Trading public parking space

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Abstract

Our paper investigates normative abstractions for the way drivers pursue parking space and respond to pricing policies about public and private parking facilities. The drivers are viewed as strategic agents who make rational decisions while attempting to minimize the cost of the acquired parking spots, under deterministic or probabilistic information for the overall parking demand. We propose auction-based systems for realizing centralized parking allocation schemes, whereby drivers bid for public parking space and a central authority coordinates the parking assignments and payments. These are compared against the conventional uncoordinated parking search practice under fixed parking service cost, formulated as a resource selection game instance. Inline with intuition, the auctioning system increases the revenue of the public parking operator exploiting the drivers' differentiated interest in parking. Less intuitively, the centralized mechanism does not necessarily induce higher cost for the drivers; instead, by eliminating the cruising cost, it emerges as the preferable option under various combinations of parking demand and pricing policies.

Index Terms

Parking games, auctions, social cost, revenue

I. Introduction

The high demand for parking space in city centers has always been a challenge in the process of city planning. The city authorities draw on both public and private parking facilities and more recently deploy parking assistance systems [1] [2], to respond to the parking needs of the car volumes that daily

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visit popular in-city destinations. Under the conventional parking search practice, drivers choose between the cheap but scarce on-street parking spots and the more expensive option of private parking space. In fact, drivers selfishly pursue to minimize the cost of access to parking facilities. However, the intuitive decision to head for the cheaper or free-of-cost on-street parking space, combined with the scarcity of public parking capacity in urban curbside of typical center areas, give rise to *tragedy of commons* effects and highlight the game-theoretic dynamics behind the parking spot selection problem.

In earlier work [3], we have formulated and studied the game that arises from the conventional parking search behavior under a fixed parking cost model. The drivers in search for parking space are viewed as rational strategic agents that choose either to compete for the cheaper but scarce on-street parking spots or head for the more expensive private parking lot. In the first case, they run the risk of failing to get a spot and having to *a posteriori* take the more expensive alternative, this time suffering the additional *cruising cost* in terms of time and fuel consumption. Drawing on real charging schemes, we have derived the equilibria strategies of the drivers and assessed their (in)efficiency via game-theoretic measures such as the social cost and Price of Anarchy. We summarize these results in Section II of this paper.

In this paper, we ask whether and how much can centralized parking assistance systems combined with more aggressive pricing schemes improve the outcome for both the on-street parking space operator *and* the drivers. More specifically, in Section III we propose different *auction* mechanisms for the assignment of on-street parking space. In fact, auction mechanisms have been used under various concepts in different disciplines. In network science, research efforts on node transactions devise auction-based schemes to address the challenge of resource (energy, bandwidth and storage space) sharing among multiple networking users [4]. Our paper approaches the process of parking space selection in urban environments as a network resource allocation problem. Indeed, the auctioning of parking spots is a promising key-idea that has only recently started to gain interest [5] [6]. The number of available auctioned spots is announced to the drivers, who submit their bids for them, expressing what are they willing to pay for a parking spot in that particular occasion with complete or probabilistic information for the overall parking needs. As central mechanisms, auctions determine who gets a parking spot and at what cost, saving the additional expenses of cruising in the non-assisted, uncoordinated parking search, while unleashing the conventional buying rules in public parking operation. Indeed, the analytical results in Section IV show that, as expected, auctions always raise the revenue of the public parking operator since they adapt payments to what drivers are willing to pay for on-street parking space. Nevertheless, this does not come necessarily at the expense of drivers, who save the cruising cost and find the auction-based system less expensive on average, under various combinations of parking demand and pricing policies.

II. THE PARKING SPOT SELECTION GAME

In the parking spot selection game, the set of players consists of drivers who circulate within a city area in search of parking space. The players have to decide whether to drive towards the scarce low-cost on-street parking spots or the more expensive private parking lot. All parking spots that lie in the same public or private area are assumed to be of the same value for the players. Thus, the decisions are made on the two *sets* of parking spots rather than individual set items. The two sets jointly suffice to serve all parking requests.

The collective decision making on parking space selection can be formulated as an instance of the strategic *resource selection games*, whereby N players (*i.e.*, drivers) compete against each other for a finite number of R on-street parking spots[1]. More specifically, drivers who decide to compete for the cheaper on-street public parking space undergo the risk of not being among the R winner-drivers to get a public spot. In this case, they have to eventually resort to private parking space, only after wasting extra time and fuel (plus patience supply) on the failed attempt. The expected cost of competing for public parking space, w_{pub}, is therefore a function of the number of competing drivers k, and is given by

$$w_{pub}(k) = min(1, R/k)c_{pub,s} + (1 - min(1, R/k))c_{pub,f} \tag{1}$$

where $c_{pub,s}$ is the fixed cost of successfully competing for public parking space, whereas $c_{pub,f} = \gamma \cdot c_{pub,s}, \gamma > 1$, is the cost of competing, failing, and eventually paying for private parking space.

On the other hand, the cost of private parking spots is fixed

$$w_{priv}(k) = c_{priv} = \beta \cdot c_{pub,s} \tag{2}$$

where $1 < \beta < \gamma$, namely the excess cost $\delta \cdot c_{pub,s}$, with $\delta = \gamma - \beta > 0$, reflects the cruising cost in terms of wasted time and fuel till eventually heading to the private parking space.

