

# Διακριτό πρόβλημα σακιδίου (Discrete Knapsack)

Input:  $|X| = n$   $c_i, a_i, b$  integers

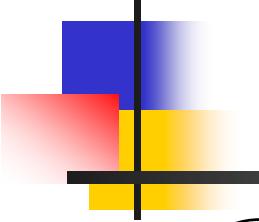
Output:  $Y \subseteq X$  s.t.  $\sum_{x_i \in Y} a_i \leq b$   
and *MAX profit*

Instance:

c: 10, 5, 8

a: 3, 2, 2

b=4, n=3



# Knapsack

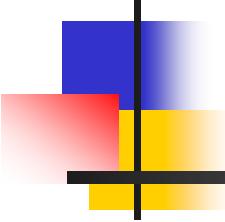
$$x_i \in \{1, 0\}$$

$$\text{Max} \sum_{i=1}^n c_i x_i$$

$$\sum_{i=1}^n a_i x_i \leq b$$

**Instance:**

$$\begin{aligned} & \max 10 x_1 + 5 x_2 + 8 x_3 \\ & 3 x_1 + 2 x_2 + 2 x_3 \leq 4 \end{aligned}$$



# Greedy Knapsack (n, c, a, b)

$c_i/a_i$  in non-increasing order

$$\frac{c_{j1}}{a_{j1}} \geq \frac{c_{j2}}{a_{j2}} \geq \dots \geq \frac{c_{jn}}{a_{jn}}$$

$Y := \emptyset$

for  $i := 1$  to  $n$  do

if  $b \geq a_i$  then

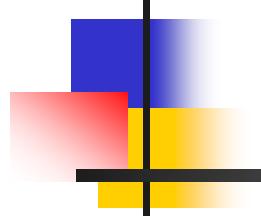
begin

$Y := Y \cup \{x_i\}$

$b := b - a_i$

end

return  $Y$



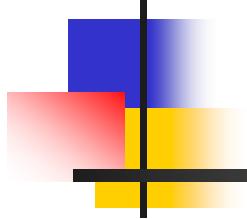
# Knapsack

Παράδειγμα

$$\text{Max } (k+1)x_1 + kx_2 + kx_3$$

$$(k+1)x_1 + kx_2 + kx_3 \leq 2k$$

$$k = 1, 2, \dots$$



# Knapsack

Παράδειγμα

$$\text{Max } (k+1)x_1 + kx_2 + kx_3$$

$$(k+1)x_1 + kx_2 + kx_3 \leq 2k$$

$$k = 1, 2, \dots$$

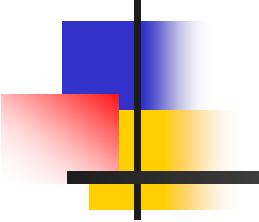
Greedy:  $\frac{k+1}{k+1} \geq \frac{k}{k} \geq \frac{k}{k}$

$$\begin{aligned} x_1 &= 1 \\ x_2 &= 0 \\ x_3 &= 0 \end{aligned} \quad \left. \right\}$$

$$G = \{k + 1\}$$

but

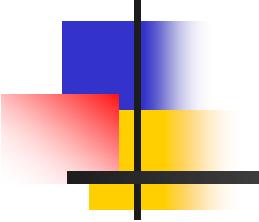
$$x^*_1 = 0, x^*_2 = 1, x^*_3 = 1$$



# Knapsack

$$\text{Max } 2x_1 + kx_2 + kx_3$$

$$x_1 + kx_2 + kx_3 \leq k$$



# Knapsack

$$\text{Max } 2x_1 + kx_2 + kx_3$$

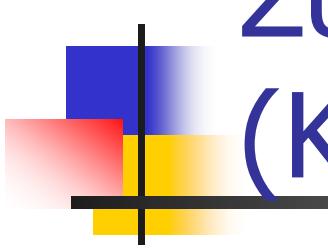
$$x_1 + kx_2 + kx_3 \leq k$$

$$G: \frac{2}{1} \geq \frac{k}{k} \geq \frac{k}{k}$$

$$x_1 = 1$$

$$x_2 = 0$$

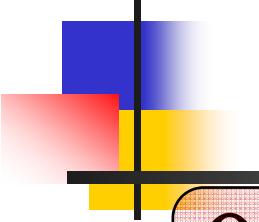
$$x_3 = 0$$



# Συνεχές πρόβλημα σακιδίου (Knapsack)

Input  $/ X \models n \quad c_i, a_i, b$  integers

Output  $Y \subseteq X$  s.t.  $\sum_{x_i \in Y} a_i \leq b$   
and *MAX profit*



# Συνεχές Knapsack

$$0 \leq x_i \leq 1$$

$$\begin{aligned} & \text{Max} \sum_{i=1}^n c_i x_i \\ & \sum_{i=1}^n a_i x_i \leq b \end{aligned}$$

# Συνεχές Knapsack

**ContinuousKnapsack (n, c, a, b)**

Ταξινόμησε τα στοιχεία κατά φθίνουσα σειρά των λόγων  $c_i/a_i$

**For**  $i=1$  to  $n$  **do**  $x_i = 0$   
**i=1, Z=0 (αντικειμενική συνάρτηση)**

**while**  $a_i \leq b$  **do**

$x_i = 1, b = b - a_i, z = z + c_i$

$i = i + 1$

**end while**

$x_i = b/a_i, z = z + c_i x_i$