PTAS:

A αλγόριθμος $\rightarrow f_A(I)$

Είναι ένας αλγόριθμος ή σειρά αλγορίθμων που δίδουν

$\forall I$ και $\forall \varepsilon > 0$: $\frac{|f_A(I)-f_A^*(I)|}{f_A^*(I)} < \varepsilon$

σε πολυωνυμίκό χρόνο $O(n^{1/\varepsilon})^k = O(n^{k/\varepsilon})$

Για $\varepsilon = \frac{1}{100}$ έχουμε $O(n^{100k})$

FPTAS:

$O(\frac{n^{k_1}}{\varepsilon^{k_2}})$

Για $\varepsilon = \frac{1}{100}$ έχουμε $O(100^{k_2} n^{k_1})$

knapsack, bin packing, independent tasks σε επεξεργαστές

Dynamic Programming

?

Exact (optimum) → efficiency !

?

efficiency → accuracy

Approximate using Dynamic Programming

j	1	2	3	4	5	6	7
m_i	4	1	2	3	2	1	2
c_i	299	73	159	221	137	89	157

b=10

Opt = {1,2,3,6,7}

$$Z^* = \sum_{i \in opt} Ci = 777$$

Ignore the last decimal digit of Ci

j	1	2	3	4	5	6	7
m_i	4	1	2	3	2	1	2
$\overline{c}_i$	290	70	150	220	130	80	150

b=10

Opt={1,3,4,6}, $\overline{Z} = \sum_{i \in \overline{opt}} \overline{Ci} = 740$

<u>5% of Opt</u>

Efficiency! (+size ↗)

AND even better: $\sum_{i \in \overline{opt}} Ci = 768$

Truncating/error

- Opt → original problem
- $\overline{\text{Opt}}$ → truncated version

$$\sum_{i \in opt} Ci \geq \sum_{i \in \overline{opt}} Ci \geq \sum_{i \in \overline{opt}} \overline{Ci} \geq \sum_{i \in opt} \overline{Ci} \geq \sum_{i \in opt} (Ci - 10) \geq$$

$$\geq \sum_{i \in opt} Ci - n * 10$$

$$\Rightarrow \sum_{i \in opt} Ci - \sum_{i \in \overline{opt}} \overline{Ci} \leq n * 10$$

Truncating/error

t decimal digits truncated

$\Downarrow$

deviation $\leq$ $n*10^t$

Gain: Cm=max{Ci}

DP:$O(n^2Cm)$

$\downarrow$ truncation

DP:$O(n^2Cm10^{-t})$

An Approximate Algorithm

$$\varepsilon - approximate\ a\lg orithm(truncation)$$

$$\varepsilon = n * 10^t / C_m$$

$$\left(\sum_{i \mathrm{E} opt} Ci - \sum_{i \mathrm{E} \overline{opt}} Ci\right) / \sum_{i \mathrm{E} opt} Ci \leq n * 10^t / C_m = \varepsilon$$

$$O(n^2 C_m 10^{-t}) = O(n^3 / \varepsilon)$$

$$t = \left(\log_{10}(\varepsilon c_m / n)\right($$