

A game theory modeling approach for 3G operators

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Abstract This paper presents the technoeconomic evaluation of a third generation rollout scenario followed by the identification of the market conditions for two operators in a simple game theory model. The considered scenarios reflect the point of view of both dominant operators and new entrants. Technoeconomic results are presented in terms of net present value (NPV), acting like the pay-off function in the proposed theoretical Game Theory model. The business case presented here was studied by using a tool, developed by European Projects IST-25172 TONIC/ECOSYS, a model developed by the EURESCOM P901 project and the Game Theory Tool named Gambit.

Keywords Technoeconomics · Techno-economics · Game theory · NPV · Investment UMTS · 3G · Market conditions · Telecommunications

1 Introduction

The investigation of a 3G rollout investment case (UMTS) is one of the hot topics in the last decade for the telecommunication operators and investors. This is true in both technical and economical points of view. In fact, these all pertain to global liberalization of Telecommunication and to the competition

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raise when the operators purchase the licenses. So, one of the main viewpoints here should be necessarily the competition and its impacts on business, through technoeconomic (TE) analysis.

Technoeconomic analysis of telecommunication systems and services, combining the economical and business aspects with comprehensive technical parameters, has attracted the interest of many researchers worldwide and several studies have been published, especially during the last decades aiming to offer quantitative and qualitative answers to questions about the telecom technology and business strategy [1–5]. However, especially towards the quantitative modeling, the business modeling as well as the business or making-profit strategies has to be combined with market forecasts, willingness-to-pay data, and cost modeling of both operational and capital expenditures in a right combination in order to produce balanced and therefore reliable results.

In order to incorporate competition analysis in every technoeconomic analysis, of a game theory provides a solid mathematical methodology for such purposes although the combination of game theory approach with traditional discount cash flow (DCF) analysis has not been widely published. Most of the articles available in the literature are based on the traditional DCF combined only with sensitivity and risk analysis, aiming to include uncertainty modeling. On the other hand, game theory has been widely used in telecommunications in radio resource management problems [6–10]. In [11], a general methodology to quantify the option value in technology investments based on option and game theory is proposed with possible application to capacity expansion problems of telecommunications, but without any specific application and numerical results. In conclusion, to the best knowledge of the authors, there is no complete analysis of any telecommunications investment case using game theory on top of traditional DCF analysis.

In this paper a simple UMTS investment case model following a game theory approach is presented, which was based upon near real business conditions and figures. This model could be easily expanded and used with modified market conditions and tariff schemes. The rest of the paper is organized as follows. In Section 2, the methodology used is presented while in Section 3, the parameters and the assumptions of the technoeconomic modeling along with the necessary mathematical background are described. Section 4 summarises the results of the modeling and finally Section 5 concludes the analysis and introduces future topics.

2 Methodology

The case presented within this article offers numerical results aiming to quantify the 3G competition problem. This investment analysis was carried using the tool, which was developed by IST TONIC [12–14], and the ECOSYS project [15] and a game theory model developed by EURESCOM (European Institute for Research and Strategic Studies in Telecommunications GmbH)

P901 project [16]. A 10-year time frame, spanning the period 2004 to 2014 has been chosen. In order to calculate discounted cash flows, a discount rate of 10% has been selected. Tariffs, market penetration of both fixed broadband and mobile services in various European countries and existing forecasts for UMTS services were gathered, and an analytical market model was developed [2]. The main assumptions regarding tariff policy as well as market conditions are analytically described in [2]. The area study represents a central or southern European country with higher population density (60 million) and lower mobile penetration (40%). The areas are composed of dense downtown zones surrounded by less dense peripheral zones, and larger suburban areas. The UMTS network is assumed to cover 80% of the total population and total mobile penetration is assumed to saturate at 80% (phone owners, not number of terminals) in 2009. Service classes are described in [2] and [3] as well as analytical TE results for the regular financial analysis.

2.1 Structure of the tool for technoeconomic evaluation

Figure 1 analyses the main principles of the methodology used in this paper. The cost figures for the network components have been collected in an integrated cost database, which is the “heart” of the model. This database is frequently updated with data obtained from the major telecommunication operators, suppliers, standardization bodies and other available sources. These data correspond to the initial prices for the future commercial network components as well as to the projection for the future production volume of them. The cost evolution of the different components derives from the cost at a given

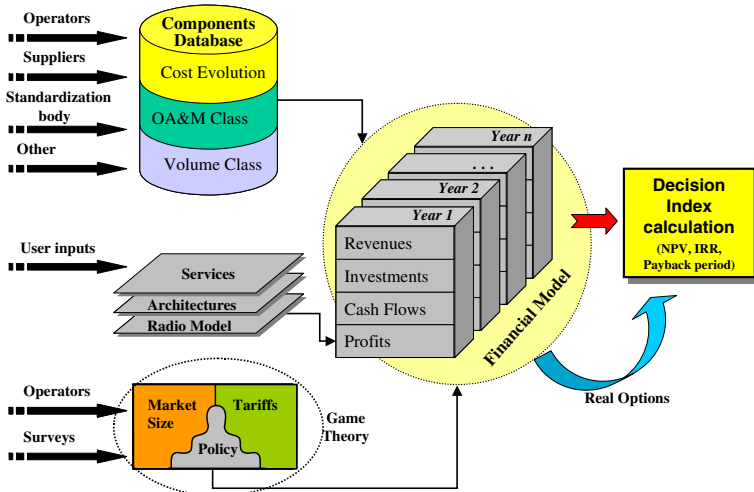


Fig. 1 Technoeconomics methodology and the connections with the game theory and real options modeling [2]

reference year and a set of parameters, which characterizes the basic principles of the component. For each component in the database, the cost evolution is estimated according to a cost evolution model [1] and [17]. In addition, estimations for the operation administration and maintenance cost as well as the production volume of the component are incorporated in the database. As a next step in the network evaluation, the services to be provided to the consumers should be specified. The network architectures for the selected set of services will be defined, and a radio model, has been used in order to calculate base transceiver station positions as well as the civil works for their installation (database data). The future market penetration of these services and the tariffs associated, according to each operator's policy, will be used for the construction of the market evolution model. The operator tariff policy could be taken into account by modifying the tariff level in conjunction with the expected penetration of the offered services. Results from statistics or surveys can be easily integrated into the tool when formulas measuring the impact of tariff level to the saturation of the services are available.

By entering the data into a financial model we calculate the revenues, investments (and IFC) cash flows and profits (or other financial results) of the studied network architectures for each year throughout a project's study period. In the final evaluation of the technoeconomic model, critical indexes are calculated in order to decide about the profitability of the investment. The methodology is illustrated in Fig. 1. The adoption of alternative financial and strategic methods, e.g. real options approach and game theory, can be included in the tool.