If σ_{pub} denotes the number of drivers that decide to compete for on-street parking space, then the aggregate cost paid by the total drivers' population (*social cost*) is given by

$$C(\sigma_{pub}) = \begin{cases} c_{pub,s}\left[N\beta - \sigma_{pub}(\beta - 1)\right], & \text{if } \sigma_{pub} \le R \\ c_{pub,s}\left[\sigma_{pub}\delta - R(\gamma - 1) + \beta N\right], & \text{if } R < \sigma_{pub} \le N \end{cases} \tag{3}$$

whereas the revenue for the public parking space operator by:

$$R(\sigma_{pub}) = \begin{cases} \sigma_{pub} c_{pub,s}, & \text{if } \sigma_{pub} \le R \\ R c_{pub,s}, & \text{if } R < \sigma_{pub} \le N \end{cases} \tag{4}$$

In [3], we have analyzed the parking spot selection game assuming both complete and probabilistic knowledge of the parking demand, *i.e.*, the number of drivers seeking for parking space, as well as

[1] The formal definition of the parking spot selection game is given in [3].

complete uncertainty about it. Following, we outline the main results.

A. Deterministic knowledge of competition

The main finding in [3] for the strategic parking spot selection game is that, for parking demand exceeding the supply ($N > R$), the number of competing drivers in the equilibria states $\sigma_{pub,eq} = \min(N, \sigma_0)$, with $\sigma_0 = \frac{R(\gamma-1)}{\delta}$, exceeds the optimal number R that would compete for and succeed in getting an on-street parking spot in the ideal scenario. In other words, an expected number of $\sigma_{pub,eq} - R$ ends up wasting time, fuel, and psychological resources on needless cruising without eventually saving the more expensive private parking fee. On the contrary, when $N \leq R$, all drivers head to the area of public parking.

The resulting social cost C_d in the game equilibria states amounts to

$$C_d \equiv C(N) = c_{pub,s}\left[N\gamma - \min(N,R)(\gamma-1)\right], \text{ if } N \leq \sigma_0 \text{ and } C_d \equiv C(\sigma_0) = c_{pub,s}\beta N, \text{ if } N > \sigma_0 \tag{5}$$

which, for $N > R$, exceeds the optimal cost value $C_{d,opt} \equiv C(R) = c_{pub,s}\left[R + \beta(N-R)\right]$, the ratio $C_d/C_{d,opt}$ expressing the price of anarchy of the game and quantifying the penalty of lack of coordination across the drivers. On the other hand, the revenue R_d for the public parking space operator becomes

$$R_d \equiv R(N) = \min(N,R)c_{pub,s}, \text{ if } N \leq \sigma_0 \text{ and } R_d \equiv R(\sigma_0) = Rc_{pub,s}, \text{ if } N > \sigma_0 \tag{6}$$

B. Probabilistic knowledge of competition

Instead of knowing exactly the number of competing vehicles, *i.e.*, a fairly strong and unrealistic assumption, the drivers may share common probabilistic knowledge about the overall parking demand, originated by multimode systems that advertise relevant information. Specifically, the assumption is that everyone has a rough knowledge of the average probability p_{act} with which each user is "active", in search for a parking spot. Therefore, while the overall population of vehicles is constant, N, the number of active vehicles, N_{act} varies stochastically. The prior probability distribution function $p_{(n,N)}$ of active vehicles is given by

$$p_{(n,N)} \doteq Pr(N_{act} = n) = B(n; N, p_{act}) = \binom{N}{n} p_{act}^n (1-p_{act})^{N-n} \tag{7}$$

Each vehicle that searches for a parking spot, maintains an estimate $p_{(n,N-1)}$ of the probability that there are n other competing vehicles. It then pursues to minimize the expected cost, given the cost functions (1) and (2), over all possible values of n in $[0, N-1]$[2]. The analysis on the equilibrium and optimal conditions results in conclusions similar to those in the strategic game, yet now we account for

[2]In [3], we investigate this particular game formulation of the problem under the term "*Bayesian* parking spot selection game".

the expected number of players Np_{act} instead of the deterministic knowledge of the total number of players in Section II-A.

III. Centralized parking search assistance

Parking assistance schemes are one way to overcome the efficiency limitations that result from the uncoordinated selfish behavior of drivers. These systems rely on wireless communication solutions for delegating the parking space assignment task to a central server, which: a) gets informed about the status of on-street parking spots; b) collects the requests and bids of drivers for parking space; and c) determines who is assigned a parking spot and at what cost, and notifies the drivers. In this paper, in particular, we propose and analyze an *auction-based* system for the management of the public parking space drawing on the theory of *multi-unit auctions with single-unit demand* [7].

In particular, N drivers (buyers) bid in a single auction for no more than one of R spare on-street parking spots (non-divisible, physically identical goods[3]). Drivers (bidders) are assumed to be symmetric: their valuations of parking spots are i.i.d RVs continuously distributed in the same interval $[v_{min}, v_{max}]$ and $F_V()$, $f_V()$ their cumulative and probability distribution functions, respectively. An intuitive choice for this interval is $[c_{pub,s}, c_{priv}]$. In other words, the operator of the public parking resources will typically impose a threshold on the selling price, *i.e.*, a reserve price, that will be no less than the on-street parking spot price under fixed cost. Drivers, in turn, will account for this lower bound in their bidding decisions; whether they will not be willing to pay more than what the private parking operator charges. Although each driver is aware of the distributions that her competitors' valuations follow, upon bidding, she can only know the realization of her own RV. Bidders are also assumed to be risk-neutral, *i.e.*, for valuations they seek to maximize their profit from bidding, and free of budget constraints [7].

