In this paper, we are dealing with the important problem of minimizing the overall energy required for the accomplishment of complex tasks in WSNs. Firstly, we provide a modeling framework for tasks, networks, and task assignment process. We formulate the assignment of tasks as an optimization problem and we apply 0-1 Integer Linear Programming (ILP) to solve the problem optimally. The optimality of results and the performance of the proposed approach are validated through extensive simulation experiments.

**Problem Formulation**

Definition (Optimal Assignment). Let W be a WSN and let T be a complex task. Given the set of all mappings \( \Phi = \{ \phi : T \to W \} \) where the capabilities of the network fulfill the task and energy requirements, we call an assignment as energy-optimal assignment \( \phi^* \) if and only if \( (\forall \phi \in \Phi) \) \( \phi^* \leq \phi \).

**Modeling**

Definition (Task). Given a set \( S \) of subtask vertices and a set \( N \) of directed edges among the subtask vertices, we define a task \( T \) as a directed acyclic graph (DAG) represented by the tuple \( T = (S, C) \).

Definition (Network). Let \( N \) be a set of network nodes and let \( A \) be a set of directed network edges. We define a WSN \( W \) as a strongly connected directed graph represented by the tuple \( W = (N, A) \).

Definition (Assignment). Let \( T = (S, C) \) be a complex task and \( W = (N, A) \) be a WSN with \( \Pi = \{\Pi_a\}_{a \in N} \) representing the set of all paths among the network nodes. We define the assignment \( \zeta : T \to W \) as the pair of mappings \( X : S \to N \) and \( Y : C \to \Pi \) that satisfy the consistency constraint:

\[
(\forall C \in C) (\forall \Pi_a \in \Pi) (C_{\Pi_a}) = \Pi_a \land (X(p) = \text{Src}(\Pi_a) \land (X(q) = \text{Dst}(\Pi_a)) )
\]

and we represent it by \( \zeta = (X, Y) \in \mathcal{Z} \) where \( \mathcal{Z} \) is the set of all possible assignments between \( T \) and \( W \).

**Methodology**

**Algorithm 1. ILP Formulation**

Input: tasks \( C, S, \Pi, \Pi_a, f_{pq} \); Variables \( \beta_{pq}, \rho_{ab}, \epsilon_{pq}, \epsilon_{ab} \).

Output: Variables \( \beta_{pq}, \rho_{ab} \).

Objective function: minimize \( \sum_{(p \in S)} \sum_{c \in C} \beta_{pq} \) \( + \sum_{(p \in S)} \sum_{c \in C} \rho_{ab} \) \( + \sum_{(p \in S)} \sum_{c \in C} \epsilon_{pq} \) \( + \sum_{(p \in S)} \sum_{c \in C} \epsilon_{ab} \).

Constraints set:

\[
(\forall c \in C) \sum_{p \in S} \beta_{pq} = 1 \quad \text{(Unique subtask vertex assignment)}
\]

\[
(\forall p \in S) \sum_{c \in C} \rho_{ab} = 1 \quad \text{(Unique subtask edge assignment)}
\]

\[
(\forall c \in C) (\forall a \in A) \sum_{p \in S} \rho_{ab} = 1 \quad \text{(Energy-to-source vertex consistency)}
\]

\[
(\forall c \in C) (\forall b \in B) \sum_{p \in S} \rho_{ab} = 1 \quad \text{(Energy-to-destination vertex consistency)}
\]

\[
(\forall a \in A) \sum_{p \in S} \epsilon_{pq} = 1 \quad \text{(Node energy conservation)}
\]

\[
(\forall p \in S) \sum_{c \in C} \epsilon_{pq} + \sum_{c \in C} \epsilon_{ab} \leq \epsilon_{\Pi_a} \quad \text{(Node energy conservation)}
\]

\[
(\forall C \in C) (\forall \Pi_a \in \Pi) s_{pq} = 1 \quad \text{(Vortex compatibility)}
\]

\[
(\forall C \in C) (\forall \Pi_a \in \Pi) \beta_{pq} = 0 \quad \text{(Edge compatibility)}
\]

### Results

**Grid network topology:** The optimal solutions of the task assignment problem take full advantage of the nodes close to the network sinks and, after a few steps, these nodes are drained.

**Mesh network topology:** As the number of network nodes increases, the ILP algorithm manages to execute more tasks in the trace of experiments.

### Discussion

The execution time increases exponentially to the number of nodes. For instance, finding an optimal solution for a network of 64 nodes requires less than 1 ms while running the ILP algorithm for a network of 1024 nodes requires more than 25 s. Peaks and valleys in the curves of Fig. 4 correspond to different levels of easiness to fulfill the task requirements. Gurobi optimizer [6] was adopted to solve the ILP formulation of the task assignment problem.

### Conclusions

The paper discusses an energy-optimized scheme for the execution of complex application tasks within a WSN. The optimization model is based on ILP. The proposed ILP scheme achieved excellent in terms of quality of simulation results. Not only did it manage to provide the optimal solutions but it also secured the execution of a long run of consecutive experiments. As expected, the ILP algorithm does not scale for large networks due to the NP-hardness of the task assignment problem [1].

### References


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