2.2 Game theory modeling

To keep simplicity, the initial technoeconomic model described in [2] has been used. The new model [16] has been build, based on the following main assumptions:

1. We investigated an *Oligopoly*, or more precisely *Duopoly* case. As it was considered to be realistic (in terms of modeling) the number of players would be only two, and one of them would be the incumbent operator with high market share, whereas the other would be a new company with relatively small market share and with Greenfield investment needs.
2. For not being complicated we decided to see the market for UMTS services as a *homogenous* market with only *one product*. It means we "merged" all services and market segments to one, independently of different frequencies used and tariffs applied.
3. During calculation we have not taken into account the feedback that general price levels have on the *total market*, so the total market was *stable or insensitive*. We supposed transmigration (churn effects) only between competitors, which means *dynamic market inside*.
4. The game was built as being a *non-cooperative game*, which means both players know everything of importance about the other, but they make

- decisions approximately at the same time and independently. In this case players are not allowed to form any kind of coalition.
5. At this stage of the work we “voted” to keep the game *static* with only *one decision point* at the beginning of the estimated time period (in case of longer time horizon this could fail to be “realistic”). This one decision point might be e.g. just before the national auctions or beauty contest for frequency allocation.
 6. In this UMTS example companies *play price-adjusting* game and quantities (quantity here refers to the size and costs of investment for network and infrastructure) do not produce an effect on the results considerably (this seems to be realistic, as a simple investment in the global coverage might be sufficient through the investigated period).
 7. The *pay-off function* of the game was decided to be the *NPV* of a 10-years long investment period, and in the first step we let out the calculations with real options values [18], so skipping the “adjusted” NPV that could have been produced.

3 Technoeconomic modeling

The calculations were based on two main inputs from the TE model results and the hypothetical (empirical) market parameters. The TE model served as the main root for calculating inputs of our calculations. So we accepted all the suppositions built into the TE model, and different models for “incumbent” and “newcomer” were prepared. We followed the 10-year time horizon, and naturally all the investments, costs and revenue figures. All the other basic parameters were set to illustrate an “average western European case”.

To be able to calculate the pay-off, two important parameters were picked up, namely:

- The market share and
- The tariff multiplier or in other words the “price”

The tariff multiplier parameter was used in order to increase or decrease the tariff of the offered product by a certain percentage. With immense running of the TE model we changed market shares in the 5–75% range in this project, while tariff multiplier changed within the 0.5–1.5 ranges in case of both players (incumbent or newcomer operators). This kind of model supposes that operators (players) make their decisions only at the beginning of the investigated time period. Results of the TE model are illustrated on Table 1 for incumbent (Player 1) and on Table 2 for the newcomer operator (Player 2). In the tables, the top heading row contains the tariff multiplier, while the left heading column contains the market share values (in percentages). Even if the game is static, the players will always have a reference to the profitability tables derived from this analysis (Tables 1 and 2) as they are continuously adopting

Table 1 Operator1-player 1 (incumbent) NPVs in B€

Market share (%)	Tariff multiplier										
	0.5 (-50%)	0.6 (-40%)	0.7 (-30%)	0.8 (-20%)	0.9 (-10%)	1	1.1 (+10%)	1.2 (+20%)	1.3 (+30%)	1.4 (+40%)	1.5 (+50%)
5	1.69	2.39	3.12	3.81	4.51	5.20	5.89	6.58	7.28	7.97	8.66
10	2.38	3.18	3.99	4.77	5.55	6.33	7.12	7.90	8.68	9.46	10.24
15	3.08	3.96	4.86	5.73	6.60	7.47	8.34	9.21	10.08	10.95	11.81
20	3.77	4.74	5.73	6.69	7.65	8.61	9.56	10.52	11.48	12.44	13.39
25	4.46	5.52	6.60	7.64	8.69	9.74	10.79	11.84	12.89	13.93	14.97
30	5.16	6.31	7.47	8.60	9.74	10.88	12.01	13.15	14.29	15.42	16.55
35	5.85	7.11	8.34	9.56	10.79	12.01	13.24	14.47	15.69	16.91	18.13
40	6.54	7.89	9.21	10.52	11.84	13.15	14.47	15.78	17.10	18.40	19.71
45	7.24	8.67	10.08	11.48	12.89	14.29	15.70	17.10	18.50	19.90	21.30
50	7.93	9.45	10.95	12.44	13.94	15.43	16.92	18.42	19.91	21.39	22.88
55	8.63	10.24	11.82	13.40	14.99	16.57	18.15	19.74	21.31	22.89	24.46
60	9.32	11.02	12.69	14.36	16.04	17.71	19.38	21.05	22.72	24.38	26.05
65	10.01	11.80	13.56	15.32	17.08	18.85	20.61	22.37	24.12	25.88	27.63
70	10.71	12.58	14.43	16.28	18.13	19.98	21.84	23.68	25.53	27.37	29.21
75	11.42	13.36	15.30	17.24	19.18	21.12	23.06	25.00	26.93	28.86	30.80

Table 2 Operator2-player 2 (newcomer) NPVs in B€

Market share (%)	Tariff multiplier										
	0.5 (-50%)	0.6 (-40%)	0.7 (-30%)	0.8 (-20%)	0.9 (-10%)	1	1.1 (+10%)	1.2 (+20%)	1.3 (+30%)	1.4 (+40%)	1.5 (+50%)
5	-2.32	-2.07	-2.00	-1.83	-1.18	-0.93	-0.69	-0.45	-0.20	0.04	0.28
10	-1.58	-1.24	-1.03	-0.75	-0.09	0.25	0.58	0.91	1.25	1.58	1.91
15	-0.84	-0.41	-0.07	0.33	1.00	1.43	1.85	2.27	2.69	3.12	3.54
20	-0.10	0.41	0.89	1.42	2.09	2.61	3.12	3.63	4.14	4.66	5.17
25	0.63	1.24	1.85	2.50	3.19	3.79	4.39	4.99	5.59	6.19	6.80
30	1.37	2.07	2.81	3.59	4.28	4.97	5.66	6.35	7.04	7.73	8.42
35	2.11	2.92	3.80	4.59	5.37	6.15	6.93	7.71	8.49	9.27	10.05
40	2.84	3.82	4.72	5.59	6.46	7.33	8.20	9.07	9.94	10.81	11.67
45	3.64	4.67	5.63	6.59	7.55	8.51	9.47	10.43	11.39	12.34	13.29
50	4.43	5.49	6.54	7.59	8.64	9.69	10.74	11.78	12.83	13.87	14.92
55	5.17	6.31	7.45	8.59	9.73	10.87	12.00	13.14	14.27	15.40	16.54
60	5.90	7.13	8.36	9.59	10.82	12.04	13.27	14.49	15.71	16.94	18.16
65	6.63	7.95	9.27	10.59	11.90	13.22	14.53	15.85	17.16	18.47	19.78
70	7.36	8.77	10.18	11.59	12.99	14.40	15.80	17.20	18.60	20.00	21.40
75	8.09	9.59	11.09	12.59	14.08	15.57	17.06	18.55	20.04	21.53	23.02

their prices to competition. So the profit margin could be always defined via the numerical results from the sensitivity analysis (Tables 1 and 2).

3.1 Hypothetical–empirical market parameters

Only for emphasizing assumptions regarding the market, we supposed that the total market is stable, and only transmigration between competitors was taken into account. This is combined with having only one “product,” which on one hand means that all services were merged to one, and on the other part there is no additional real market segmentation except the segmentation described in [2].