In general, a selling auction mechanism consists of three components: the set of *bids* $\mathcal{B}_i$ (increasing functions of valuations) for each driver $i \in \mathcal{N}$; an *allocation rule* $\pi : \mathcal{B}_1 \times ... \times \mathcal{B}_N \rightarrow \Delta(\mathcal{N})$, where Δ is the set of probability distributions over $\mathcal{N}$ determining who are awarded parking spots, and a *payment rule* $p : \mathcal{B}_1 \times ... \times \mathcal{B}_N \rightarrow \mathbb{R}^N$ for the selling price of each allocated spot. Out of the variety of options, hereafter we consider the three most thoroughly analyzed implementations, the *uniform price*, *discriminatory price* and *Vickrey* auctions. All three auction formats are *standard* in that they assign the parking spots to the users that submit the highest bids. Under single-item demand and symmetric risk-neutral bidders, all three auctions are also *efficient* and assign the parking spots to the users that

[3]In [3] we also discuss the case that different parking spots bear different values for the competing drivers.

value them most[4]. In other words, they induce equilibria states, whereby the top-bids are submitted by the drivers that value the parking spots most. On the other hand, whereas all three auctioning mechanisms follow the same allocation rule, they differ in the payment rule they apply.

- Under the *Uniform Price Auction* (UPA) and the *Vickrey Auction* (VA), all parking spots are sold at the same price, the "market-clearing price", which is equal to the first losing bid, *i.e.*, the $(R+1)^{th}$ highest over all drivers' bids.
- Under the *Discriminatory Price Auction* (DPA), the winning drivers pay an amount equal to their individual bids.

A. Deterministic knowledge of competition

The assumption in this Section is that the drivers are aware of how many they compete against; for instance, because the parking assistance system provides them with this information. We first define the equilibrium bidding strategies and then discuss their efficiency from bidders' and operator's interim perspective, given that the auctioned parking spots do not suffice to fulfil the entire parking demand. Otherwise, it is trivial to show that the centralized auction's and the distributed practice's outcomes coincide.

1) Uniform price and Vickrey auction: Both the single-demand UPA and VA mechanism come under the broader category of incentive-compatible (truthful) mechanisms in that the equilibrium strategy, $\beta(v)$ for the drivers is to bid their real values v,

$$\beta_{UPA}(v) = \beta_{VA}(v) = v \tag{8}$$

For $N > R$, the expected driver's payment, conditional on its value v, is given by

$$\begin{aligned} p_{UPA}(v) = p_{VA}(v) &= Pr(V_{(N-R,N-1)} < v) \cdot E[V_{(N-R,N-1)} | V_{(N-R,N-1)} < v] \\ &= \int_{v_{min}}^{v} y f_{V_{(N-R,N-1)}}(y) dy \end{aligned} \tag{9}$$

where $V_{(k,n)}$ is the k^{th} order statistic of the n competing valuations (*i.e.*, the k^{th} smallest out of n samples drawn from RVs $V_1, ..., V_n$) with probability density function $f_{V_{(k,n)}}(y) = \{B(k, n-k+1)\}^{-1}\{F(y)\}^{k-1}\{1-F(y)\}^{n-k} f_V(y)$, where $B(\cdot,\cdot)$ stands for the complete Beta function [8].

[4] In general, reserve prices introduce a positive probability that the auctioned object remains unsold and cause the efficiency property of the mechanism. Herein, however, this event is excluded, since drivers' bids range in $[v_{min}, v_{max}]$.

Therefore, its unconditional (*ex ante*) expected payment can be written

$$\begin{aligned} p_{UPA} = p_{VA} &= \int_{v_{min}}^{v_{max}} p_{UPA}(v) f_V(v) dv \\ &= \frac{R}{N} E[V_{(N-R,N)}] \end{aligned} \tag{10}$$

while the *expected* revenue of the public parking service provider becomes

$$\begin{aligned} R_c = E[R_{UPA}] = E[R_{VA}] &= N p_{VA} \\ &= R E[V_{(N-R,N)}] \end{aligned} \tag{11}$$

and is collected from the drivers with the top R bids.

On the other hand, drivers with the $N - R$ lowest bids resort to private parking facilities, all paying the fixed cost $c_{priv} = v_{max}$. Thus, the *expected* social cost turns out to be

$$C_c = E[C_{UPA}] = E[C_{VA}] = R E[V_{(N-R,N)}] + (N - R) v_{max} \tag{12}$$

For $N \leq R$, it is trivial to show that,

$$p_{UPA} = p_{VA} = v_{min}$$

$$R_c = E[R_{UPA}] = E[R_{VA}] = N v_{min} \tag{13}$$

$$C_c = E[C_{UPA}] = E[C_{VA}] = N v_{min} \tag{14}$$

2) Discriminatory price auction: The Discriminatory Price auction mechanism is the multi-item counterpart of the single-demand First Price auctions. Vickrey, already in [9], showed that the expected revenue for all multi-unit auctions with single unit demand featuring the same allocation rule, is the same, a demonstration of the *revenue equivalence principle*. Therefore,

$$p_{DPA}(v) = p_{UPA}(v) = p_{VA}(v),$$

$$R_c = E[R_{DPA}] \text{ and } C_c = E[C_{DPA}] \tag{15}$$

For $N > R$, the equilibrium bidding strategy equals

$$\begin{aligned} \beta_{DPA}(v) &= E[V_{(N-R,N-1)} | V_{(N-R,N-1)} < v] \\ &= \frac{1}{F_{V_{(N-R,N-1)}}(v)} \int_{v_{min}}^{v} y \cdot f_{V_{(N-R,N-1)}}(y) dy \end{aligned} \tag{16}$$

Otherwise,

$$\beta_{DPA}(v) = v_{min} \tag{17}$$

B. Probabilistic knowledge of competition

In this Section we relax the assumption about complete awareness of the number of bidders and discuss how the uncertainty for the competition level affects the equilibrium bidding strategies and the corresponding payments.

