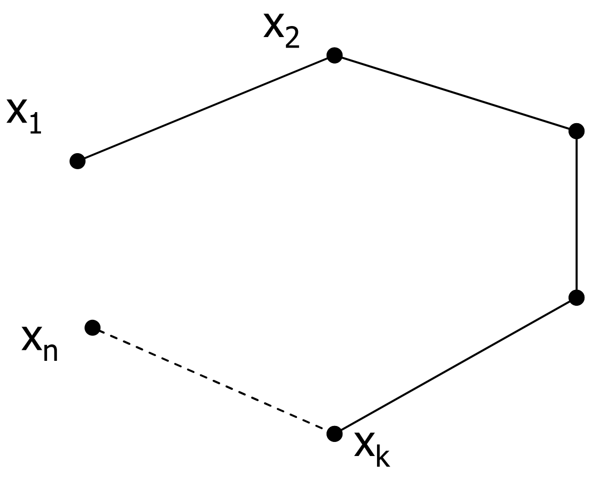
Hamiltonian path

- $G=(X, A)$ weighted, non-directed
  - $x_i \rightarrow d_i^G$
  - $|x|=n$



Hamiltonian path

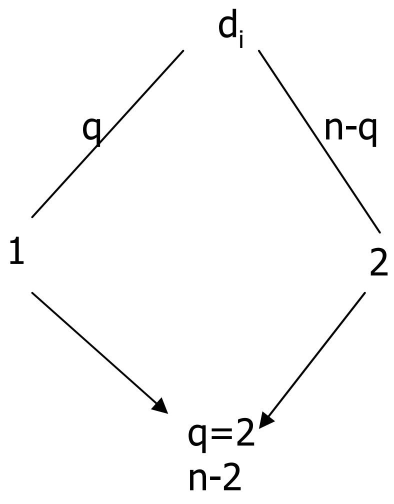
- T=(X, AT) of G (spanning tree)
  - $x_i \rightarrow d_i^T$
- E_T: closeness of T to HP

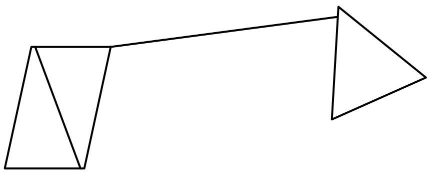
$$E_T = \sum_{d_i^T > 2} (d_i^T - 2)$$

(E_T=0 for a HP)

Problem

- Find the shortest spanning tree $T^*=(X,A^*)$ of G so that the degree of no vertex exceeds 2





G: connected

Problem

- T tree

$$d_i \neq 0 \Rightarrow d_i = \begin{cases} 1 & (q) \\ 2 & (n-q) \end{cases}$$

$$m_T = \frac{1}{2}\sum_{i=1}^{n} d_i^T$$

$$m_T = \frac{1}{2}[q + 2(n-q)] = n - \frac{q}{2}$$

$$n - 1 = n - \frac{q}{2} \Rightarrow \begin{matrix} q = 2 & (d_i^T = 1) \\ n-2 & (d_i^T = 2) \end{matrix}$$

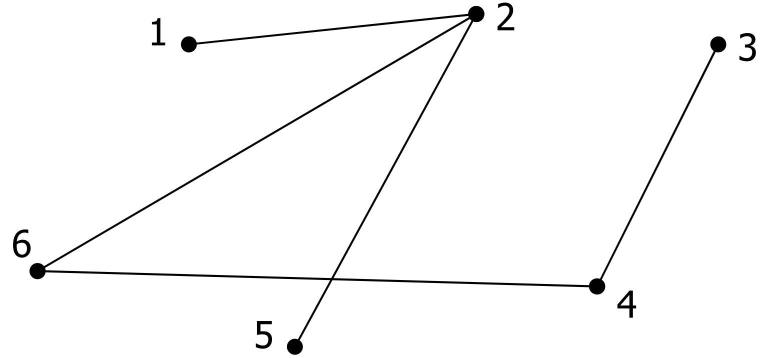
Branch and Bound (Ham.path)

- G=(X,A,C)