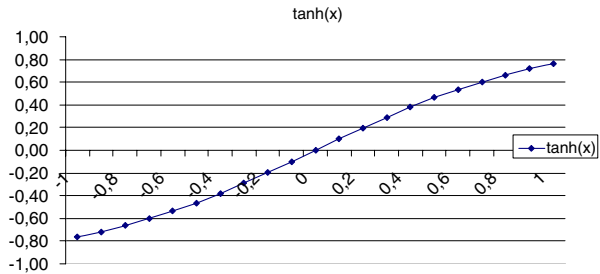
In the price-adjusting model, “price-war” was assumed, and we tried to depict this “war” by a hypothetical transmigration market function. The simple reason behind this is that when one of the players decreases his price related to the other’s, the cheaper operator will reach the higher portion of the market share, namely when lowering the price compared to the competitor’s, a gain market share will be obtained.

The difference between tariff multipliers (denoted by x) should be limited by the interval defined for the tariff multipliers for the two operators (denoted by TM_1 and TM_2). So TM_1 and TM_2 should be chosen from the $[0.5, 1.5]$ and the difference of the tariff multipliers ($TM_1 - TM_2$) will be limited in the $[-1, 1]$ interval. When x is 0, the tariffs are equal and when $x < 0$ then Player 1 offers more competitive product (with lower price). In general for the telecommunication market we are not able to collect data for the x values near to -1 or 1 since operators usually are not able to sell products in triple price than the competitor’s. (In case of $x = 1$ or $x = -1$ there is a three times difference in the product prices regarding the players, i.e. one product 50 and the other 150 euros).

There are a number of problems with the construction of the function that controls the market share of the operator. In our calculation the market function and plots are built from the newcomer operator’s (player 2) point of view. The definition of the market behavior function due to the price changes (from the point of view of player 2) follows some “logical” rules described as follows:

1. Value domains $[MSD_{Min}, MSD_{Max}]$ minimum and maximum variation of the market share (defined by the user each time)
2. The function applied in the value domain (MSD_{Max}) should be limited in the interval $[0, 1]$ and $-1 < x < 1$, where x is the relation of the tariffs (which shows the increase or decrease of tariffs for the offered services regarding the two operators). This constraint relies on the fact that an operator cannot lack more than 100% of its initial market share
3. Slope: for large values of x the function quickly reaches the saturation level, which means that the maximum market share cannot change further.
4. For $x < 0$ the function derives to 1 and actually to MSD_{Min} in the final implementation.

Fig. 2 Shape of the hyperbolic tangent function



- The slope should be steeper near zero, which means that the marker (subscribers) is sensitive with respect to small changes of tariffs. Furthermore, this slope depends on different market behaviours. Thus, it is desirable to be able to adjust this slope. Different market reaction will be tested in the implementation of the model.

Some example functions that could meet some of the above criteria are well-known and can be generated from the hyperbolic tangent function ($\tanh(x)$) (or from the $-x$ function when rule 4 is applied) and are defined by:

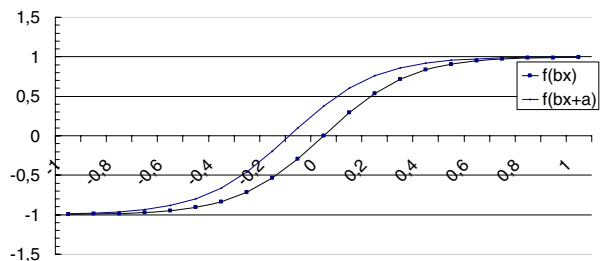
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{1}$$

$$f(bx + a) = \frac{e^{bx+a} - e^{-bx-a}}{e^{bx+a} + e^{-bx-a}} \tag{2}$$

Figure 2 illustrates the function of Eq. 1 for player 2 i.e. $f(-x)$. If we define some external parameters a and b we can derive a function which is more close to our market modeling needs with an adequate slope near zero. An example is illustrated in Fig. 3 and Eq. 2 ($a = 0.4, b = 3$). Different market conditions (rule 5) can be achieved mainly by changing parameter b that affects the steepness of the slope.

The general problem with these functions is that the result is symmetric (and can take negative values). A competition market model cannot be symmetric as far as it concerns the customer behavior. The slope of the market penetration should give a certain advantage or disadvantage to one

Fig. 3 Shape of modified hyperbolic tangent function ($a = 0.4, b = 3$)



of the competitors. The market forecasts within the projects TONIC [14] and ECOSYS [15] suggest that in the tariff strategy the incumbent operator (i.e. Finnish, France, German market) has a small advantage, allowing him to offer more expensive products to the market compare to the newcomer without loosing significant market share.

In another similar example the Boltzmann equation defined as

$$y = \frac{A_1 - A_2}{1 + e^{(x-x_0)/dx}} + A_2 \quad (3)$$

can be used where,

x_0 is the centre or then mean value

dx is the width

A_1 is the initial value of y (left horizontal asymptote, $y - \infty$)

A_2 the final value of y (right horizontal asymptote, $y + \infty$)

The $y(x_0)$ is the mean value between the two limiting values A_1 and A_2 and therefore is defined as:

$$y(x_0) = \frac{(A_1 + A_2)}{2} \quad (4)$$

The y value changes drastically within the range of the x variable (the change in x corresponding to the most significant change in y values). The width of this range is approximately dx .

By using the data illustrated in Table 3 which are related to a competitive market with two operators and which are original data from the GSM

Table 3 Market share variation

Tariff multiplier variance	Market share variance (original data)
-1	0.02
-0.9	0.08
-0.8	0.25
-0.7	0.67
-0.6	1.58
-0.5	3.33
-0.4	6.35
-0.3	11.06
-0.2	17.74
-0.1	26.42
0	36.79
0.1	48.24
0.2	59.97
0.3	71.1
0.4	80.86
0.5	88.7
0.6	94.36
0.7	97.86
0.8	99.53
0.9	99.98
1	100

Market according to EURESCOM [16], a non linear squares (NLS) regression has been performed and the results are quite adequate (Fig. 4). In this figure Eq. 3 results and NLS plot are depicted. Table 3 shows the percentage change of of the maximum market share. As the tariff multiplier variance moves to 1 market share reaches its maximum value (player 2). In this example it has been assumed that the operator can retain a minimum market share (5) and with the parameters of Table 4 the market share could change negatively.

Within the EURESCOM P901 [16] an improved model for the definition of the market behavior has been proposed. The criteria 1 to 5 are satisfied and a general “S-shaped” curve was supposed, with three sort of market behavior regarding price-reaction of customers. This kind of function stands for both companies, but with different parameters. As we mention before, in our calculation the market function is built from the newcomer operator’s (player 2) point of view. All the parameters refer to the newcomer (player 2). The market share of the newcomer is given by:

$$MS = MS_{Start} + \Delta MSD_{Min} + \Delta MSD_{Max} \cdot e^{-e^{\left(\frac{a+b}{\sqrt{1+TM_2-TM_1}}\right)}} \tag{5}$$

where,

- MS is the market share
- MS_{Start} is the market share at the beginning, where decision is made
- ΔMSD_{Min}, ΔMSD_{Max} are the minimum and maximum values of market share changes (delta)
- TM₁, TM₂ are the tariff multipliers of player 1 and player 2 respectively.