1) Uniform price and Vickrey auction: The equilibrium strategy under UPA and VA remains the same, irrespective of drivers' uncertainty on competition level; each user bids her true value [10],

$$\beta_{UPA,B}(v) = \beta_{VA,B}(v) = v \tag{18}$$

Given the function (7), the expected payment per user, conditioned on the value v, is

$$\begin{aligned} p_{UPA,B}(v) = p_{VA,B}(v) &= v_{min}\sum_{n=0}^{R-1} p_{(n,N-1)} + \sum_{n=R}^{N-1} p_{(n,N-1)} Pr(V_{(n-R+1,n)} < v) \cdot E[V_{(n-R+1,n)} | V_{(n-R+1,n)} < v] \\ &= v_{min}\sum_{n=0}^{R-1} p_{(n,N-1)} + \sum_{n=R}^{N-1} p_{(n,N-1)} \int_{v_{min}}^{v} y \cdot f_{V_{(n-R+1,n)}}(y) dy \end{aligned} \tag{19}$$

Notice that under low parking demand (*i.e.*, $N \leq R$, for fixed N), every driver that is deemed winning pays the reserved price v_{min}, irrespective of her valuation/bid. Likewise, the expected per user payment, revenue and social cost become

$$\begin{aligned} p_{UPA,B} = p_{VA,B} &= v_{min}\sum_{n=0}^{R-1} p_{(n,N-1)} + \sum_{n=R}^{N-1} p_{(n,N-1)} \frac{R}{n+1} E[V_{(n-R+1,n+1)}], \\ R_{c,B} = E[R_{UPA,B}] = E[R_{VA,B}] &= v_{min}\sum_{n=0}^{R} p_{(n,N)} n + R\sum_{n=R+1}^{N} p_{(n,N)} E[V_{(n-R,n)}] \text{ and} \end{aligned} \tag{20}$$

$$C_{c,B} = E[C_{UPA,B}] = E[C_{VA,B}] = R_{c,B} + v_{max}\sum_{n=R+1}^{N} p_{(n,N)}(n-R) \tag{21}$$

2) Discriminatory price auction: By (16) and (17), for fixed N, the equilibrium bid equals the first losing bid if $N > R$, or the minimum value v_{min}, otherwise. Thus, the revenue equivalence principle remains valid for variable number of bidders as well. Therefore, as with fixed N, it holds that

$$R_{c,B} = E[R_{DPA,B}] \text{ and } C_{c,B} = E[C_{DPA,B}] \tag{22}$$

Furthermore, since the expected per user payments are equal, we get

$$p_{UPA,B}(v) = p_{VA,B}(v) = p_{DPA,B}(v) = \beta_{DPA,B}(v) \left[\sum_{n=0}^{R-1} p_{(n,N-1)} + \sum_{n=R}^{N-1} p_{(n,N-1)} Pr(V_{(n-R+1,n)} < v) \right] \quad (23)$$

Hence, replacing the left-hand side with (19) we get $\beta_{DPA,B}(v)$.

IV. Numerical results

In Sections II and III we have outlined the game formulations of the two main practices in managing public (on-street) parking space and derived the equilibria behaviors they induce. Under conventional uncoordinated search for on-street parking, drivers have the chance to pay a lower parking fee when they succeed in getting a public on-street parking spot. However, they run the risk of paying a normalized per-hour cruising cost $\delta c_{pub,s}$, on top of the private parking fee, when they eventually drive to private parking lot, after having competed without success to get an on-street parking spot. On the other hand, the centralized auctioning of on-street parking spaces exploits the, generally variable, user need for parking space and allows for higher payments for on-street parking space but saves the "price of anarchy", paid in the absence of central coordination in the first case. In this Section, we explore how different pricing schemes and the users' interest in parking (as captured in their valuations' distributions) affect a) the revenue achievable by the public parking service operator; and b) the resulting per-user expected cost of the parking service, under the two radically different paradigms of parking space management.

For the pricing policy, we adopt values used in the municipal parking system in the city of Athens [11]. In particular, $c_{pub,s} \leq 2$ €, and $\beta \leq 7$, for 60-minute period. The cruising cost parameter δ is let range in $(0, 10]$. On the other hand, we consider three alternatives for the distributions of the drivers' valuations, $f_V(v)$. In all three of them, V lies within an interval $[v_{min}, v_{max}] = [c_{pub,s}, \beta c_{pub,s}]$, yet the mass of the distribution is spread differently over this interval (see Fig. 1):

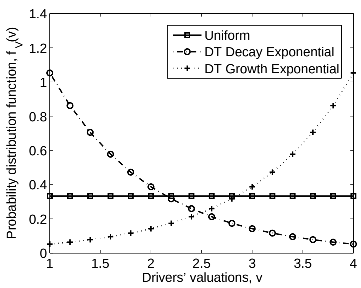
Doubly-truncated decay exponential valuations: This instance of valuation function corresponds to scenaria, whereby drivers are not willing to pay high for a parking spot. It could model driving in the center during leisure hours, where the retrieval of a parking spot is less urgent. The moments of the $(N-R)^{th}$-order statistics can be computed numerically through the recurrence relations derived by Joshi in [12].