$C = [c_{ij}] =$

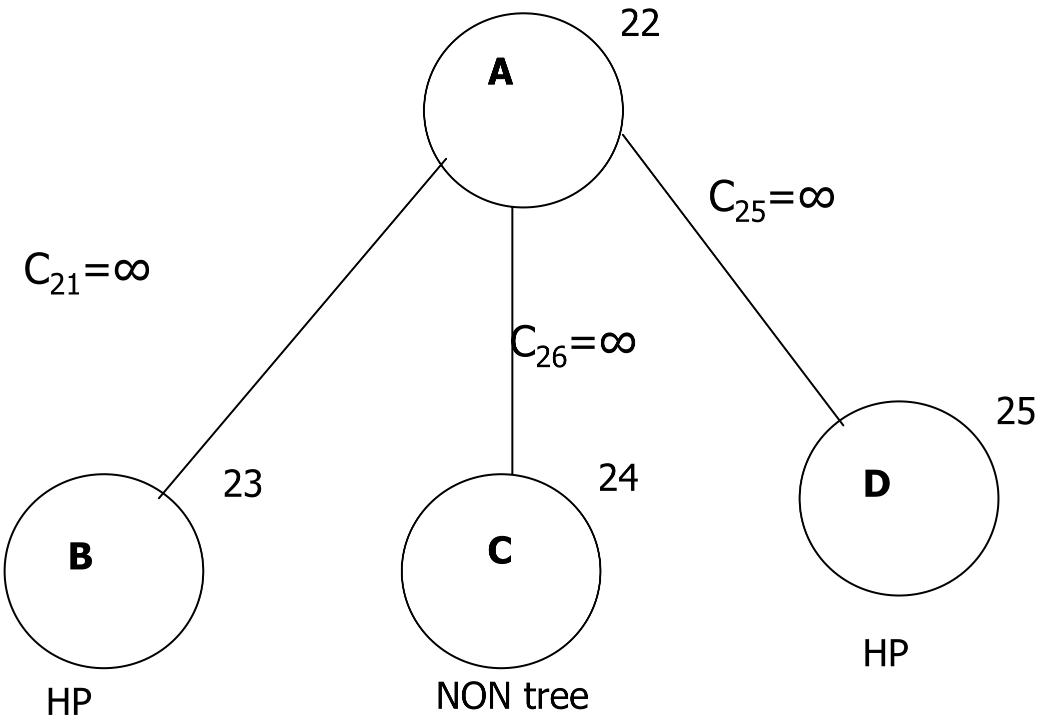
	1	2	3	4	5	6
1	0	4	10	18	5	10
2	4	0	12	8	2	6
3	10	12	0	4	18	16
4	18	8	4	0	14	6
5	5	2	18	14	0	16
6	10	6	16	6	16	0

Minimum spanning tree

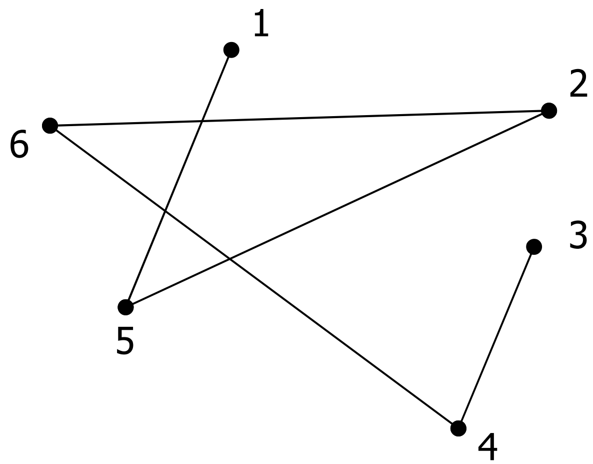


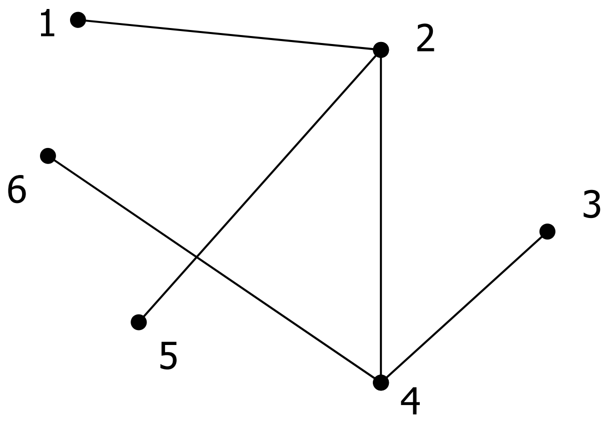
- $T^*(A)$: cost=22 ($d_2^T=3$)
- "Shortest spanning tree with all $d_i^T \leq 2$" => At least one of the links (2,1), (2,6), (2,5) must be absent from the final answer

Branch and Bound



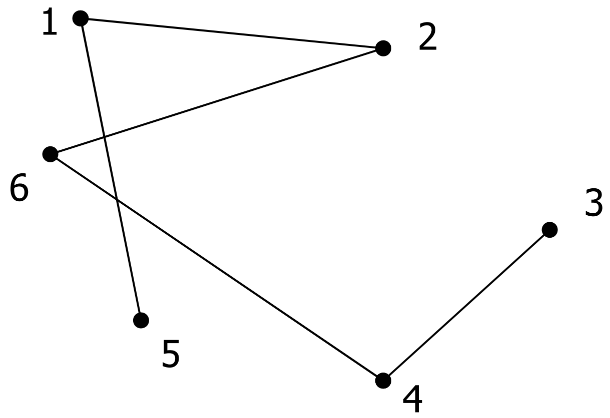
Nodes B and C of the Tree Search





- $T^*(B)$ cost=23
- $T^*(C)$ cost=24

Node D of the Tree search



- $T^*(D)$ cost=25
- 4 Spanning Trees
- Near optima HP! (practice)