# AI Tutorial

## Resolution ($3^d$ Inference Method)

Charalampos S. Nikolaou
charnik@di.uoa.gr

Department of Informatics and Telecommunications

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# Monty Python's Holy Grail World

1. Every woman that can be burnt is a witch.
2. Everything that is made of wood can be burnt.
3. Everything that floats is made of wood.
4. Everything that weighs the same as something that floats, does float too.
5. This girl is a woman.
6. This girl weighs the same as this duck.
7. This duck floats.

? Is the girl a witch?

# Representing Monty Python's Holy Grail World using FOL

1. $(\forall x)(Burns(x) \wedge Woman(x) \Rightarrow Witch(x))$.
2. $(\forall x)(Ismadeofwood(x) \Rightarrow Burns(x))$.
3. $(\forall x)(Floats(x) \Rightarrow Ismadeofwood(x))$.
4. $(\forall x)(\forall y)((Floats(x) \wedge Sameweight(x, y)) \Rightarrow Floats(y))$.
5. $Woman(Girl)$.
6. $Sameweight(Duck, Girl)$.
7. $Floats(Duck)$.

? $Witch(Girl)$

# Resolution (1/3)

## Transform KB into conjuctive normal form (CNF)

1. $\neg Burns(x_1) \vee \neg Woman(x_1) \vee Witch(x_1)$.
2. $\neg Ismadeofwood(x_2) \vee Burns(x_2)$.
3. $\neg Floats(x_3) \vee Ismadeofwood(x_3)$.
4. $\neg Floats(x_4) \vee \neg Sameweight(x_4, y_1) \vee Floats(y_1)$.
5. $Woman(Girl)$.
6. $Sameweight(Duck, Girl)$.
7. $Floats(Duck)$.

We shall insert in KB the complement of the proposition that we want to prove. If the initial proposition is true according to our KB then we will end up in a refutation.

$$? \ \neg Witch(Girl)$$

# Resolution (2/3)
## Resolution method application (1/2)

$$\boxed{\neg \text{Witch}(\textbf{Girl})} \qquad \boxed{\neg \textit{Burns}(x_1) \vee \neg \textit{Woman}(x_1) \vee \text{Witch}(\mathbf{x_1})}$$

# Resolution (2/3)

## Resolution method application (1/2)

$\boxed{\neg \text{Witch}(\textbf{Girl})}$ $\boxed{\neg \mathit{Burns}(x_1) \vee \neg \mathit{Woman}(x_1) \vee \text{Witch}(\mathbf{x_1})}$

$\boxed{\neg \mathit{Ismadeofwood}(x_2) \vee \text{Burns}(\mathbf{x_2})}$ $\boxed{\neg \text{Burns}(\textbf{Girl}) \vee \neg \mathit{Woman}(\mathit{Girl})}$

# Resolution (2/3)
## Resolution method application (1/2)

$\boxed{\neg \text{Witch}(\textbf{Girl})}$ $\boxed{\neg \textit{Burns}(x_1) \vee \neg \textit{Woman}(x_1) \vee \text{Witch}(\mathbf{x_1})}$

$\boxed{\neg \textit{Ismadeofwood}(x_2) \vee \text{Burns}(\mathbf{x_2})}$ $\boxed{\neg \text{Burns}(\textbf{Girl}) \vee \neg \textit{Woman}(\textit{Girl})}$

$\boxed{\textbf{Woman}(\textbf{Girl})}$ $\boxed{\neg \textit{Ismadeofwood}(\textit{Girl}) \vee \neg \textbf{Woman}(\textbf{Girl})}$

# Resolution (2/3)

## Resolution method application (1/2)

$\neg \text{Witch}(\textbf{Girl})$

$\neg \textit{Burns}(x_1) \vee \neg \textit{Woman}(x_1) \vee \text{Witch}(\mathbf{x_1})$

$\neg \textit{Ismadeofwood}(x_2) \vee \text{Burns}(\mathbf{x_2})$

$\neg \text{Burns}(\textbf{Girl}) \vee \neg \textit{Woman}(\textit{Girl})$

$\textbf{Woman}(\textbf{Girl})$

$\neg \textit{Ismadeofwood}(\textit{Girl}) \vee \neg \textbf{Woman}(\textbf{Girl})$

