MIS – Δ/2 προσεγγίσιμο

- Αλγόριθμος
  - ευριστικός “smallest degree”¨: Λύση $a'_2(G)$
  -“exposed vertices”: Λύση $a'_1(G)$

  $a\{G\}$: maximum $\{a'_1(G), a'_2(G)\}$

- τ(G):ελάχιστη διαμέριση σε κλίκες του G

Matching-Exposed vertices

- $a(G) \le \tau(G)$
- $\text{maximum matching}(\mathrm{m}^{'})$

$\sigma\tau o\nu \;\; G \Rightarrow \mathrm{m}^{'} \le n/2 \Rightarrow$

$\Rightarrow a(G) \le \tau(G) \le \underbrace{\mathrm{m}^{'} + \text{Expo.Vert}}$

$\le n/2$

$\mathrm{m}^{'} = n/2 * (1-\gamma), \gamma \notin [0,1]$

$|E| = n - 2\mathrm{m}^{'} = \gamma n (\text{Ind.Set})$

$\mathrm{a}_1^{'}(G) = \gamma n$

Upper Bound for MIS

- Max $\{a'_1(G), a'_2(G)\}$

- τ(G):ελάχιστη διαμέριση σε κλίκες του G

$$a(G) \leq m' + \gamma * n = (n/2) * (1+\gamma) \Rightarrow$$
$$\Rightarrow a(G) \leq (n/2) * (1+\gamma)$$

Approximation ratio (Exposed vertices)

$$a(G)/a_1^{'}(G) \leq (n/2)(1+\gamma)/n\gamma =$$

$$= (\gamma+1)/2\gamma = \rho(\gamma)$$

Approximation ratio (smallest degree)

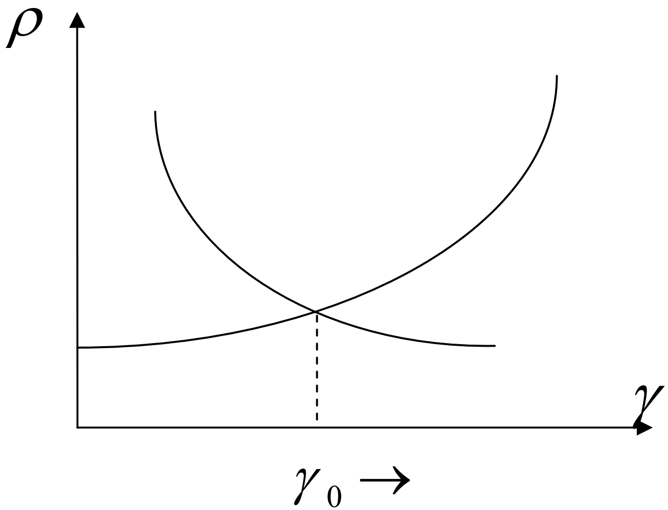
$a_2'(G) \geq [(n-(\delta+1))/\Delta]+1$ (G:connected)

$a(G)/a_2'(G) \leq [(n/2)(1+\gamma)]/\{[(n-(\delta+1))/\Delta]+1\} \leq$

$\leq [\Delta n/2(\gamma+1)]/[n-(\delta+1)+\Delta] \leq [\Delta n(\gamma+1)]/[2(n-1)] =$

$= \rho'(\gamma)$

Break point-approximation ratio



$$\rho(\gamma_0) = \rho'(\gamma_0) \Rightarrow \gamma_0 = \frac{n-1}{\Delta n}$$

$$\rho(\gamma_0) = \frac{\Delta}{2}\left(1 + \frac{1}{n-1}\right) + \frac{1}{2}$$

$$\text{for } n \uparrow \ (\Delta \uparrow) \qquad \rho = \rho(\gamma_0) = \frac{\Delta}{2}$$

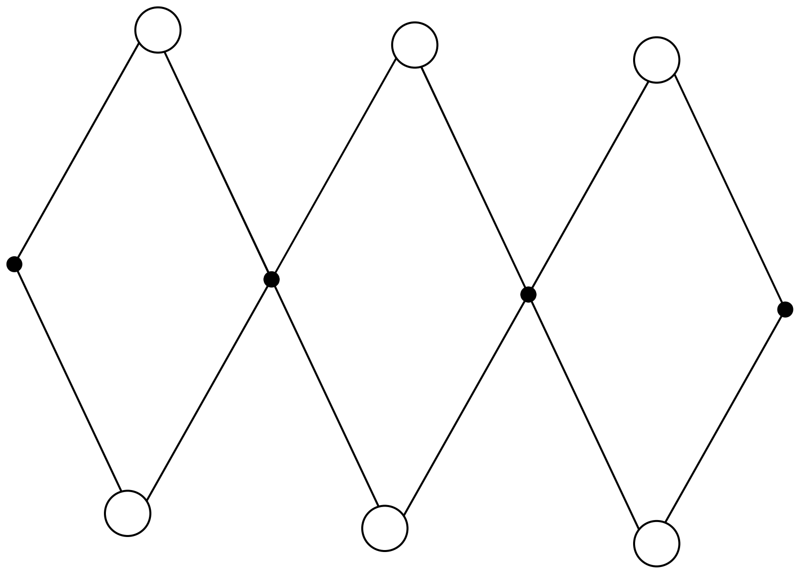
MIS Non Approximated

- Minimum couloring
- Δ_I=2, Δ=4
- Kk-IS-v ---→

MIS: Polynomial cases

- Chains
- Cycles
- $\Delta=2$
- Interval graphs
- Bipartite graphs
- Chordal graphs

