Optimal Task Assignment in Sensor Networks

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Introduction

In this paper, we are dealing with the important problem of **minimizing the overall energy** required for the accomplishment of complex tasks in WSNs. Firstly, we provide a **modeling framework for tasks, networks, and task assignment process**. We formulate the assignment of tasks as an optimization problem and we apply **0-1 Integer Linear Programming (ILP)** to solve the problem optimally. The optimality of results and the performance of the proposed approach are validated through extensive simulation experiments.

Problem Formulation

Definition (Optimal Assignment). Let *W* be a WSN and let *T* be a complex task. Given the set of all mappings $\Phi = \{\phi : T \to W\}$ where the capabilities of the network fulfill the task and energy requirements, we call an assignment as *energy-optimal assignment* ϕ_{opt} if and only if $(\forall \phi \in \Phi)\, \varepsilon_{W,T}^{\phi_{opt}} \le \varepsilon_{W,T}^{\phi}$.

Modeling

Definition (Task). Given a set S of subtask vertices and a set N of directed edges among the subtask vertices, we define a task *T* as a *directed acyclic graph (DAG)* represented by the tuple $T \equiv \langle S, C \rangle$.

Definition (Network). Let N be a set of network nodes and let A be a set of directed network edges. We define a WSN W as a *strongly connected directed graph* represented by the tuple $W \equiv \langle N, A \rangle$.

Definition (Assignment). Let $T \equiv \langle S, C \rangle$ be a complex task and $W \equiv \langle N, A \rangle$ be a WSN with $\Pi = \{\Pi_{ab}\}, \forall a, b \in N$ representing the set of all paths among the network nodes. We define the assignment $\zeta : T \to W$ as the pair of mappings $X_\zeta : S \to N$ and $Y_\zeta : C \to \Pi$ that satisfy the *consistency constraint*:

$$(\forall C_{pq} \in C)(\forall \Pi_{ab} \in \Pi) \quad Y_\zeta(C_{pq}) = \Pi_{ab} \Leftrightarrow$$
$$\left(\left(X_\zeta(p) = Src(\Pi_{ab}) \right) \wedge \left(X_\zeta(q) = Dst(\Pi_{ab}) \right) \right)$$

and we represent it by $\zeta \equiv \langle X_\zeta, Y_\zeta \rangle \in Z$ where Z is the set of all possible assignments between T and *W*.

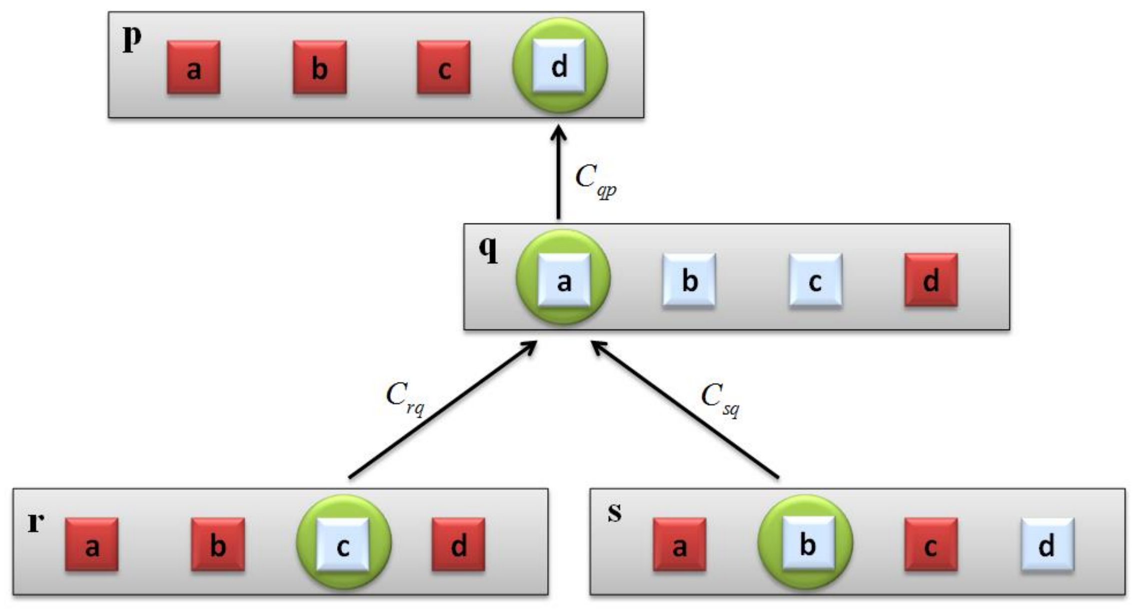

Figure 1. Graphical representation of an assignment ζ.