Doubly-truncated growth exponential valuations: The mass in this valuation distribution is concentrated towards the rightmost values of its support. Compared with the doubly-truncated decay exponential distribution, this one can model driving in the city center during busy hours for business purposes.

Uniform valuations: This is the intermediate scenario, where the value of parking spots for individual users may lie anywhere in $[v_{min}, v_{max}]$ equiprobably. In this case, the expected value of the $(N-R)^{th}$-order statistic can be also computed through the mean value of the generalized Beta distribution $f(v; N-R, R+1)$, for $v \in [v_{min}, v_{max}]$, that is,

$$E[X_{N-R,N}] = v_{min} + \frac{N-R}{N+1}(v_{max} - v_{min}) \tag{24}$$

We consider medium to high parking demand levels (up to 160 drivers) and limited public parking supply ($R = 20$ spots) during the time window over which the parking requests are issued.



Distribution of drivers' valuations	$f_V(v)$
Uniform	$\frac{1}{v_{max}-v_{min}}$
Doubly-truncated *decay* exponential	$\frac{e^{-v}}{e^{-v_{min}}-e^{-v_{max}}}$
Doubly-truncated *growth* exponential	$\frac{e^{v}}{e^{v_{max}}-e^{v_{min}}}$

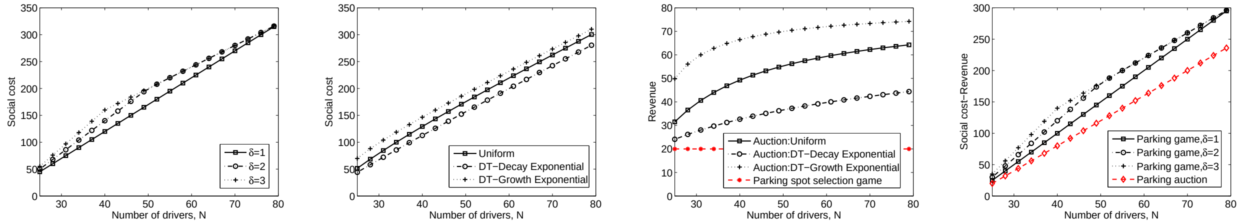
Fig. 1. Probability density functions for drivers' valuations of public parking spots, $c_{pub,s} = 1$, $\beta = 4$.

A. Comparing the revenue achievable by the public parking service operator under the two parking space management practices

Figure 2 analyzes the aggregate social cost over all drivers in the distributed (Fig. 2a) and centralized system (Fig. 2b), into the revenue achievable by the public parking operator (Fig. 2c) and the total cost paid by the drivers that end up in private parking facilities (Fig. 2d), against different parking demand levels and fixed charging for private space.

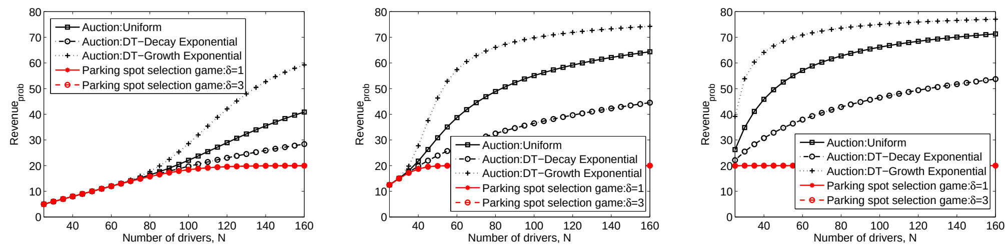
Inline with intuition, the social cost increases with parking demand, irrespective of the applied parking allocation system (see Fig. 2a, b). Yet, in the absence of central coordination, the total cost paid by the drivers includes the actual cost of cruising in search of available on-street space[5]. Under the auction-based system, the incurred social cost under the three valuation distributions is strictly ordered, as we prove in Appendix and plot in Figure 2b. Furthermore, all bidders that are not awarded public parking spots, end up paying the same fixed cost for private parking allocation (see Fig. 2d), irrespective of their valuations. Thus, the revenue follows the ordering of social cost for the three cases, as Figure 2c suggests, as well. As expected, the revenue from auctioning the public parking spots exceeds the corresponding

[5]The impact of different charging practices on the social cost incurred by drivers in the parking spot selection game, is discussed in [3].



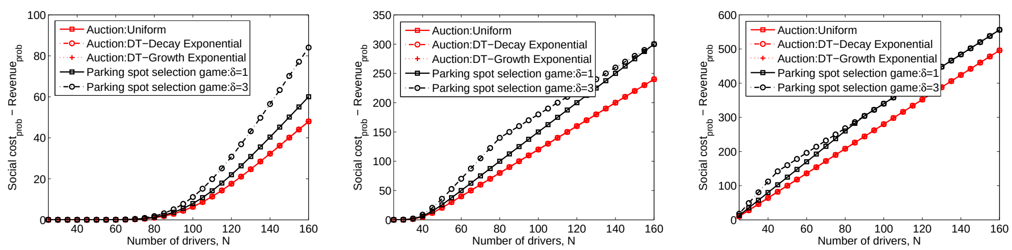
a. b. c. d.