Greedy MIS

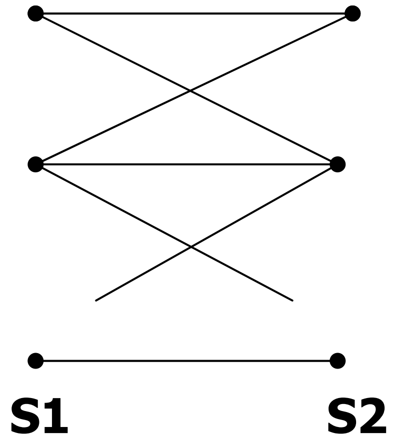


$$\rho = \frac{6}{4} = \frac{3}{2}$$

MIS (efficient algorithms)

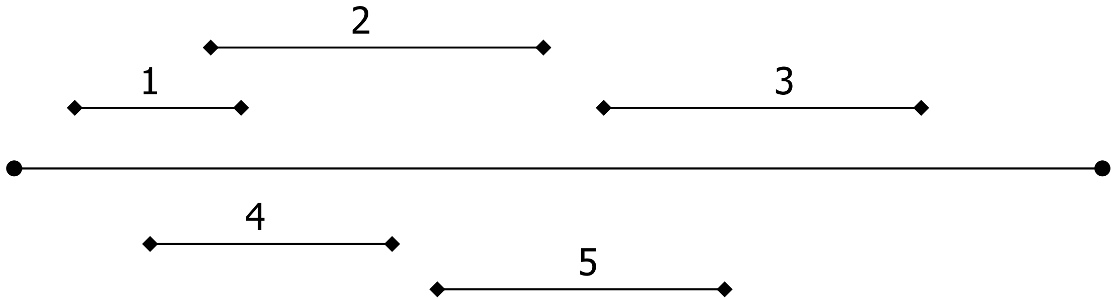
- G=(V,E), d(v)=2, $\forall v \in V$
  - Find polynomial algorithm
- G bipartite
  - Pb polynomial ($\exists$ polynomial algorithm)

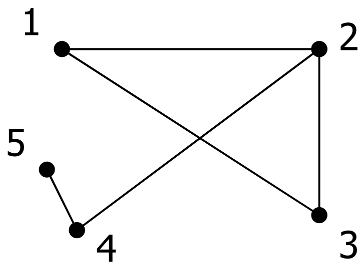
MIS (efficient algorithms)



- S_1 feasible solution
- S_2 feasible solution

- Interval graphs





Weighted IS Pb

- G=(V, E)

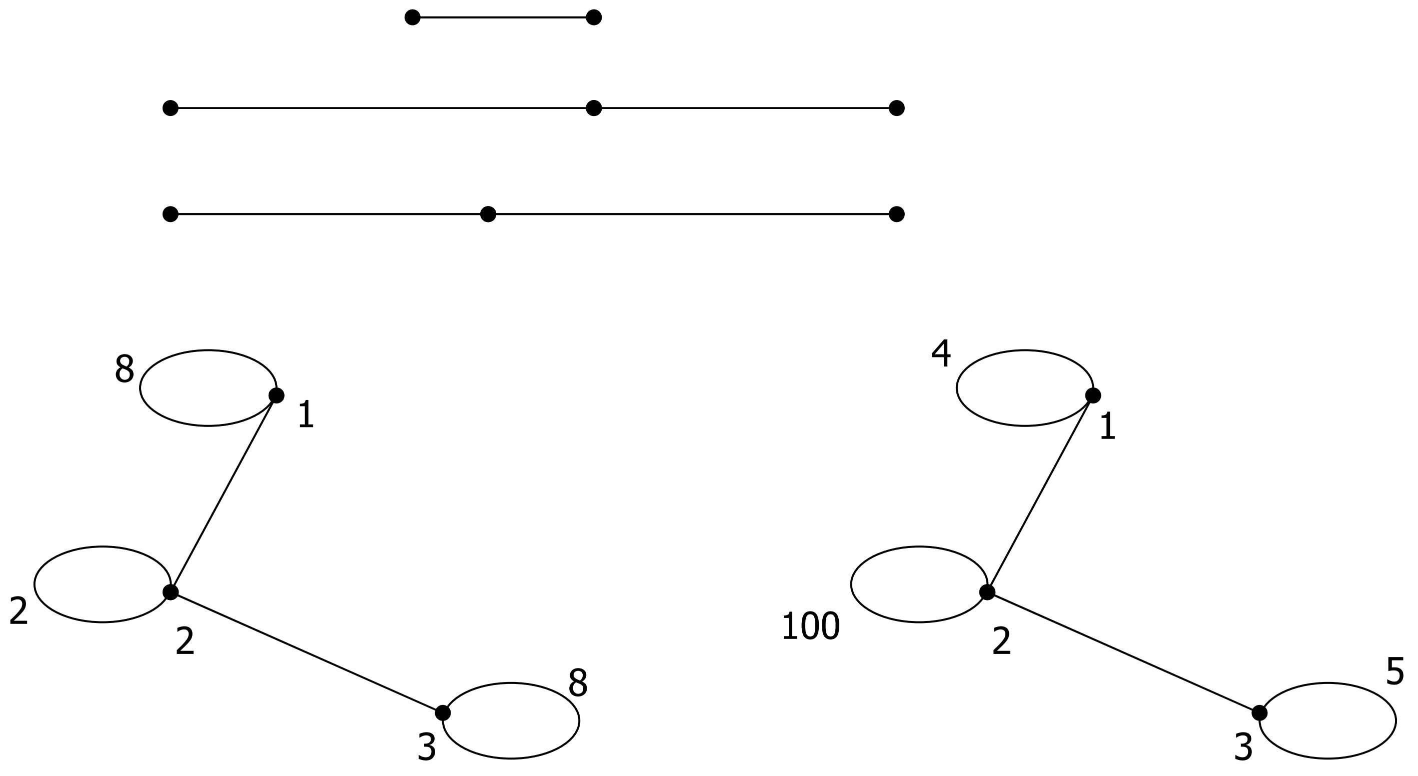
$$\forall v \in V \Rightarrow \exists w(v) \qquad weight$$

