without intersecting any other circle, $E_j^{(1)} = p_3|A(j)|$;
- 13. if a circle j lies in \bar{S} without intersecting any other circle, $E_j^{(1)} = p_2|A(j)|$.

In summary, the term of E dealing with requirement 1 is $E^{(1)} = \sum_{i=1}^{m-1} \sum_{j>i} E_{ij}^{(1)} + \sum_{j=1}^m E_j^{(1)}$.

Let now $E_i^{(2)}$ be the basic component of the term of E dealing with requirement 2. A measure of how close to the bottom-left corner of R a circle i lies (and, consecutively, how many circles are placed inside R) is the value of the expression $D_i = r_i(|y_i - r_i| + |x_i - r_i|)$ (where (x_i, y_i) are the coordinates of the center of circle i), notice that if $(x_i, y_i) = (r_i, r_i)$, then $D_i = 0$, while if $(x_i, y_i) \neq (r_i, r_i)$, the non-zero value of $(|y_i - r_i| + |x_i - r_i|)$ is amplified by its multiplication by r_i . In other words, informally, the fact that a circle i is far from the bottom-left corner of R induces a large value for D_i , and consequently, for the energy function. Therefore, we form the second part of the energy function as:

$$E_i^{(2)} = p_4(r_i(|y_i - r_i| + |x_i - r_i|))$$

and the term of E dealing with this requirement is $E^{(2)} = \sum_{i=1}^m E_i^{(2)}$. Finally,

$$E = E^{(1)} + E^{(2)} = \left(\sum_{i=1}^{m-1} \sum_{j>i} E_{ij}^{(1)} + \sum_{j=1}^m E_j^{(1)} \right) + \sum_{i=1}^m E_i^{(2)} \tag{1}$$

We now show how to adjust the penalty parameters p_1, p_2, p_3 , and p_4 in order to obtain feasible configurations. As we mentioned before, we first concentrate a maximum number of circles in the strip $S_0 \cup S_r$ (in order to be introduced later on in R) by allowing some overlapping. For this, we have to discourage the circles to take places in \bar{S} . We therefore

consider a large value for p_2 . In this way, since E (expression (1)) is to be minimized, the terms with factor p_2 must be the least possible in number or/and to have values close to 0. Since we have decided to allow some overlapping, we consider $p_1 < p_2$.

In order to avoid overlapping inside R , we encourage a certain shifting to the right (even if this shifting will produce the introduction of some circles from S_0 to S_r); to achieve this we set $p_3 < p_1$. Thus, finally, $p_3 < p_1 < p_2$, and the term $\sum_{i=1}^{m-1} \sum_{j>i} E_{ij}^{(1)} + \sum_{j=1}^m E_j^{(1)}$ encourages the introduction of circles in R , avoiding both overlapping between circles and overflow of circles with respect to the edges of R .

We now complete the adjustment by trying to further strain the circles to be placed in the strip $S_0 \cup S_r$ in order to ensure that a maximum number of circles will finally be introduced in the strip without overlapping. This can be obtained using parameter p_4 . At the initial step of the algorithm, we consider a value for p_4 as large as the one for p_2 (by allowing high initial-energy values). In other words, we attribute the same importance to both the overlapping elimination and the concentration of the circles to the bottom-left corner of R . Progressively, the value of p_4 will be reduced. This means that we accept the shifting of some circles from S_0 to S_r in order to avoid overlapping into S_0 . So, if p_4 becomes close to 0 and if no overlapping is produced, then the value of the energy becomes small.

2.2. Initialization of the method and achievement of consecutive neighborhoods

The initial configuration is performed by placing inside the rectangle R as many circles as possible, taking care to the feasibility of the configuration; in other words by fully obeying criterion 1. To achieve that we take randomly a circle and we put it as near as possible to the lower left corner of the rectangle. As soon as a circle has been placed, we “subtract” from the rectangle R a square that enscripts exactly this circle. We continue this way horizontally until we reach the rightmost edge. At this point a strip with height the maximum diameter between the (randomly) already chosen circles has been extracted from the initial rectangle. We continue this way by placing randomly chosen circles until available area of the rectangle is exhausted. The remaining cycles are placed around the three edges of the rectangle in the \bar{S} .

