$$\min Z = -cx$$

$$Ax = b$$

$$x \geq 0$$

$$\left.\begin{array}{l} c_j, \quad 1 \leq j \leq n \\ a_{ij}, \quad 1 \leq i \leq m \\ \qquad\quad 1 \leq j \leq n \\ b_i, \quad 1 \leq i \leq m \end{array}\right\} \in N$$

$$P = \left\{ x \middle| x \in \mathfrak{R}^n, Ax = b, x \geq 0 \right\}$$

P φραγμένο

$$P \neq \varnothing$$

$$P = \left\{ x \mid x \in \mathfrak{R}^n, Ax = b, x \geq 0 \right\}$$

P φραγμένο

$P \neq \varnothing$

$a_j \leq x_j \leq b_j$

$\Uparrow$

$\max x_j$

$Ax = b$

$x \geq 0$

$x_j \rightarrow k+1$ ακέραιες τιμές

$x_j \in \{0,1\}$

$x_j = y_0 + 2y_1 + 4y_2 + \cdots + 2^p y_p$

$k \leq 2^{p+1} - 1$

$$\left.\begin{array}{l} \min Z = 3x_1 - 4x_2 \\ x_1 + 2x_2 \leq 6 \\ -2 \leq x_1 \leq 4 \\ 1 \leq x_2 \leq 3 \\ x_1, x_2 \in N \end{array}\right\} \Rightarrow 5 \text{ δυαδικές μεταβλητές}$$

$$x_1 = -2 + y_1 + 2y_2 + 4y_3$$
$$x_2 = 1 + y_4 + 2y_5$$

$$\min Z = -10 + 3y_1 + 6y_2 + 12y_3 - 4y_4 - 8y_5$$
$$y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 \leq 6$$
$$y_1 + 2y_2 + 4y_3 \leq 6$$
$$y_4 + 2y_5 \leq 2$$
$$y_i \in \{0,1\}$$

$$\min Z = -10x_1 - 11x_2$$

$$10x_1 + 12x_2 \le 59$$

$$x_1, x_2 \in N$$

$$\begin{aligned}\min Z &= cx \\ Ax &= b \\ x &\ge 0 \\ x_j \ \text{ακέραιοι}\end{aligned} \Bigg\} (IP)$$

$$\left.\begin{aligned}x_1 &= 5,9 \\ x_2 &= 0\end{aligned}\right\} \Rightarrow Z = -59$$