- WMIS: find IS maximum weight

V′ $\subseteq$ V|V′ IS and

$$\max \sum_{v \in V'} w(v)$$

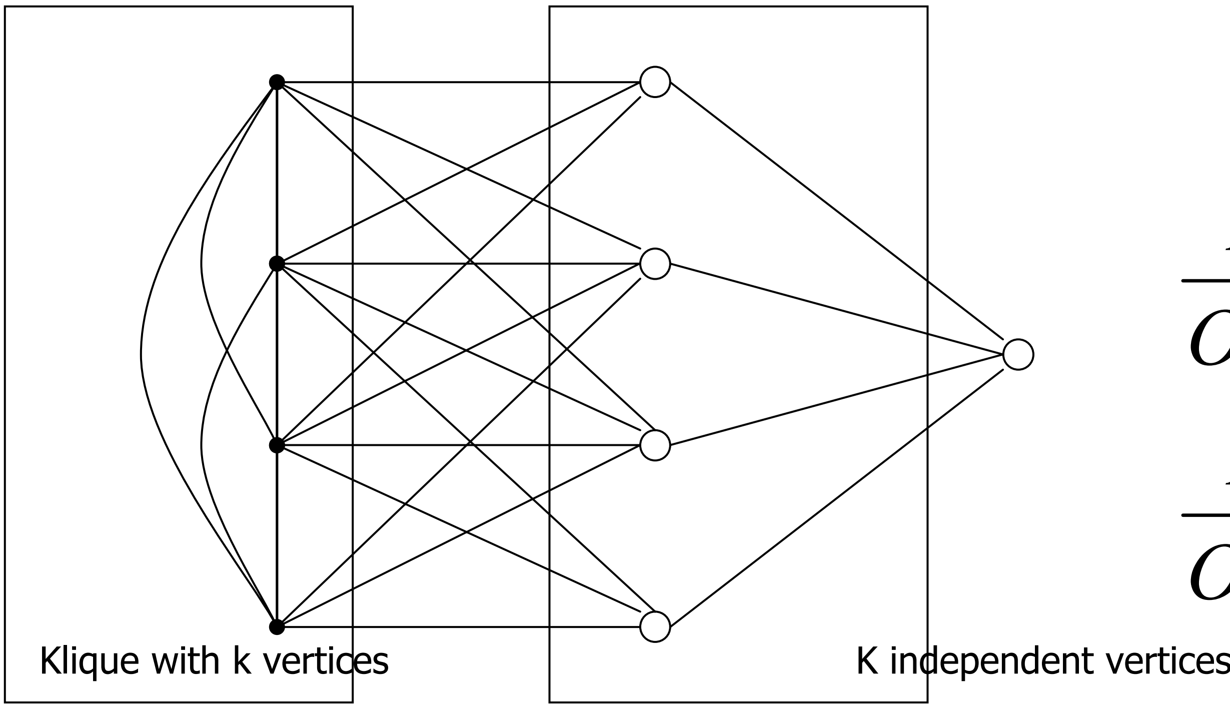
Παράδειγμα (max duration)



MIS non approximate

- MIS not so good as Max Knapsack! not in the method but difficulty of MIS
- Unless P=NP No Approximation Algorithm can be exist for MIS

Non approximability of MIS (example for smallest degree algorithm)



$$\frac{A}{Opt} \geq \frac{2}{4}$$

$$\frac{A}{Opt} \geq \frac{2}{k} \to 0$$