Fig. 2. Decomposition of the social cost in the distributed (a, c, d) and the centralized system (b, c, d), under different pricing schemes with $c_{pub,s} = 1, \beta = 4, \delta \in \{1, 2, 3\}$, and various probability distributions for drivers' valuations.



a. $p_{act} = 0.2$ b. $p_{act} = 0.5$ c. $p_{act} = 0.9$

Fig. 3. Revenue accruing to the operator of the public parking space with $c_{pub,s} = 1, \beta = 4, \delta \in \{1, 3\}$, under different scenaria with respect to the users' interest in parking as well as users' awareness of the overall parking demand.



a. $p_{act} = 0.2$ b. $p_{act} = 0.5$ c. $p_{act} = 0.9$

Fig. 4. Cost incurred by drivers who end up in private parking space, under $c_{pub,s} = 1, \beta = 4, \delta \in \{1, 3\}$, and different scenaria with respect to the users' interest in parking as well as users' awareness of the overall parking demand.

gains under fixed parking service cost. Overall, despite the similar social cost results for both practices, in the distributed scheme the excess cost (in terms of fuel and time) due to the lack of coordination is wasted on cruising, whereas in the centralized system the excess cost from bidding over the fixed minimum cost $c_{pub,s}$, is collected by the operator.

By Figures 2, 3 and 4, scenaria with different probabilistic knowledge of the parking demand (*i.e.*, binomial distribution with different p_{act} parameter), are equivalent to settings whereby Np_{act} drivers complete for parking space, all possessing deterministic knowledge of the competition. Notice that when the expected number of drivers is less than the public parking supply the two practices for parking space management coincide. Overall, drivers' (seller's) preference over the two approaches is indifferent between availing (revealing) the first moment of the probability distribution of active vehicles or the probability distribution per se.

B. Comparing drivers' welfare under the two parking space management practices

The drivers' welfare is quantified by the expected cost they end up paying for parking space. Hence, their preference for the distributed or centralized parking assignment practices is determined by the difference

$$\Delta = \frac{1}{N}(C_d - C_c) \tag{25}$$

between the expected per driver cost C_d/N under the distributed parking spot assignment and its counterpart C_c/N under the centralized allocation. For $\Delta > 0$ ($\Delta < 0$), this difference expresses the *excess* cost that drivers pay in the conventional (auction-based) compared to the auction-based (conventional) parking assignment paradigm. Drivers are indifferent over the two paradigms for $\Delta = 0$. Thus, by (5) and (25), for $N \leq \sigma_0$, the two options considered equivalent when

$$\Delta = \frac{1}{N}\left[c_{pub,s}[N\gamma - R(\gamma - 1)] - C_c\right] \quad = \quad 0 \tag{26}$$

or for $\delta \leq \frac{R(\beta-1)}{N-R}$,

$$\delta \quad = \quad \frac{1}{N-R}\left[\frac{C_c}{c_{pub,s}} + R(\beta - 1) - N\beta\right] \tag{27}$$

Therefore, the tie is possible as the cruising cost decreases with parking demand (see Fig. 5d). Additionally, the higher the social cost the auctioning process induces, the more the cruising between the area of public and private parking should cost to counterbalance the higher payments of drivers under the auction-based system. Indeed, as we show in the Appendix, the valuation distribution induces the ordering

$$C_c^g \geq C_c^u \geq C_c^d \tag{28}$$

where C_c^g, C_c^u, C_c^d stand for the resulting social cost under growth exponential, uniform and decay exponential valuations, respectively. This causal relation between valuations and cruising cost parameter is clearly seen in Figure 5d.

On the contrary, when $N > \sigma_0$, from (5) and (25) we have that

$$\begin{aligned}
\Delta &= \frac{1}{N}\left[c_{pub,s}N\beta - C_c\right] \qquad (29)\\
&= \frac{1}{N}\left[c_{pub,s}N\beta - [R_c + (N-R)\beta c_{pub,s}]\right]\\
&= \frac{R}{N}\left[\beta c_{pub,s} - E[V_{N-R,N}]\right] > 0
\end{aligned}$$

since the per-spot expected payment $E[V_{N-R,N}]$ is strictly smaller than the cost of private parking space. Therefore, for $N > \sigma_0$, drivers are always better off with the centralized auctioning process.

In what follows, we iterate on the impact of the number of drivers, N, and the cost parameters $c_{pub,s}$ (public parking cost), β (private parking cost) and δ (cruising cost), on the expected per user cost taking as reference the most randomized, *i.e.*, uniformly distributed, valuations. In the Appendix we show that

$$\Delta_u(N,\beta,\delta;R) = \begin{cases} c_{pub,s}\frac{(N-R)}{N}\left[\delta-(\beta-1)\frac{R}{N+1}\right], & \text{if } N \le \sigma_0 \\ c_{pub,s}(\beta-1)\frac{R(R+1)}{N(N+1)}, & \text{if } N > \sigma_0 \end{cases} \tag{30}$$

Impact of number of drivers: For given public parking supply and charging parameters, if $N > \sigma_0$, drivers always prefer the centralized system (*i.e.*, $\Delta_u > 0$). However, the difference Δ_u is strictly decreasing with N since

$$\frac{\vartheta \Delta_u}{\vartheta N} = -c_{pub,s}(\beta-1)\frac{R(R+1)(2N+1)}{[N(N+1)]^2} < 0 \tag{31}$$

Hence, as Figure 5 shows, drivers become indifferent between the two alternatives under high competition, irrespective of the applied charging scheme.