Methodology

ALGORITHM 1. ILP Formulation

Input: Sets $C, S, N, \Pi, I_p, J_{pq}$, Variables $\beta_{pq}, \rho_{ab}, \rho_{ab,c}, \varepsilon_{c,p}, \varepsilon_c$

Output: Variables $x_{p,a}$, $y_{pq,ab}$

Objective function: *minimize* $\left\{ \sum_{c \in N} \left(\sum_{p \in S} \varepsilon_{c,p} \cdot x_{p,c} + \sum_{C_{pq} \in C} \sum_{\Pi_{ab} \in \Pi} \beta_{pq} \cdot y_{pq,ab} \cdot \rho_{ab,c} \right) \right\}$

Constraints set:

$(\forall p \in S) \quad \sum_{a \in N} x_{p,a} = 1$ //Unique subtask vertex assignment

$(\forall C_{pq} \in C) \quad \sum_{\Pi_{ab} \in \Pi} y_{pq,ab} = 1$ //Unique subtask edge assignment

$(\forall C_{pq} \in C)(\forall a \in N) \quad \sum_{\Pi_{ab} \in Src(a)} y_{pq,ab} = x_{p,a}$ //Edge-to-source-vertex consistency

$(\forall C_{pq} \in C)(\forall b \in N) \quad \sum_{\Pi_{ab} \in Dst(b)} y_{pq,ab} = x_{q,b}$ //Edge-to-destination-vertex consistency

$(\forall c \in N) \quad \sum_{p \in S} \varepsilon_{c,p} \cdot x_{p,c} + \left\{ \sum_{C_{pq} \in C} \sum_{\Pi_{ab} \in \Pi} \beta_{pq} \cdot y_{pq,ab} \cdot \rho_{ab,c} \right\} \le \varepsilon_c$ //Node energy conservation

$(\forall p \in S)(\forall a \in N / I_p) \quad x_{p,a} = 0$ //Vertex compatibility

$(\forall C_{pq} \in C)(\forall \Pi_{ab} \in \Pi \setminus J_{pq}) \quad y_{pq,ab} = 0$ //Edge compatibility

Table 1. Experimental parameters - Tasks.

Parameter	Value
$\lvert CI \rvert$	10
$\lvert Alg \rvert$	15
$\lvert Sens_{aver} \rvert$	4
$\lvert Alg_{aver} \rvert$	2
$Quant_{aver}$	6000
$\#SinkSubtasks$	1
$\beta_{pq,aver}$	320 Kbit
Spatial constraint	1:5 sensing subtasks

Table 2. Experimental parameters - Networks.

Parameter	Value
$\lvert Sens_{aver} \rvert$	5
$\lvert Alg_{aver} \rvert$	10
$\#SinkNodes$	1
$M_{aver}(op_i)$	0.005mJ
$\rho_{trx,aver} = \rho_{rcv,aver} = \rho_{aver}$	0.0001mJ/bit
ε_{aver}	300J

Results

Grid network topology: The optimal solutions of the task assignment problem take full advantage of the nodes close to the network sinks and, after a few steps, these nodes are drained.

Mesh network topology: As the number of network nodes increases, the ILP algorithm manages to execute more tasks in the trace of experiments.