$\neg \textit{Floats}(x_3) \vee \text{Ismadeofwood}(\mathbf{x_3})$

$\neg \text{Ismadeofwood}(\textbf{Girl})$

# Resolution (3/3)

## Resolution method application (2/2)

$$\neg \mathit{Floats}(x_4) \vee \neg \mathit{Sameweight}(x_4, y_1) \vee \mathrm{Floats}(\mathbf{y_1})$$

$$\neg \mathrm{Floats}(\mathbf{Girl})$$

# Resolution (3/3)

## Resolution method application (2/2)

$$\neg \mathit{Floats}(x_4) \vee \neg \mathit{Sameweight}(x_4, y_1) \vee \mathrm{Floats}(\mathbf{y_1})$$

$$\neg \mathrm{Floats}(\mathbf{Girl})$$

$$\mathrm{Sameweight}(\mathbf{Duck}, \mathrm{Girl})$$

$$\neg \mathrm{Sameweight}(\mathbf{x_4}, \mathrm{Girl}) \vee \neg \mathit{Floats}(x_4)$$

# Resolution (3/3)

## Resolution method application (2/2)

$\neg \mathit{Floats}(x_4) \vee \neg \mathit{Sameweight}(x_4, y_1) \vee \mathrm{Floats}(\mathbf{y_1})$

$\neg \mathrm{Floats}(\mathbf{Girl})$

$\mathrm{Sameweight}(\mathbf{Duck}, \mathrm{Girl})$

$\neg \mathrm{Sameweight}(\mathbf{x_4}, \mathrm{Girl}) \vee \neg \mathit{Floats}(x_4)$

$\mathbf{Floats(Duck)}$

$\mathbf{\neg Floats(Duck)}$

# Resolution (3/3)

## Resolution method application (2/2)

$\boxed{\neg \mathit{Floats}(x_4) \vee \neg \mathit{Sameweight}(x_4, y_1) \vee \mathrm{Floats}(\mathbf{y_1})}$ $\boxed{\neg \mathrm{Floats}(\mathbf{Girl})}$

$\boxed{\mathrm{Sameweight}(\mathbf{Duck}, \mathrm{Girl})}$ $\boxed{\neg \mathrm{Sameweight}(\mathbf{x_4}, \mathrm{Girl}) \vee \neg \mathit{Floats}(x_4)}$

$\boxed{\mathbf{Floats(Duck)}}$ $\boxed{\neg \mathbf{Floats(Duck)}}$

$\boxed{\emptyset}$

# Conclusions

1. We used the resolution method to prove that the **Girl** is a **Witch**. Even if the concluding fact and the rules (of KB) don't reflect our perception in real life, resolution managed to come to a solution.

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2. Why?
   Because resolution method is an inference rule, that is, it can be applied mechanically. Inference rules do not discriminate between different perceptions of different worlds. Inference rules treat every such world equally.

# Conclusions

1. We used the resolution method to prove that the **Girl** is a **Witch**. Even if the concluding fact and the rules (of KB) don't reflect our perception in real life, resolution managed to come to a solution.
2. Why?
   Because resolution method is an inference rule, that is, it can be applied mechanically. Inference rules do not discriminate between different perceptions of different worlds. Inference rules treat every such world equally.
3. So, we can surely say that Holy Grail's World thinks Logically!

# Roman World

1. Marcus was a man.
2. Marcus was a Pompeian.
3. All Pompeian were Romans.
4. Ceasar was a ruler.
5. All Romans were either loyal to Ceasar or hated him.
6. Everyone is loyal to someone.
7. Men only try to assassinate rulers that are not loyal to.
8. Marcus tried to assassinated Ceasar.

? Was Marcus hating Ceasar?

# Representing Romean's World using FOL

1. $man(Marcus)$.
2. $Pompeian(Marcus)$.
3. $(\forall x)(Pompeian(x) \Rightarrow Roman(x))$.
4. $ruler(Ceasar)$.
5. $(\forall x)(Roman(x) \Rightarrow loyalto(x, Ceasar) \vee hate(x, Ceasar))$.
6. $(\forall x)(\exists y) loyalto(x, y)$.
7. $(\forall x)(\forall y)(man(x) \wedge ruler(y) \wedge tryassassinate(x, y) \Rightarrow \neg loyalto(x, y))$.
8. $tryassassinate(Marcus, Ceasar)$.