On the contrary, under lower parking demand ($N \le \sigma_0$), no scheme dominates over the other. Drivers end up paying less on average under the auction-based scheme if $\delta > \frac{R(\beta-1)}{N+1}$; otherwise, it is the conventional non-assisted parking search practice that becomes favorable. In particular, for all realistic values of the cruising cost δ ($\delta < R(\beta-1)$), Δ_u obtains a minimum at

$$N_0 = \frac{\tau + \sqrt{\tau(\beta-1)(R+1)}}{\beta-1+\delta}, \tag{32}$$

where $\tau = R(\beta-1)-\delta$, which increases with β and decreases with δ since

$$\frac{\vartheta N_0}{\vartheta \beta} = \frac{\delta(R+1)+\sqrt{\tau(\beta-1)(R+1)}}{(\beta-1+\delta)^2}\left[\frac{1}{2}\frac{[2R(\beta-1)-\delta](\beta-1+\delta)}{\tau(\beta-1)} - 1\right] > 0 \tag{33}$$

and

$$\frac{\vartheta N_0}{\vartheta \delta} = \frac{(\beta-1)(-R-1)-\sqrt{\tau(\beta-1)(R+1)}}{(\beta-1+\delta)^2}\left[1+\frac{1}{2}\frac{\beta-1+\delta}{\tau}\right] < 0 \tag{34}$$

On a last note, the analysis of the convexity of Δ_u function with N, expressed via

$$\frac{\vartheta^2 \Delta_u}{\vartheta N^2} = \begin{cases} c_{pub,s}\frac{2R}{N^3}\left[\frac{-(\beta-1)}{(N+1)^3}\left[N^3-R-3RN(N+1)\right]-\delta\right], & \text{if } N \le \sigma_0 \\ c_{pub,s}\frac{2R(\beta-1)}{[N(N+1)]^3}[1+3N(N+1)] > 0, & \text{if } N > \sigma_0 \end{cases} \tag{35}$$

shows that although it starts convex, turns concave, and finishes convex, as Figure 5 illustrates, as well.

Impact of cruising, public and private parking costs: Firstly, as Figure 5 and (30) suggest, the shape of Δ_u function is primarily determined by the relation between the number of drivers N and the number $\sigma_0 = \frac{R(\gamma-1)}{\delta}$. Indeed, the turning point at $N = \sigma_0$ is shifted to the left as (a) the public parking capacity decreases; or (b) the cruising cost increases; or (c) the cost of private parking space drops.

For given public parking demand and supply, the centralized auctioning system presents a cheaper alternative to the drivers as a) the distance between public and private parking facilities grows and/or the fuel prices increase, thus inflating the cruising cost; or b) the private parking cost gets higher, thus motivating more drivers to compete for the scare on-street parking space and increasing the "price of anarchy" of the uncoordinated parking practice, under high parking demand. However, under medium parking demand, any increase in private cost, raises the payments in the auction system at the expense of drivers' welfare and hence, reduces the advantage of saving the cruising cost (see Fig. 5a, c). Finally, when the public parking operator increases the public cost, he allows for higher variation in the difference between drivers' welfare under the two systems, as Figure 5b illustrates, as well. Analytically, Δ increases as the cruising cost parameter increases, while the distance between public and private parking area remains close,

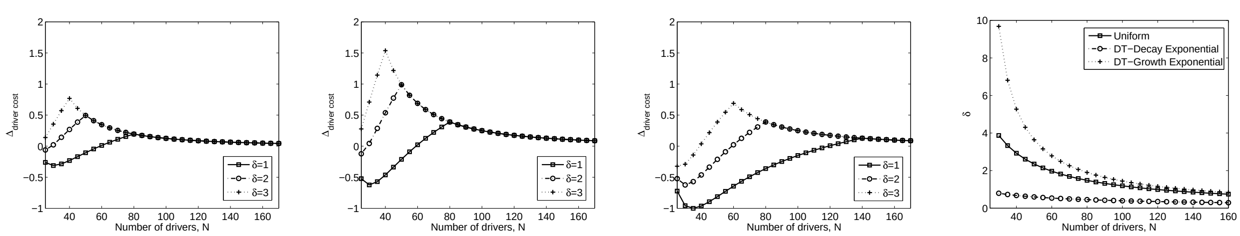
$$\frac{\vartheta \Delta_u}{\vartheta \delta} = \begin{cases} c_{pub,s}\frac{(N-R)}{N} > 0, & \text{if } \delta \le \frac{R(\beta-1)}{N-R} \\ 0, & \text{if } \delta > \frac{R(\beta-1)}{N-R} \end{cases} \tag{36}$$

or the private cost increases (drops), while remains comparable to (much more expensive than) its public counterpart,

$$\frac{\vartheta \Delta_u}{\vartheta \beta} \begin{cases} c_{pub,s}\frac{-R(N-R)}{N(N+1)} < 0, & \text{if } \beta \ge 1 + \frac{\delta(N-R)}{R} \\ c_{pub,s}\frac{R(R+1)}{N(N+1)} > 0, & \text{if } \beta < 1 + \frac{\delta(N-R)}{R} \end{cases} \tag{37}$$