Fig. 4 Illustration of shape of Boltzmann equation (fitting, dot lines)

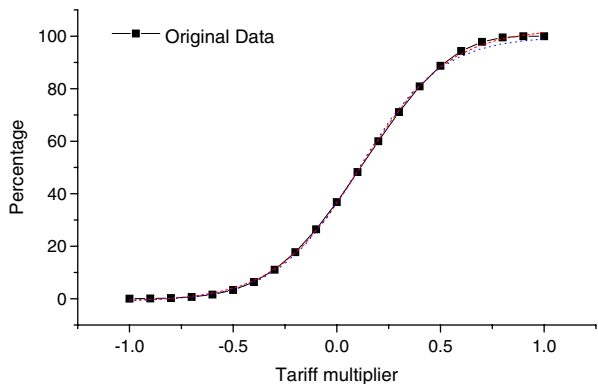


Table 4 Different versions of market parameters used in the calculation for newcomer (Player 2)

Versions: operator2 (newcomer) parameters			
MS _{Start}	15% (S:Small)	20% (M:Medium)	25% (H:High)
ΔMSD _{Min}	-3% (I:Insensitive)	-5% (A:Average)	-7% (S: Sensitive)
ΔMSD _{Max}	37% (I:Insensitive)	45% (A:Average)	53% (S: Sensitive)
<i>a</i>	0	0.1	0.3
<i>b</i>	-3	-4	-5

$x = TM_1 - TM_2$ is the tariff multiplier variance

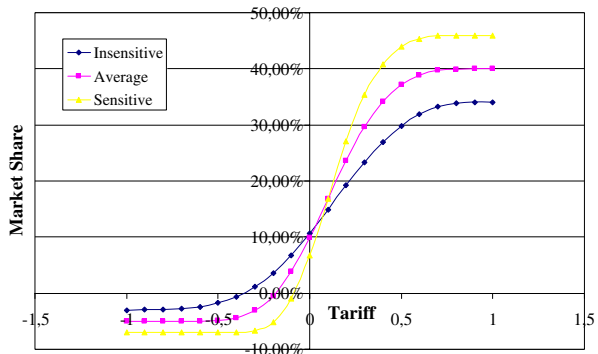
$$y(x) = \left(a + b \frac{x}{\sqrt{1-x}} \right)$$

$e^{-e^{y(x)}}$ is the market share variance

The three sorts of market share ranged from insensitive to sensitive with different parameters of the function (MSD_{min}, MSD_{max}, *a* and *b* values are presented in Table 4). Additionally, three different measures of the newcomer’s (player 2) initial or start market share were assumed, namely 15%, 20% and 25%. Table 4 shows the different versions of calculation, and Fig. 5 illustrates the shape of the used function (without the term MS Start) for all the values from Table 4 (three different shapes of the market function). The level of sensitivity is taken into account by increasing the limits ±2% and ±7% of the MSD_{min}, MSD_{max} from the basic case (average market). Parameter *b* influences the steepness of the slope namely the transition of customers when a small change of tariff occurs.

It is important to note, that “hypothetical” or empirical corresponds to experts’ opinions and feelings. So, in reality, the market punishes definitely more heavily high-priced operators. The reader should tailor this kind of functionality to his own market, both in shape and measure. To fit this kind of model into the reality, taking interdependencies of price and total market

Fig. 5 Shape of the hypothetical market function (without the term MS_{Start} of Eq. 5)



size into consideration would be necessary. The parameters for the greenfield operator/newcomer are illustrated below:

Our hypothetical market function could be termed unfettered. It has been fabricated in order to be realistic following the market condition already proven in the GSM market. The way of achieving this kind of function may rely on time series of similar products/services history. In case of services of UMTS we might consider GSM services. If there were analytical GSM prices–market shares (for at least two providers) time series, we could be able to construct a simplest function for the description of the market conditions. Probably, the derived function would be simpler. In our UMTS example we assumed only one decision point in the beginning and a 10-years long study. One could disagree with this approach. If we would like the approach to be closer to reality, the decision point should follow one after the other on a yearly or monthly basis. In that case, we might use a simple function, reached from time series year by year. Furthermore, in gaming, the higher the number of decision points, the more complex the model becomes.

The sensitivity analysis that we used and applied in UMTS BC has not been used in constructing the hypothetical market function, but has been greatly used in estimating the pay-offs (i.e. NPV). Following a different approach, another kind of sensitivity analysis would be (price level)/(market share), which might fruit into a simple linear function and can be used in limited circumstances (time frame, price-cap, market size). In our paper we have not investigated sensitivity in that way, but it might have fitted in market issues when assuming e.g. yearly decision points in the TE and GT model, and by using results from market sensitivity functions. The next paragraph explains thoroughly the outputs.

3.2 Implementation of the game

The main body of the calculations is made in MS Excel worksheets. This is also true for market functions, tables as well as for pay-off tables. Pay-off tables stood as input of a simple translator program written on C language. The output of this little program served the GAMBIT solver with input. GAMBIT software is quite deeply explained in [19]. The results of GAMBIT were the places of Nash-equilibrium points (in this game). These points were then drawn back to pay-off matrixes in MS Excel and TE tool.

Analytically in Fig. 6 the data flow between the TE tool (3G operator model) and the Game theory tables are illustrated. For each point defined by a specific tariff policy via the tariff multipliers exist's one market share for each player (player1—incumbent, player 2—newcomer). In the next step for each value of the tariff multiplier and market share one value of NPV per player can be calculated by the TE tool (NPV_{PL1} and NPV_{PL2}). These NPV values are stored in a cell, consisting as a step for the players for GAMBIT Tool. The process continues until all cells are completed with NPV values, (last step $[TM_{PL1}, TM_{PL2}] = [1.5, 1.5]$). These results are basically all the sensitivity results for all combinations. The process will be repeated for all defined market

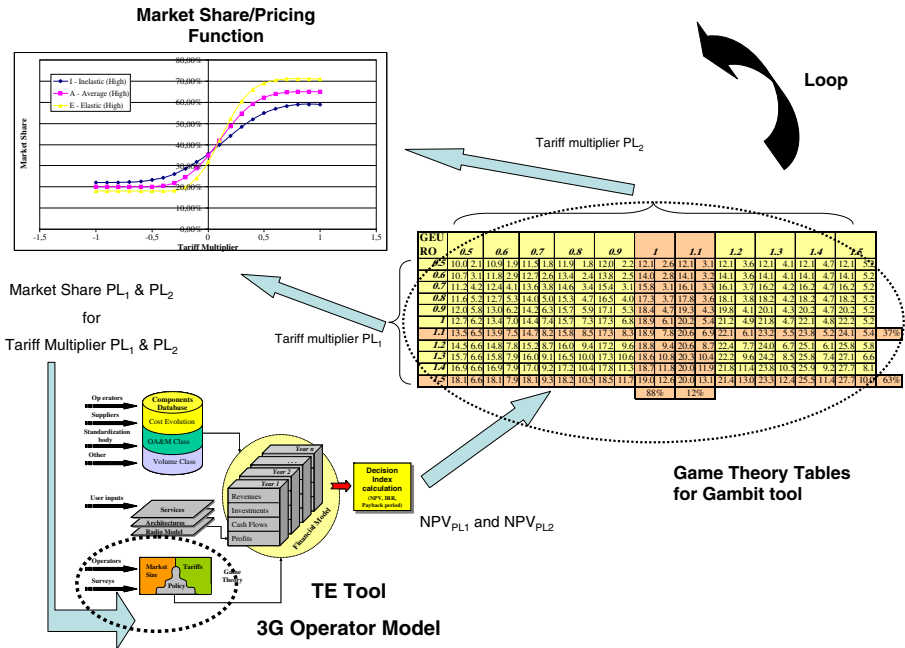


Fig. 6 Data flow between the technoeconomic tool and the game theory tables

types [Insensitive—Small, Medium, High], [Average—Small, Medium, High] and [Sensitive—Small, Medium, High], nine times in total.