$?\ hate(Marcus, Ceasar)$

# Resolution (1/2)
## Transform KB into conjuctive normal form (CNF)

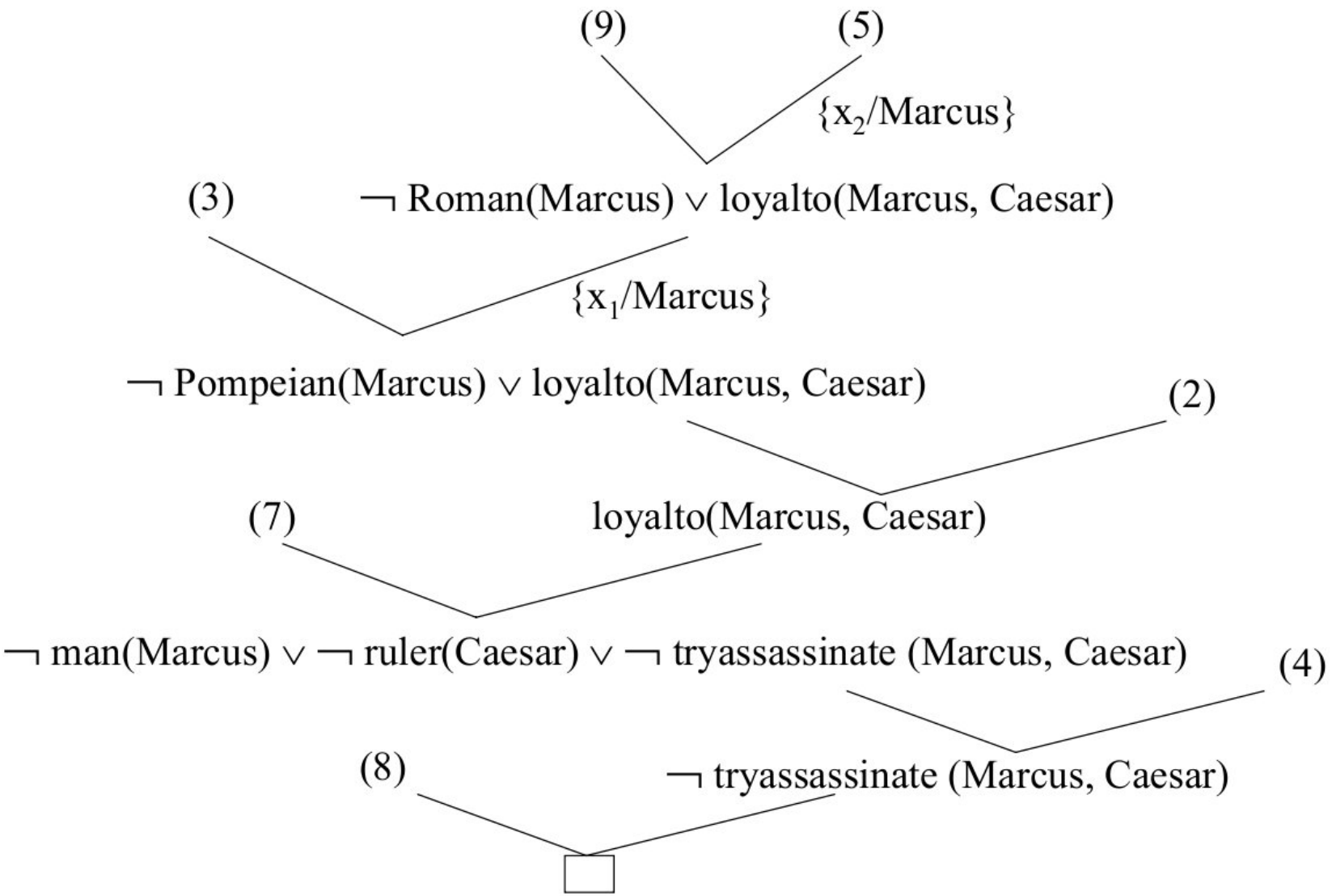
1. $man(Marcus)$.
2. $Pompeian(Marcus)$.
3. $\neg Pompeian(x_1) \vee Roman(x_1)$.
4. $ruler(Ceasar)$.
5. $\neg Roman(x_2) \vee loyalto(x_2, Ceasar) \vee hate(x_2, Ceasar)$.
6. $loyalto(x_3, F(x_3))$.
7. $\neg man(x_4) \vee \neg ruler(y_1) \vee \neg tryassassinate(x_4, y_1) \vee \neg loyalto(x_4, y_1)$.
8. $tryassassinate(Marcus, Ceasar)$.

We shall insert in KB the complement of the proposition that we want to prove. If the initial proposition is true according to our KB then we will end up in a refutation.

(9) $\neg hate(Marcus, Ceasar)$

# Resolution (2/2)

## Resolution method application



# References


Monty Python and the Holy Grail (1975).
*You can find the verbatim dialogue at*
*http://en.wikiquote.org/wiki/Monty_Python_and_the_Holy_Grail#The_Witch.*
*You can also find the corresponding movie scene at*
*http://www.youtube.com/watch?v=zrzMhU_4m-g.*

The application of the resolution method in Roman's World has been taken from the
Elaine Rich and Kevin Knight, Artificial Intelligence, $2^{nd}$ edition, McGrawHill, 1990.

The LaTeX Beamer Class Homepage.
*http://latex-beamer.sourceforge.net/.*

# The End

Thank you!