Set covering

Κάλυψη Συνόλου

Set Partitionig

Set Packing

$$\left.\begin{array}{l} X = \{x_1, x_2, \ldots, x_m\} \\ F = \{s_1, s_2, \ldots, s_n\} \end{array}\right\} \Rightarrow X = \bigcup_{i=1}^{n} s_i$$

Να βρεθεί $C \subseteq F$ τέτοιο ώστε

$X = \bigcup_{S_i \in C} s_i$ και $|C|$ ελάχιστο

X	s_1	s_2	s_3	s_4	s_5	s_6
x_1	×		×			
x_2	×			×		
x_3	×				×	
x_4	×		×			
x_5	×	×		×		
x_6	×	×			×	
x_7			×	×		
x_8		×		×		
x_9		×			×	
x_{10}			×			×
x_{11}				×		×
x_{12}					×	

$$C = \{s_1, s_2, s_3, s_4, s_5\}$$

$$|G| = 4$$

$$C = \{s_3, s_4, s_5\}$$

$$|C|^* = 3$$

$$x_j = \begin{cases} 1, & \text{αν } s_j \text{ στη λύση} \\ 0, & \text{αλλιώς} \end{cases}$$

$$A = (a_{ij})_{\substack{1 \le j \le m \\ 1 \le i \le n}} = \begin{cases} 1, & \text{αν } x_i \in \mathrm{s_j} \\ 0, & \text{αλλιώς} \end{cases}$$

$$(SC) = \begin{cases} \min \sum_{j=1}^{n} x_j \\ Ax \ge 1 \end{cases} \qquad (wSC) = \begin{cases} \min \sum_{j=1}^{n} c_j x_j \\ Ax \ge 1 \end{cases}$$

Vertex Covering

- Special case of SC

procedure SetCover(X, F)

$U = X$

$C = \varnothing$

while $U \neq \varnothing$ do

    select $S \in F$ that ~~minimizes~~ $|S \cap U|$

    $U = U - S$

    $C = C \cup \{S\}$

return S

Set Partitionig

$$\left.\begin{array}{l} X = \{x_1, x_2, \ldots, x_m\} \\ F = \{s_1, s_2, \ldots, s_n\} \end{array}\right\} \Rightarrow X = \bigcup_{i=1}^{n} s_i$$

Να βρεθεί $C \subseteq F$ τέτοιο ώστε

$X = \bigcup_{S_i \in S} s_i$ και $|C|$ ελάχιστο

$S_i \cap S_j = \varnothing$

Set Partitionig

$$x_j = \begin{cases} 1, \ \text{αν } s_j \ \text{στη λύση} \\ 0, \ \text{αλλιώς} \end{cases}$$

$$A = (a_{ij})_{\substack{1 \le j \le m \\ 1 \le i \le n}} = \begin{cases} 1, \ \text{αν } x_i \in \text{s}_\text{j} \\ 0, \ \text{αλλιώς} \end{cases}$$

$$(SPP) = \begin{cases} \min \sum_{j=1}^{n} x_j \\ Ax = 1 \end{cases} \quad (wSPP) = \begin{cases} \min \sum_{j=1}^{n} c_j x_j \\ Ax = 1 \end{cases}$$

Set Packing

(MIS)

$$x_j = \begin{cases} 1, & \text{αν } s_j \text{ στη λύση} \\ 0, & \text{αλλιώς} \end{cases}$$

$$A = (a_{ij})_{\substack{1 \le j \le m \\ 1 \le i \le n}} = \begin{cases} 1, & \text{αν } x_i \in \mathrm{s_j} \\ 0, & \text{αλλιώς} \end{cases}$$

$$(SP) = \begin{cases} \min \sum_{j=1}^{n} x_j \\ Ax \le 1 \end{cases} \quad (wSP) = \begin{cases} \min \sum_{j=1}^{n} c_j x_j \\ Ax \le 1 \end{cases}$$