4 Results and discussion

The following tables (Tables 5, 6, 7, 8, 9, 10, 11, 12, and 13 in B€) show the results reached by calculating pay-off functions and in addition the Nash-equilibrium points are illustrated. Each cell contains two figures. The first column is the tariff multiplier of player 1 (PL_1) and the first row belongs to player 2 (PL_2). Each cell defined by the tariff multipliers contains two values. The first value is the NPV of Operator 1 (NPV_{PL_1}) and the second is the NPV of Operator 2 (NPV_{PL_2}) for a specific tariff policy (i.e. first cell [0.5, 0.5] means 50% reduction in tariff compared to the base tariff for both players and $NPV_{PL_1} = 9.9$ B€, $NPV_{PL_2} = 2.2$ B€). The “position” of each player can be improved if its financial result is higher.

The crossings of the vertically and horizontally colored different cells show the Nash-equilibrium points. The Nash-equilibrium points are bordered differently and so do strategies of competitors bound to the equilibrium. In case of having solutions with mixed strategies probabilities of specific strategies, these are put at the end of rows or columns (i.e. Table 8 82% and 12% probability). It is important to notice that these strategy combinations and equilibrium points

Table 5 Pay-off matrix of HI version (high market share, insensitive market; P11/P12)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5											
0.5	9.9	2.2	10.5	2.4	10.9	2.5	11.2	2.7	11.5	3.0	11.7	3.4	11.8	3.8	11.8	4.2	11.8	4.8	11.8	5.3	11.8	5.8
0.6	11.0	2.8	11.7	3.0	12.3	3.2	12.8	3.3	13.2	3.4	13.5	3.6	13.6	3.9	13.7	4.3	13.8	4.8	13.8	5.3	13.8	5.8
0.7	12.0	3.5	12.7	3.8	13.5	3.9	14.1	4.0	14.7	4.0	15.1	4.0	15.4	4.2	15.6	4.5	15.7	4.9	15.8	5.3	15.8	5.8
0.8	12.8	4.2	13.6	4.6	14.4	4.8	15.2	4.7	15.9	4.7	16.6	4.6	17.0	4.7	17.4	4.8	17.6	5.1	17.7	5.5	17.8	5.9
0.9	13.5	4.7	14.3	5.2	15.2	5.5	16.1	5.6	17.0	5.5	17.8	5.4	18.4	5.3	19.0	5.3	19.3	5.4	19.6	5.6	19.7	6.0
1	14.3	5.2	15.0	5.8	15.8	6.2	16.8	6.4	17.7	6.4	18.7	6.3	19.6	6.1	20.3	6.0	20.9	5.9	21.3	6.0	21.5	6.2
1.1	15.2	5.5	15.7	6.3	16.5	6.9	17.3	7.2	18.4	7.4	19.4	7.3	20.5	7.1	21.4	6.8	22.2	6.6	22.8	6.5	23.2	6.6
1.2	16.3	5.6	16.6	6.6	17.1	7.4	17.9	8.0	18.9	8.3	19.9	8.3	21.1	8.2	22.2	7.9	23.2	7.6	24.1	7.3	24.7	7.2
1.3	17.4	5.7	17.6	6.8	18.0	7.8	18.6	8.5	19.4	9.1	20.4	9.3	21.5	9.3	22.8	9.0	24.0	8.7	25.0	8.3	25.9	8.0
1.4	18.7	5.8	18.8	6.9	18.9	8.0	19.3	9.0	20.0	9.7	20.8	10.1	21.9	10.3	23.1	10.2	24.4	9.9	25.7	9.5	26.8	9.0
1.5	20.0	5.8	20.0	7.0	20.1	8.1	20.3	9.2	20.7	10.1	21.4	10.8	22.3	11.2	23.4	11.3	24.7	11.2	26.1	10.8	27.4	10.3

Table 6 Pay-off matrix of MI version (medium market share, insensitive market)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5											
0.5	10.6	1.5	11.2	1.5	11.6	1.6	12.0	1.7	11.5	2.0	11.7	2.2	11.8	2.5	11.8	2.9	11.8	3.3	11.8	3.7	11.8	4.2
0.6	11.8	2.1	12.5	2.2	13.1	2.2	13.6	2.2	14.0	2.3	13.5	2.5	13.6	2.7	13.7	3.0	13.8	3.3	13.8	3.8	13.8	4.2
0.7	12.8	2.7	13.6	2.9	14.3	2.9	15.0	2.9	15.6	2.9	16.0	2.9	15.4	3.0	15.6	3.2	15.7	3.4	15.8	3.8	15.8	4.2
0.8	13.7	3.3	14.5	3.6	15.4	3.7	16.2	3.7	16.9	3.6	17.5	3.5	18.0	3.4	17.4	3.5	17.6	3.6	17.7	3.9	17.8	4.3
0.9	14.6	3.9	15.3	4.4	16.2	4.6	17.1	4.6	18.0	4.4	18.8	4.2	19.5	4.0	20.0	3.9	19.3	4.0	19.6	4.1	19.7	4.4
1	15.5	4.4	16.1	5.0	17.0	5.3	17.9	5.4	18.9	5.3	19.8	5.1	20.7	4.8	21.4	4.6	22.0	4.5	21.3	4.5	21.5	4.6
1.1	16.5	4.7	17.0	5.5	17.7	6.0	18.6	6.2	19.6	6.3	20.6	6.1	21.7	5.8	22.6	5.5	23.4	5.2	24.0	5.0	23.2	5.0
1.2	17.6	4.9	17.9	5.8	18.5	6.5	19.2	7.0	20.2	7.2	21.3	7.1	22.4	6.9	23.5	6.5	24.5	6.1	25.4	5.8	26.0	5.5
1.3	18.8	5.0	19.0	6.0	19.4	6.9	20.0	7.6	20.8	8.0	21.8	8.1	22.9	8.0	24.2	7.7	25.4	7.2	26.4	6.7	27.3	6.3
1.4	20.2	5.1	20.2	6.1	20.4	7.1	20.8	8.0	21.4	8.6	22.3	9.0	23.4	9.0	24.6	8.8	25.9	8.4	27.2	7.9	28.3	7.4
1.5	21.6	5.1	21.6	6.1	21.7	7.2	21.9	8.2	22.3	9.1	22.9	9.6	23.9	10.0	25.0	10.0	26.3	9.7	27.7	9.2	29.0	8.6