A placement of cycles around and inside R determines a *configuration* characterized by an energy value. Obviously any configuration coincides with a solution (feasible or unfeasible) of ANTENNAS PREASSIGNMENT. The *neighbor* of a configuration C is a new configuration C' obtained by applying on C one of the transformations we will describe just below. In what follows, the neighborhood of a circle i is the set of circles, the center of which lies into a cycle of center (x_i, y_i) and radius $2r_i$.

Transformation T1. This transformation is performed either horizontally or vertically (in both cases parallel either to axis $\vec{0x}$, or to axis $\vec{0y}$). We randomly choose a circle i ; for a circle j neighbor of i , with $y_j = y_i$ we define $\delta_j^x = |x_i - x_j| - (r_i + r_j)$, its horizontal distance from i . For a circle l neighbor of i , with $x_l = x_i$, we define $\delta_l^y = |y_i - y_l| - (r_i + r_l)$, its vertical distance from i . Transformation **T1** consists of bringing circle $k \in \arg \min_{1 \leq j, l \leq m} \{ \delta_j^x, \delta_l^y \}$, k being a neighbor of i , closer to i .

Transformation T2. This transformation is performed diagonally with respect to axes $\vec{0x}$ and $\vec{0y}$. Given two pieces i and j , we define their diagonal distance $d = [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2}$. As previously, we try to move piece j , lying on the neighborhood of i , in such a way that the two pieces touch each other and, moreover, the center of j always lies on the straight line defined by the points (x_i, y_i) and (x_j, y_j) . This movement can be seen as two simultaneous movements, one parallel to axis $\vec{0x}$ by a distance δx , and the other parallel to axis $\vec{0y}$ by a distance δy . These two distances are defined by

$$(\delta x, \delta y) = \begin{cases} (\Delta x (1 - \frac{r_i+r_j}{d}), \Delta y (1 - \frac{r_i+r_j}{d})) & d \geq r_i + r_j \\ (\Delta x (\frac{r_i+r_j}{d} - 1), \Delta y (\frac{r_i+r_j}{d} - 1)) & \text{otherwise} \end{cases}$$

where $\Delta x = |x_i - x_j|$ and $\Delta y = |y_i - y_j|$.

Transformation T3. Let us consider two circular pieces i and j touching each other, and suppose that i lies on the left of j . Let us denote by (x_t, y_t) the coordinates of the point where a tangent of j , vertical to the axis $\vec{0x}$, touches j . Transformation **T3** consists of performing a symmetric displacement of j with respect to the axis represented by the straight line vertical to $\vec{0x}$ containing point (x_t, y_t) , and then a right shifting of the two pieces by an horizontal distance $|x_t - x_i - r_i|$. The meaning of this transformation is that if i has important overlapping surfaces on its left, and moreover, $r_j < r_i$, by **T3** we can reduce overlapping. On the other hand, if $r_j > r_i$, and there exist some “holes” (waste) on the left of i , then **T3** will reduce their surface.

Transformation T4. Let us consider a piece j into S_0 and tangent either (i) to straight line $y = W$, or (ii) to straight line $x = L$. Let us also consider a piece i tangent to j and lying out of S_0 . Then, transformation **T4** consists of a symmetric displacement of i and j with respect to the straight line $y = W$ in case (i), or $x = L$ in case (ii). This transformation allows new pieces to be introduced into S_0 .

Transformation T5. Consider a piece i lying on S_0 and a piece j out of S_0 , the radii of which verify $0 < \max\{r_i, r_j\}/|r_i - r_j| \leq \beta$ for a positive constant $\beta > 1$. Then, transformation **T5** consists of an interchanging between i and j . The meaning of **T5** is that if $r_i > r_j$, then, the performed interchanging will reduce the overlapping into S_0 , while, in the opposite case, the waste will be reduced.

Transformations **T1**, **T2** and **T3** operate in the interior of S_0 and perform a local rearrangement of pieces already introduced. Transformations **T4** and **T5** are more “global” and interchange pieces inside S_0 with pieces outside it.