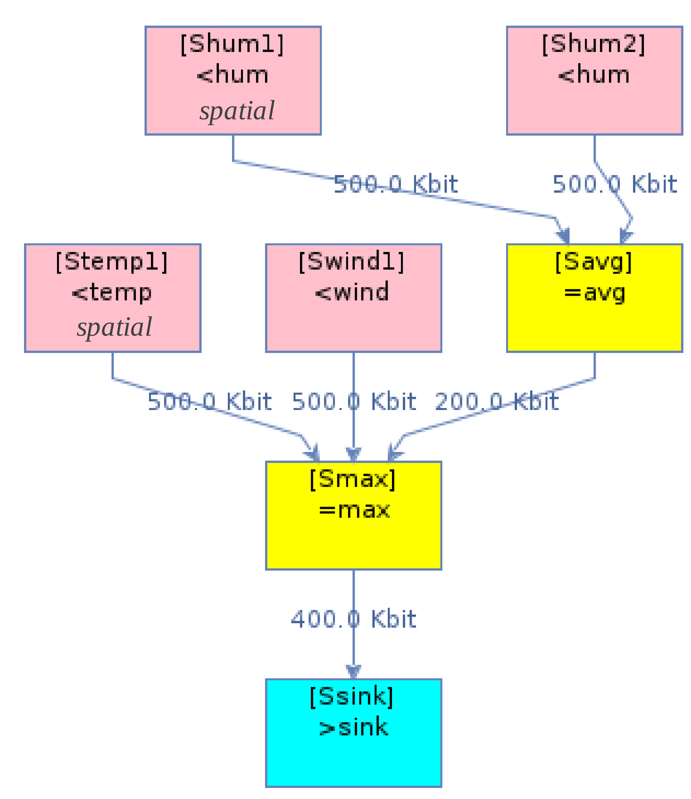

Figure 2. Example task in the experiments.

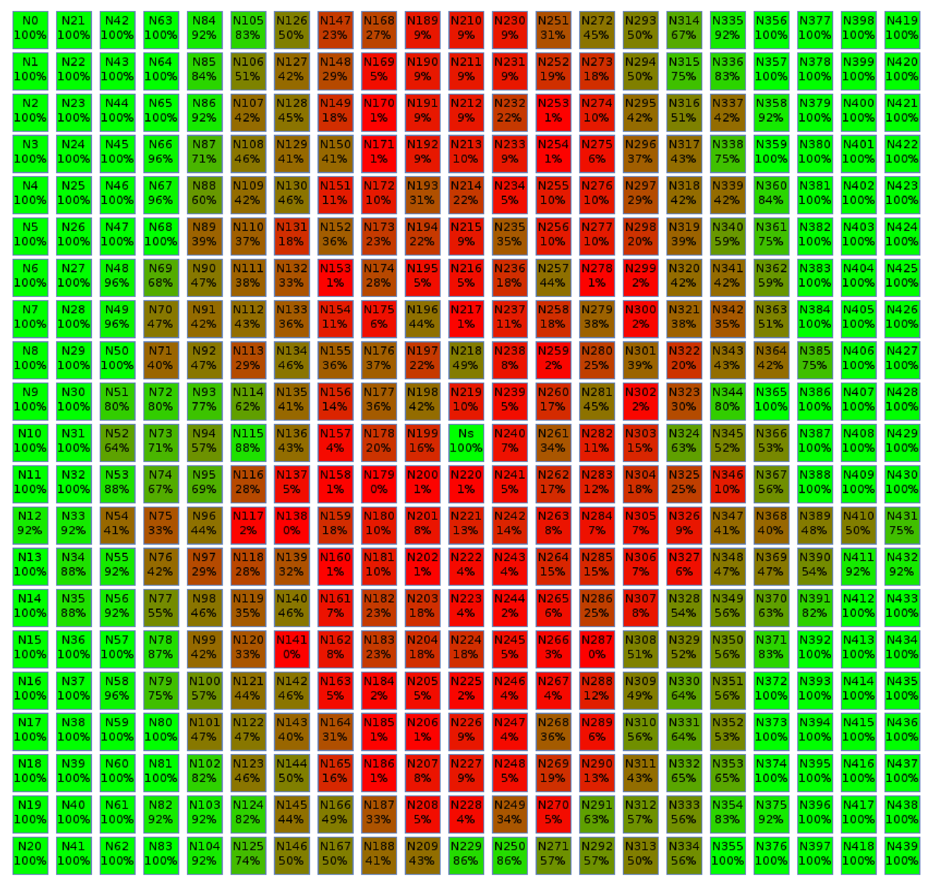

Figure 3. Energy status of the network after run #100.