Table 7 Pay-off matrix of SI version (small market share, insensitive market)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5											
0.5	11.3	0.7	11.9	0.7	11.6	0.6	12.0	0.6	11.5	0.9	11.7	1.0	11.8	1.2	11.8	1.5	11.8	1.9	11.8	2.2	11.8	2.6
0.6	12.6	1.3	13.3	1.3	13.9	1.2	13.6	1.1	14.0	1.2	13.5	1.3	13.6	1.4	13.7	1.6	13.8	1.9	13.8	2.2	13.8	2.6
0.7	13.7	2.0	14.5	2.0	15.2	2.0	15.9	1.8	15.6	1.8	16.0	1.7	15.4	1.7	15.6	1.8	15.7	2.0	15.8	2.3	15.8	2.6
0.8	14.7	2.6	15.5	2.8	16.3	2.8	17.1	2.6	17.9	2.5	17.5	2.3	18.0	2.1	17.4	2.1	17.6	2.2	17.7	2.4	17.8	2.6
0.9	15.6	3.1	16.4	3.5	17.3	3.6	18.2	3.6	19.1	3.3	19.9	3.0	19.5	2.8	20.0	2.6	19.3	2.5	19.6	2.6	19.7	2.8
1	16.6	3.6	17.3	4.2	18.1	4.5	19.0	4.5	20.0	4.2	21.0	3.9	21.9	3.6	21.4	3.2	22.0	3.0	21.3	2.9	21.5	3.0
1.1	17.7	3.9	18.2	4.7	18.9	5.1	19.8	5.2	20.8	5.2	21.9	4.9	22.9	4.5	23.9	4.1	23.4	3.7	24.0	3.5	23.2	3.3
1.2	18.9	4.1	19.2	5.0	19.8	5.6	20.5	6.0	21.5	6.1	22.6	6.0	23.7	5.6	24.8	5.2	25.8	4.7	25.4	4.2	26.0	3.9
1.3	20.2	4.2	20.4	5.2	20.8	6.0	21.4	6.6	22.2	6.9	23.2	6.9	24.4	6.7	25.6	6.3	26.8	5.8	27.8	5.2	27.3	4.7
1.4	21.7	4.3	21.7	5.3	21.9	6.2	22.3	7.0	22.9	7.5	23.8	7.8	24.9	7.8	26.1	7.5	27.4	7.0	28.7	6.4	29.8	5.7
1.5	23.2	4.3	23.2	5.3	23.3	6.3	23.4	7.2	23.9	8.0	24.5	8.5	25.4	8.7	26.6	8.6	27.9	8.3	29.3	7.7	30.6	7.0

Table 8 Pay-off matrix of HA version (high market share, average market)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5												
0.5	10.0	2.1	10.9	1.9	11.5	1.8	11.9	1.8	12.0	2.2	12.1	2.6	12.1	3.1	12.1	3.6	12.1	4.1	12.1	4.7	12.1	5.2	
0.6	10.7	3.1	11.8	2.9	12.7	2.6	13.4	2.4	13.8	2.5	14.0	2.8	14.1	3.2	14.1	3.6	14.1	4.1	14.1	4.7	14.1	5.2	
0.7	11.2	4.2	12.4	4.1	13.6	3.8	14.6	3.4	15.4	3.1	15.8	3.1	16.1	3.3	16.1	3.7	16.2	4.2	16.2	4.7	16.2	5.2	
0.8	11.6	5.2	12.7	5.3	14.0	5.0	15.3	4.7	16.5	4.0	17.3	3.7	17.8	3.6	18.1	3.8	18.2	4.2	18.2	4.7	18.2	5.2	
0.9	12.0	5.8	13.0	6.2	14.2	6.3	15.7	5.9	17.1	5.3	18.4	4.7	19.3	4.3	19.8	4.1	20.1	4.3	20.2	4.7	20.2	5.2	
1	12.7	6.2	13.4	7.0	14.4	7.4	15.7	7.3	17.3	6.8	18.9	6.1	20.2	5.4	21.2	4.9	21.8	4.7	22.1	4.8	22.2	5.2	
1.1	13.5	6.5	13.9	7.5	14.7	8.2	15.8	8.5	17.3	8.3	18.9	7.8	20.6	6.9	22.1	6.1	23.2	5.5	23.8	5.2	24.1	5.4	37%
1.2	14.5	6.6	14.8	7.8	15.2	8.7	16.0	9.4	17.2	9.6	18.8	9.4	20.6	8.7	22.4	7.7	24.0	6.7	25.1	6.1	25.8	5.8	
1.3	15.7	6.6	15.8	7.9	16.0	9.1	16.5	10.0	17.3	10.6	18.6	10.8	20.3	10.4	22.2	9.6	24.2	8.5	25.8	7.4	27.1	6.6	
1.4	16.9	6.6	16.9	7.9	17.0	9.2	17.2	10.4	17.8	11.3	18.7	11.8	20.0	11.9	21.8	11.4	23.8	10.5	25.9	9.2	27.7	8.1	
1.5	18.1	6.6	18.1	7.9	18.1	9.3	18.2	10.5	18.5	11.7	19.0	12.6	20.0	13.1	21.4	13.0	23.3	12.4	25.5	11.4	27.7	10.0	63%
											88%	12%											

Table 9 Pay-off matrix of MA version (medium market share, average market)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5												
0.5	10.7	1.4	11.6	1.1	11.5	0.8	11.9	0.7	12.0	1.1	12.1	1.5	12.1	1.9	12.1	2.3	12.1	2.7	12.1	3.1	12.1	3.5	
0.6	11.5	2.4	12.6	2.1	13.5	1.6	13.4	1.3	13.8	1.4	14.0	1.6	14.1	1.9	14.1	2.3	14.1	2.7	14.1	3.1	14.1	3.5	
0.7	12.1	3.4	13.2	3.2	14.4	2.8	15.5	2.3	15.4	2.0	15.8	1.9	16.1	2.0	16.1	2.3	16.2	2.7	16.2	3.1	16.2	3.5	
0.8	12.5	4.4	13.7	4.5	15.0	4.2	16.3	3.6	17.4	3.0	17.3	2.5	17.8	2.3	18.1	2.4	18.2	2.7	18.2	3.1	18.2	3.5	
0.9	13.1	5.1	14.0	5.4	15.3	5.4	16.7	4.9	18.2	4.3	19.4	3.5	19.3	3.0	19.8	2.8	20.1	2.9	20.2	3.2	20.2	3.5	
1	13.8	5.5	14.5	6.2	15.5	6.5	16.9	6.3	18.4	5.8	20.0	4.9	21.4	4.1	21.2	3.5	21.8	3.2	22.1	3.3	22.2	3.6	
1.1	14.7	5.7	15.2	6.7	15.9	7.3	17.0	7.5	18.5	7.3	20.2	6.6	21.9	5.6	23.3	4.7	23.2	4.0	23.8	3.7	24.1	3.7	30%
1.2	15.9	5.9	16.1	6.9	16.5	7.8	17.3	8.4	18.5	8.5	20.1	8.2	21.9	7.4	23.7	6.3	25.3	5.3	25.1	4.5	25.8	4.2	
1.3	17.1	5.9	17.2	7.1	17.4	8.2	17.9	9.0	18.7	9.5	20.0	9.6	21.7	9.1	23.6	8.2	25.6	7.0	27.2	5.9	27.1	5.0	
1.4	18.4	5.9	18.4	7.1	18.5	8.3	18.7	9.4	19.3	10.2	20.2	10.7	21.5	10.6	23.3	10.1	25.3	9.0	27.4	7.7	29.2	6.4	
1.5	18.1	5.9	19.7	7.1	19.7	8.4	19.8	9.5	20.1	10.6	20.6	11.4	21.6	11.8	23.0	11.7	24.9	11.0	27.1	9.8	29.2	8.4	70%
											39%	61%											