2.3. The overall algorithm

We now give an overall specification of the simulated annealing method devised, denoted by SA_ALG in what follows:

1. begin with an initial configuration and an initial temperature $T_k = 10$; define two thresholds: the first one being the length of the Markovian chain for a given temperature (fixed to $3m$, where $m = \sum_{i=1}^l n_i$), and the second one being a fraction of the length of the chain (in our case this threshold is half the length of the Markovian chain, i.e., $\lfloor 3m/2 \rfloor$);
2. repeat the following steps:
 - (a) choose (randomly) a circle lying inside the rectangle; for *any* circle that belongs to the neighborhood of the chosen circle, apply one of the transformations **T1**, **T2**, **T3**; for the new configuration calculate its energy E ; if $\Delta E \leq 0$ or if $\exp(-\Delta E/T_k) > \text{random}[0, 1)$, then keep the most recent configuration;
 - (b) for the piece chosen at the previous step (step 2a), apply one of the transformations **T4**, **T5**; for the new configuration obtained, calculate its energy E ; if $\Delta E \leq 0$ or if $\exp(-\Delta E/T_k) > \text{random}[0, 1)$, then keep the most recent configuration;
 until the number of transformations that do not produce a better configuration exceeds the small threshold *or* the number of transformations performed so far for the current temperature exceeds the big threshold;
3. reduce temperature T_k by multiplying it by a constant smaller than 1 (in our case this constant has been fixed to 0.95);
4. repeat steps 2 and 3, until one of the exit criteria is satisfied.

In the simulated annealing algorithm sketched just above, we use the following two exit criteria:

- (i) a threshold for the temperature (fixed for our algorithm to 10^{-3});
- (ii) a threshold ϵ for the quantity $|E^* - (\sum_{k=1}^{\rho} E(k)/\rho)|$ where E^* is the smallest energy achieved so far, $E(k)$ is the energy for the last configuration produced for a temperature T_k , and ρ is the number of configurations tested for the temperature T_k (here, we have taken $\epsilon = 10^{-3}$).

3. Some computational studies

As we have mentioned in the beginning of the paper, our initial purpose was to face ANTENNAS PREASSIGNMENT problem as it has been posed to us by industrial decision makers. Solution computed was completely satisfactory for them and fulfilled all of their requirements.

We then have performed a limited computational experience on instances which, even they did not correspond to real-world configurations, they were validated by our partners. More precisely, we have generated 20 instances for which optimal solutions have been computed by exhaustive search. Dimensions L and W for R are randomly drawn from interval $[25, 300]$. For any instance, we have assumed equal circles the radii of which have been randomly drawn from interval $[0.5, 7.85]$. For any test (using circles of radius r), the number of circles available was taken equal to $3\lfloor L \times W / (4r^2) \rfloor$. Penalties values have been considered as follows:

- p_1 and $p_3 \in [0.1, 1]$;
- $p_2 = 1$;
- p_4 has been initialized to 1 (as the tenth of the initial temperature); for any change for temperature T_k , p_4 was set to $p_4 = T_k/10$.

Even if algorithm SA_ALG, may produce a number of configurations corresponding to unfeasible solutions for ANTENNAS PREASSIGNMENT, no such configurations have been observed in our tests (this, with respect to [5], maybe due to a more exhaustive study of unfeasibility causes performed in the current paper, or to a different choice of the several simulated annealing parameters). In all,

- the average differential ratio (i.e., the ratio of the total coverage produced by the circles placed by our algorithm to the total coverage produced by the circles of the optimal solution) over the twenty instances tested and over all runs for the different values of penalties p_1 , p_2 and p_3 , is equal to 0.954 (this ratio varies from 0.78 to 1);
- the average ratio of the coverage produced by SA_ALG over the whole surface of the rectangle is equal to 0.89 and varies from 0.72 to 0.95;
- the average CPU-time is equal 7 min on a Pentium 4 at 2.4 GHz with memory of 512 Mb.

4. Research in progress

Our first computational experiments are quite promising and represent an improvement (to be confirmed) with respect to the results presented in [5]. However, more systematic experiments have to be performed in order to establish the pertinence of our model. Our actual research carries over two directions already (partially) dealt in [5].

The first direction is in connexion with the very interesting strip-packing method of [7]. Strip-packing problem is a kind of “dual” of circular cutting, where one is given a set of circular pieces and she/he wishes to place them, without overlapping, in a rectangle of dimensions L and W , so that that either L , or W is minimized. We are working in testing our method on data-sets kindly sent to us by the authors of [7].

The second direction deals with the work of [4] about strip-packing. There, authors consider a fixed number of disks of the same radius and search to determine the side length of the smallest square that contains the disk centers. Testing our method on results of [4] is our second short terms research direction.

Acknowledgements

“Development Agency of Karditsa” (ANKA) is gratefully acknowledged for having proposed us the problem and for financial support.

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