$$(x_1 = 6, x_2 = 0 \Rightarrow ?)$$

$$x_1^* = 1, x_2^* = 4 \Rightarrow Z = -54$$

$$\min 15x_1 + 2x_2 - 4x_3 + 10x_4$$

$$x_3 - x_4 \leq 0$$

$$x_1 + 2x_2 + 4x_3 + 8x_4 = 20$$

$$x_2 + x_4 \leq 1$$
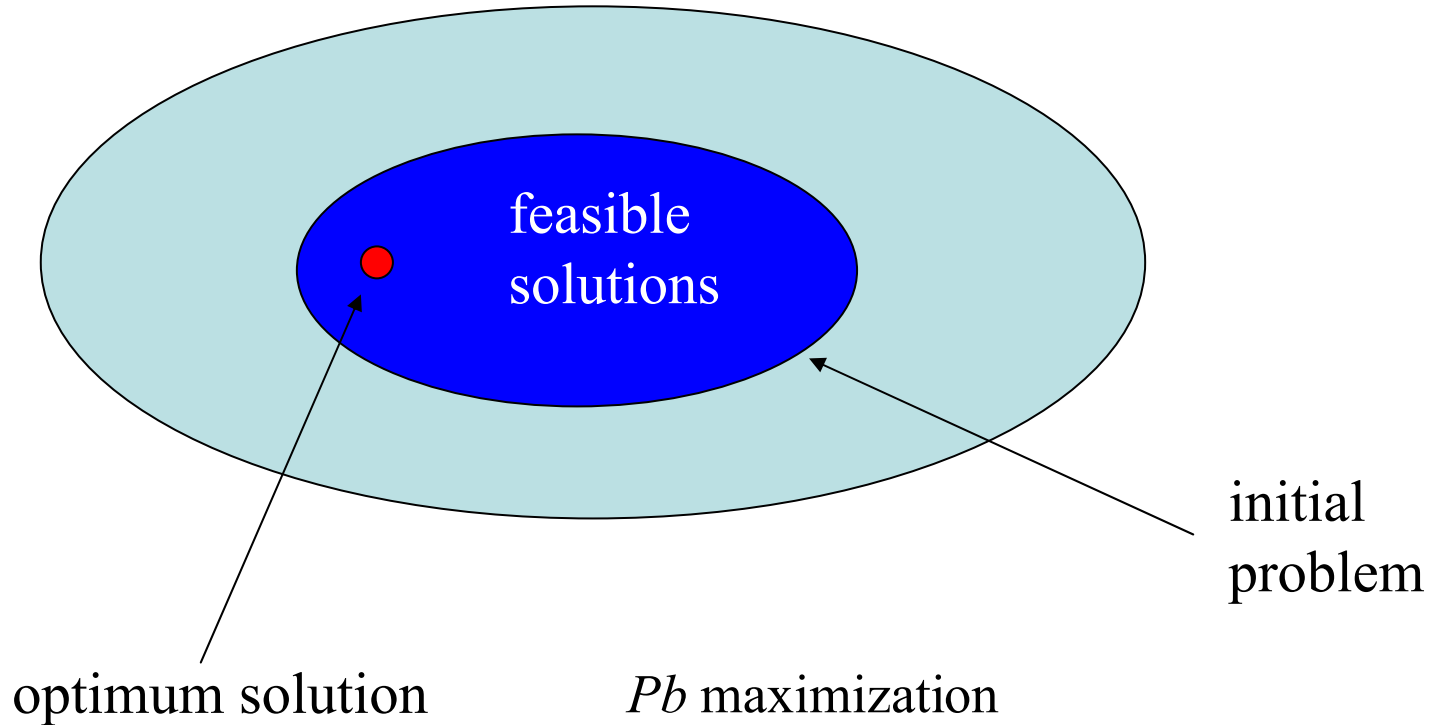
$$x_2, x_3, x_4 \in \{0,1\}$$

$$\Downarrow (LR)$$

$$0 \leq x_2 \leq 1$$

$$0 \leq x_3 \leq 1$$

$$0 \leq x_4 \leq 1$$

Φράγματα και αποδυναμώσεις

feasible solutions

initial problem

optimum solution

Pb maximization

$$\left.\begin{array}{l} R(p) \to V^*(R) \\ P \to V^*(P) \end{array}\right\} \quad V^*(R) \geq V^*(P)$$

Pb   minimization

$$V^*(R) \leq V^*(P)$$

$$\left.\begin{array}{l} \max x_1 + x_2 + x_3 \\ x_1 + x_2 \leq 1 \\ x_1 + x_3 \leq 1 \\ x_2 + x_3 \leq 1 \\ x_i \in \{0,1\} \end{array}\right\}(P)$$

$$V^*(P) = 1 \quad (\quad x^* = (1,0,0) \quad)$$

$$V^*(R) = \frac{3}{2} \quad (\quad \tilde{x} = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \quad)$$

$$\left.\begin{array}{l} \min 20x_1 + 9x_2 + 7x_3 \\ 10x_1 + 4x_2 + 3x_3 \geq 7 \\ x_i \in \{0,1\} \end{array}\right\} (P)$$

$$V^*(P) = 16 \quad (\ x^* = (0,1,1) \ )$$

$$V^*(R) = 14 \quad (\ \tilde{x} = (\frac{7}{10}, 0, 0) \ )$$

"Αν $\tilde{x}$ είναι εφικτή για το P

τότε η $\tilde{x}$ είναι ΒΕΛΤΙΣΤΗ για το P"