Table 10 Pay-off matrix of SA version (small market share, average market)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5												
0.5	11.4	0.6	11.6	0.2	11.5	-0.2	11.9	-0.3	12.0	0.0	12.1	0.3	12.1	0.6	12.1	0.9	12.1	1.2	12.1	1.6	12.1	1.9	
0.6	12.3	1.6	13.4	1.2	13.5	0.7	13.4	0.2	13.8	0.3	14.0	0.4	14.1	0.6	14.1	0.9	14.1	1.2	14.1	1.6	14.1	1.9	
0.7	12.9	2.6	14.1	2.4	15.3	1.8	15.5	1.2	15.4	0.9	15.8	0.7	16.1	0.7	16.1	1.0	16.2	1.3	16.2	1.6	16.2	1.9	
0.8	13.5	3.5	14.6	3.5	15.9	3.2	17.3	2.5	17.4	1.9	17.3	1.3	17.8	1.1	18.1	1.1	18.2	1.3	18.2	1.6	18.2	1.9	
0.9	14.1	4.3	15.1	4.6	16.3	4.5	17.8	4.0	19.2	3.2	19.4	2.4	19.3	1.7	19.8	1.4	20.1	1.4	20.2	1.6	20.2	1.9	
1	14.9	4.8	15.6	5.4	16.7	5.6	18.0	5.3	19.6	4.7	21.1	3.8	21.4	2.8	21.2	2.1	21.8	1.8	22.1	1.8	22.2	2.0	
1.1	16.0	5.0	16.4	5.8	17.1	6.4	18.3	6.5	19.7	6.2	21.4	5.4	23.1	4.4	23.3	3.3	23.2	2.6	23.8	2.2	24.1	2.1	
1.2	17.2	5.2	17.4	6.1	17.8	6.9	18.6	7.4	19.8	7.5	21.4	7.0	23.2	6.1	25.0	5.0	25.3	3.8	25.1	3.0	25.8	2.5	26%
1.3	18.5	5.2	18.6	6.3	18.8	7.2	19.3	8.0	20.2	8.4	21.4	8.4	23.1	7.9	25.0	6.8	27.0	5.6	27.2	4.3	27.1	3.4	
1.4	19.9	5.2	19.9	6.3	20.0	7.4	20.2	8.4	20.7	9.1	21.7	9.5	23.0	9.4	24.8	8.7	26.8	7.6	28.9	6.2	29.2	4.8	
1.5	19.7	5.2	21.3	6.3	21.3	7.4	21.4	8.5	21.6	9.5	22.2	10.2	23.2	10.5	24.6	10.3	26.5	9.5	28.6	8.3	30.8	6.8	74%
											7%	93%											

Table 11 Pay-off matrix of HS version (high market share, sensitive market)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5											
0.5	10.5	1.6	11.6	1.1	11.4	0.9	11.6	1.1	11.7	1.7	11.7	2.1	11.7	2.6	11.7	3.1	11.7	3.6	11.7	4.0	11.7	4.5
0.6	10.7	3.1	12.3	2.4	13.5	1.7	13.4	1.4	13.6	1.7	13.7	2.1	13.7	2.6	13.7	3.1	13.7	3.6	13.7	4.0	13.7	4.5
0.7	10.6	4.8	12.4	4.2	14.1	3.1	15.5	2.3	15.3	2.1	15.6	2.2	15.6	2.6	15.6	3.1	15.6	3.6	15.6	4.0	15.6	4.5
0.8	10.5	5.9	12.0	5.8	14.0	5.1	15.9	4.0	17.4	3.0	17.3	2.6	17.6	2.7	17.6	3.1	17.6	3.6	17.6	4.0	17.6	4.5
0.9	10.6	6.8	11.8	7.2	13.5	6.9	15.6	6.0	17.8	4.7	19.4	3.6	19.2	3.1	19.5	3.2	19.6	3.6	19.6	4.0	19.6	4.5
1	11.1	7.2	11.8	8.1	13.1	8.4	15.0	8.0	17.3	6.9	19.6	5.4	21.3	4.1	21.2	3.6	21.5	3.7	21.6	4.1	21.6	4.5
1.1	11.9	7.4	12.3	8.6	13.0	9.4	14.4	9.7	16.4	9.1	18.9	7.8	21.4	6.1	23.3	4.7	23.1	4.1	23.5	4.1	23.5	4.5
1.2	12.9	7.5	13.1	8.8	13.4	10.0	14.2	10.8	15.7	10.9	17.9	10.2	20.6	8.7	23.2	6.8	25.3	5.3	25.0	4.6	25.4	4.6
1.3	14.0	7.5	14.0	8.9	14.2	10.2	14.6	11.4	15.4	12.1	17.0	12.1	19.3	11.3	22.2	9.6	25.0	7.5	27.2	5.9	27.0	5.1
1.4	15.1	7.5	15.1	8.9	15.2	10.3	15.3	11.7	15.7	12.8	16.7	13.4	18.3	13.4	20.8	12.4	23.8	10.5	26.9	8.3	29.2	6.5
1.5	16.2	7.5	16.2	8.9	16.2	10.4	16.3	11.8	16.4	13.1	16.9	14.2	17.9	14.8	19.6	14.6	22.2	13.4	25.4	11.4	28.7	9.0
							68%		8%				24%									47%
																						3%