or the public parking gets more expensive (cheaper), while the distance between public and private parking is significant (close),

$$\frac{\vartheta \Delta_u}{\vartheta c_{pub,s}} \begin{cases} \frac{(N-R)}{N}\left[\delta - (\beta-1)\frac{R}{N+1}\right] \ge 0, & \text{if } \frac{R(\beta-1)}{N+1} \le \delta \le \frac{R(\beta-1)}{N-R} \\ \frac{(\beta-1)R(R+1)}{N(N+1)} > 0, & \text{if } \delta > \frac{R(\beta-1)}{N-R} \\ < 0, & \text{otherwise} \end{cases} \tag{38}$$



a. $\beta = 4, c_{pub,s} = 1$ b. $\beta = 4, c_{pub,s} = 2$ c. $\beta = 7, c_{pub,s} = 1$ d. $\beta = 7, c_{pub,s} = 1$

Fig. 5. Difference between the expected cost per driver in the parking game and that in the auction with uniform valuations, under various pricing schemes (a,b,c) and different charging policies that zero the excess cost Δ (d).

V. Conclusions

In this paper, we analytically and systematically explore the dynamics behind different game formulations of the parking spot allocation problem. Equilibrium behaviors under various auction settings, differing in their pricing rules and the levels of uncertainty bidders experience about the parking demand, are compared against those under the uncoordinated distributed parking spot assignment scheme, with respect to the cost that incurs to the users and the revenue accruing to the operator of the public parking space. Our analytical results show that, when the public parking service provider seeks to maximize his revenue, profits from auctioning the public parking resources, exploiting drivers' interest in on-street parking. Less intuitively, we locate specific contexts (*i.e.*, charging policy and competition intensity) in which drivers serve their self-interest also under the centralized market system.

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Appendix

A. Difference Δ of the expected per driver costs under the two parking space management practices

By (5), (12) and (24), with $[v_{min}, v_{max}] = [c_{pub,s}, \beta c_{pub,s}]$, for $N \leq \sigma_0$, the excess cost is

$$\begin{aligned}
\Delta_u &= \frac{1}{N}(C_d - C_{c,u}) \\
&= \frac{c_{pub,s}}{N}[N\gamma - R(\gamma - 1)] - \frac{c_{pub,s}}{N}\left[R + (\beta - 1)\frac{R(N-R)}{N+1} + \beta(N-R)\right] \\
&= \frac{c_{pub,s}}{N}\left[N\gamma - R\gamma - (\beta - 1)\frac{R(N-R)}{N+1} - N\beta + R\beta\right] \\
&= c_{pub,s}\frac{(N-R)}{N}\left[\delta - (\beta - 1)\frac{R}{N+1}\right]
\end{aligned} \tag{39}$$

while for $N > \sigma_0$,

$$\begin{aligned}
\Delta_u &= \frac{1}{N}(C_d - C_{c,u}) \\
&= \frac{c_{pub,s}}{N}N\beta - \frac{c_{pub,s}}{N}\left[R + (\beta - 1)\frac{R(N-R)}{N+1} + \beta(N-R)\right] \\
&= c_{pub,s}(\beta - 1)\frac{R(R+1)}{N(N+1)}
\end{aligned} \tag{40}$$

B. Expected payments and revenue under the three valuation functions

Under the auction-based parking space allocation mechanism, there is a strict ordering of the drivers' expected payments (hence, the revenues of the public parking space operators as well) under the three valuation functions.

$$C_c^g \geq C_c^u \geq C_c^d \tag{41}$$

Equivalently, we want to derive a similar relationship for the $(N-R)^{th}$ order statistics of the three evaluation functions.

The proof proceeds in three steps. Firstly, we note that there are first-order stochastic dominance relationships between the three cumulative distribution functions in Figure 1, that is

$$F_V^g(v) \prec F_V^u(v) \prec F_V^d(v) \tag{42}$$

as can be readily seen in Figure 6.

Then, we need to recall that the cumulative distribution function of the $(N-R)^{th}$ order statistic is written [8]

$$F_{(N-R,N)}(x) = \int_0^{F(x)} \frac{N!}{R!(N-R-1)!} t^{N-R-1}(1-t)^R dt \tag{43}$$

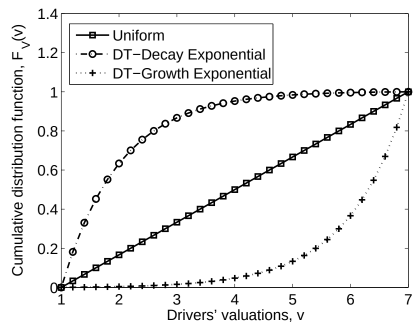

Fig. 6. Stochasting ordering of the three valuation functions $F_V(v)$, ($v_{min} = 1$, $v_{max} = 7$).

Therefore, the first-order dominance relationships in the drivers' valuations (42) is inherited by their $(N-R)^{th}$ order statistics.

$$F^g_{(N-R,N)}(x) \prec F^u_{(N-R,N)}(x) \prec F^d_{(N-R,N)}(x) \tag{44}$$

Finally, (41) emerges directly when relating the expected values of the valuations to their cumulative distribution functions through

$$E[X_{(N-R,N)}] = \int_0^\infty [1 - F_{(N-R,N)}(x)]dx \tag{45}$$

a general relation concerning nonnegative RVs [13].