Discussion

The execution time increases **exponentially** to the number of nodes. For instance, finding an optimal solution for a network of 64 nodes requires less than 1 ms while running the ILP algorithm for a network of 1024 nodes requires more than 25 s. Peaks and valleys in the curves of Fig. 4 correspond to different levels of easiness to fulfil the task requirements. Gurobi optimizer [6] was adopted to solve the ILP formulation of the task assignment problem.

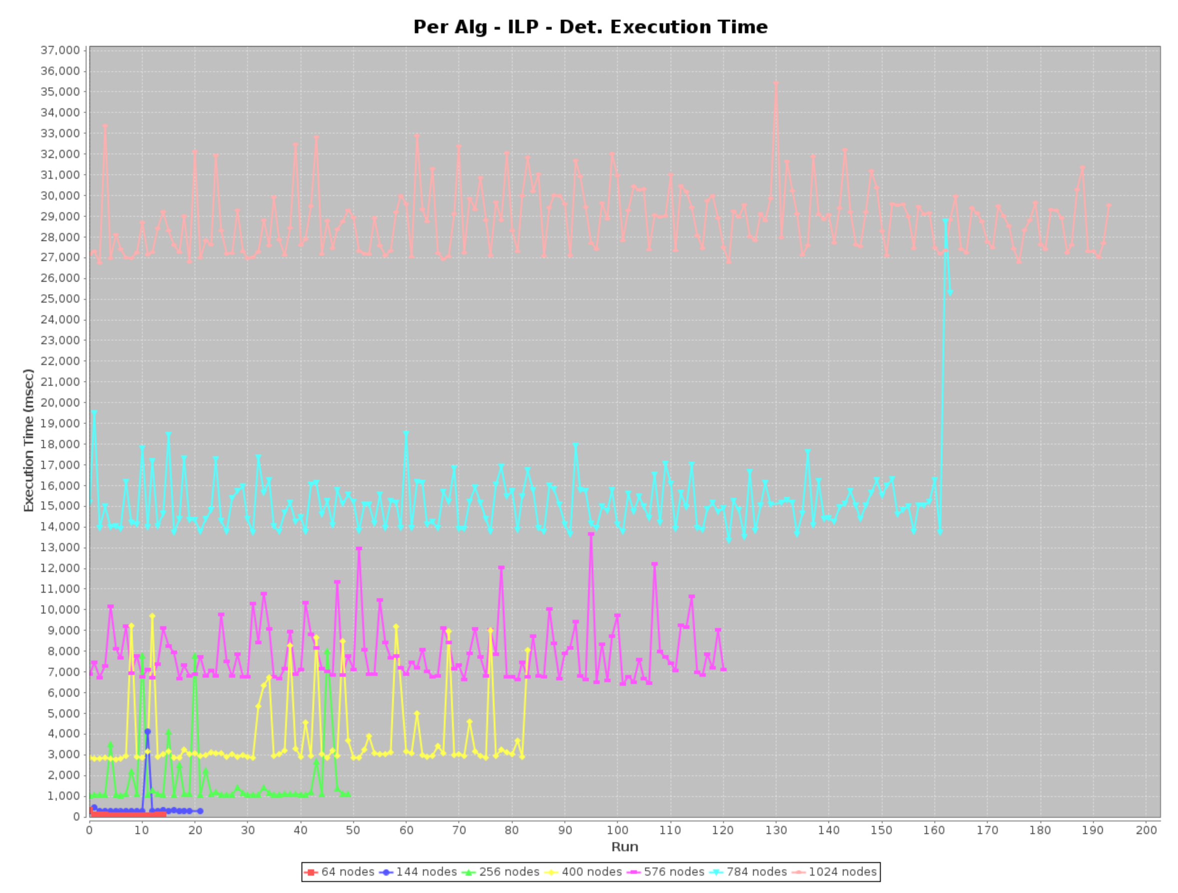

Figure 4. Execution time per run.

Conclusions

The paper discusses an energy-optimal scheme for the execution of complex application tasks within a WSN. The optimization model is based on ILP. The proposed ILP scheme achieved excellence in terms of quality of simulation results. Not only did it manage to provide the optimal solutions but it also secured the execution of a long run of consecutive experiments. As expected, the ILP algorithm does not scale for large networks due to the NP-hardness of the task assignment problem [1].

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