Table 12 Pay-off matrix of MS version (medium market. share, sensitive market)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5											
0.5	11.2	0.9	11.6	0.3	11.4	-0.1	11.6	0.0	11.7	0.6	11.7	1.0	11.7	1.3	11.7	1.7	11.7	2.1	11.7	2.5	11.7	2.9
0.6	11.5	2.4	13.1	1.5	13.5	0.7	13.4	0.3	13.6	0.6	13.7	1.0	13.7	1.3	13.7	1.7	13.7	2.1	13.7	2.5	13.7	2.9
0.7	11.5	4.0	13.2	3.2	15.0	2.2	15.5	1.2	15.3	1.0	15.6	1.0	15.6	1.4	15.6	1.7	15.6	2.1	15.6	2.5	15.6	2.9
0.8	11.4	5.2	13.0	5.0	15.0	4.2	16.9	2.9	17.4	1.9	17.3	1.4	17.6	1.4	17.6	1.7	17.6	2.1	17.6	2.5	17.6	2.9
0.9	11.7	6.0	12.8	6.4	14.5	6.0	16.7	5.0	18.8	3.6	19.4	2.4	19.2	1.8	19.5	1.8	19.6	2.1	19.6	2.5	19.6	2.9
1	12.3	6.5	13.0	7.3	14.2	7.5	16.1	7.0	18.4	5.8	20.7	4.2	21.3	2.9	21.2	2.2	21.5	2.2	21.6	2.5	21.6	2.9
1.1	13.1	6.7	13.5	7.8	14.3	8.5	15.6	8.7	17.6	8.0	20.1	6.6	22.6	4.8	23.3	3.4	23.1	2.6	23.5	2.6	23.5	2.9
1.2	14.2	6.8	14.4	8.0	14.7	9.1	15.6	9.8	17.0	9.8	19.2	9.0	21.9	7.4	24.5	5.5	25.3	3.9	25.0	3.1	25.4	3.0
1.3	15.4	6.8	15.4	8.1	15.6	9.3	16.0	10.4	16.9	11.0	18.4	10.9	20.7	10.0	23.6	8.2	26.4	6.1	27.2	4.4	27.0	3.5
1.4	16.6	6.8	16.6	8.1	16.6	9.4	16.8	10.7	17.2	11.7	18.1	12.2	19.8	12.1	22.3	11.0	25.3	9.0	28.3	6.7	29.2	4.8
1.5	17.8	6.8	17.8	8.1	17.8	9.4	17.8	10.8	18.0	12.0	18.5	13.0	19.4	13.5	21.2	13.2	23.8	12.0	27.0	9.9	30.2	7.4
							41%		59%													53%

Table 13 Pay-off matrix of SS version (small market share, sensitive market)

	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5											
0.5	11.9	0.2	11.6	-0.6	11.4	-1.1	11.6	-1.1	11.7	-0.5	11.7	-0.2	11.7	0.1	11.7	0.4	11.7	0.7	11.7	1.0	11.7	1.3
0.6	12.3	1.6	13.9	0.7	13.5	-0.3	13.4	-0.8	13.6	-0.5	13.7	-0.2	13.7	0.1	13.7	0.4	13.7	0.7	13.7	1.0	13.7	1.3
0.7	12.3	3.2	14.1	2.4	15.9	1.2	15.5	0.1	15.3	-0.1	15.6	-0.1	15.6	0.1	15.6	0.4	15.6	0.7	15.6	1.0	15.6	1.3
0.8	12.4	4.5	14.0	4.2	15.9	3.2	17.9	1.8	17.4	0.8	17.3	0.2	17.6	0.2	17.6	0.4	17.6	0.7	17.6	1.0	17.6	1.3
0.9	12.7	5.3	13.9	5.5	15.6	5.1	17.7	4.0	19.9	2.5	19.4	1.2	19.2	0.5	19.5	0.5	19.6	0.7	19.6	1.0	19.6	1.3
1	13.4	5.8	14.1	6.4	15.4	6.6	17.2	6.0	19.6	4.7	21.9	3.0	21.3	1.6	21.2	0.9	21.5	0.8	21.6	1.0	21.6	1.3
1.1	14.4	6.0	14.7	7.0	15.5	7.6	16.8	7.7	18.9	6.9	21.4	5.4	23.9	3.6	23.3	2.0	23.1	1.2	23.5	1.1	23.5	1.3
1.2	15.5	6.0	15.7	7.2	16.1	8.2	16.9	8.8	18.3	8.7	20.5	7.8	23.2	6.1	25.9	4.1	25.3	2.4	25.0	1.5	25.4	1.4
1.3	16.8	6.0	16.8	7.3	17.0	8.4	17.4	9.4	18.3	9.9	19.8	9.8	22.1	8.7	25.0	6.9	27.8	4.6	27.2	2.8	27.0	1.9
1.4	18.1	6.0	18.1	7.3	18.1	8.5	18.3	9.7	18.7	10.6	19.6	11.1	21.3	10.8	23.8	9.6	26.8	7.6	29.8	5.2	29.2	3.2
1.5	19.4	6.0	19.4	7.3	19.4	8.5	19.4	9.8	19.6	10.9	20.0	11.8	21.0	12.2	22.8	11.9	25.4	10.5	28.6	8.3	31.8	5.7
							14%		86%													57%

should be seen as being Nash-sense, and even if we might speak about strategy likely to be followed, the general meaning of this is different from a “dominant strategy”.

Having Nash-sense equilibrium practically means both competitors play their best strategy, related to the other strategies selected. This can be done only in such a way, that players know each other’s strategy in advance. Within specific limits, this could be a real situation taking also into account the regulatory obligations for specific roll-out plans. Formation of mixed strategies as solutions means that unambiguous or simple strategies do not exist in order to reach an equilibrium. Of course, the reader might define other kinds of equilibrium condition than Nash-sense, e.g. managers may be interested in strategy being “the best” independent of other players’ strategy-selection, or what happens if all the other players work the ruin of the others. In the first case, “dominant strategy” can be reached while in the second case, operators would have in their hands “the safe strategy”; so both being a kind of equilibrium.

Falling back on the Nash-equilibrium points, and having mixed strategy solutions, the operators should play on a “statistical base”, and choose strategies relying on numerical possibilities. As this seems to be rather “unrealistic”, operators would likely follow a strategy of higher probability value.

Quite independently of the initial market share of the newcomer and the type of market, the newcomer has to have always smaller prices, as the market analyzer might feel that this is “natural”. In most of our results this offers quite small tariffs compared to the incumbent operator’s ones.

As the initial market share of newcomer increases, both competitors should decrease their prices, but regarding our inelastic total market models, incumbent always has a chance of having “the highest” price, being viable at the same time (quite positive NPV). This is true due to shape of the market share function that has been derived from the original data (Table 3) and due to the fact that the incumbent operator has larger initial market share.

It is interesting that with increasing sensibility of the market the solutions exist only with mixed strategies. This means that the mutual impact of competitors on each other, assumes that players have investigated what the other player does, deeply into statistical decision making “process”.

These ideas above were only initial ones, and the reader should feel free to draw deeper experiences.

5 Conclusions and further topics

As far as game theory is concerned the investigated model served mainly as a simple but good illustrative example to show the UMTS business situation with competition among operators. It contains quite some limitations, but it succeeded in illustrating how competitors might effect on each other. Incorporation of real options into the calculation would be possible in the technoeconomic tool in the latest version [15]. In this case, NPV should be

replaced by the adjusted NPV. The hypothetical market function remains to be proved by the new statistics including data from the western European Countries, or relying on real market movement that any other type of function might be built on. The independence of the total market from price level can be true only in a certain degree. Therefore, building a model with a price feedback to total market would be important. In addition, a case study of another scenario with a third player buying an unused license (like it happened in Sweden) and competing with two dominant actors or 3G auctions in emerging markets could be an interesting extension to this model.

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