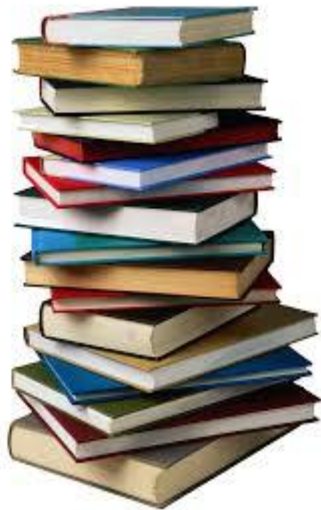
Stacks

Manolis Koubarakis

Stacks and Queues

- **Linear data structures** are collections of components arranged in a straight line.
- If we restrict the growth of a linear data structure so that new components can be added and removed only at one end, we have a **stack.**
- If new components can be added at one end but removal of components must take place at the opposite end, we have a **queue**.

Examples of Stacks in Real Life



Stacks in Computer Science

- Stacks are used in many areas of Computer Science:
  - Parsing algorithms
  - Pushdown automata
  - Expression evaluation algorithms
  - Backtracking algorithms
  - Activation records in run-time stack.

Stacks

- Stacks are sometimes called **LIFO lists** where LIFO stands for “last-in, first-out”.
- When we add a new object to the top of a stack, this is called “**pushing**”.
- When we remove an object from the top of a stack, this is called “**popping**”.
- Pushing and popping are inverse operations.

Sequences

- A **finite-length sequence** $S=(s_1, s_2, \ldots, s_n)$ is just an ordered arrangement of finitely many components $s_1, s_2, \ldots, s_n$.
- The **length** of a sequence is the number of its components.
- There is a special sequence with length *0* called the **empty sequence**.

An Abstract Data Type for Stacks

- A **stack** *S* of items of type *T* is a sequence of items of type *T* on which the following operations can be defined:
  1. Initialize the stack *S* to be the **empty stack**.
  2. Determine whether or not the stack *S* is **empty**.
  3. Determine whether or not the stack *S* is **full**.
  4. **Push** a new item onto the top of stack *S*.
  5. If *S* is nonempty, **pop** an item from the top of stack *S*.

An Interface for Stacks

- Using **separately compiled C files**, we can define **C modules** that specify the underlying representation for stacks and implement the abstract stack operations.

The Stack ADT Interface

```
/* This is the file StackInterface.h
*/
#include “StackTypes.h”

void InitializeStack(Stack *S);
int Empty(Stack *S);
int Full(Stack *S);
void Push(ItemType X, Stack *S);
void Pop(Stack *S, ItemType *X);
```

Using the Stack ADT to Check for Balanced Parentheses

- The first application of the Stack ADT that we will study involves determining whether parentheses and brackets balance properly in algebraic expressions.

- Example:

$$\{a^2 - [(b+c)^2 - (d+e)^2] * [\sin(x-y)]\} - \cos(x+y)$$

- This expression contains parentheses, square brackets, and braces in balanced pairs according to the pattern

$$\{[()()][()]\}()$$

The Algorithm

- We can start with an **empty stack** and scan a string representing the algebraic expression from left to right.
- Whenever we encounter a left parenthesis (, a left bracket [or a left brace {, we **push** it onto the stack.
- Whenever we encounter a right parenthesis), a right bracket] or a right brace }, we **pop** the top item off the stack and check to see that its type matches the type of right parenthesis, bracket or brace encountered.
- If the stack is **empty** by the time we get to the end of the expression string and if all pairs of matched parentheses were of the same type, the expression has properly balanced parentheses. Otherwise, the parentheses are not balanced properly.

The Program

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>

char *InputExpression;

int Match(char c, char d)
{
   switch (c){
       case '('   : return d==')';
          break;
       case '['   : return d==']';
          break;
       case '{'   : return d=='}';
          break;
       default    : return(0);
          break;
   }
}
```

The Program (cont'd)

```
void ParenMatch(void)
{
    int  n, i=0;
    char c, d;
    Stack ParenStack;

    InitializeStack(&ParenStack);

    n=strlen(InputExpression);
    while (i < n){
       d=InputExpression[i];
       if ( d==‘(‘ || d==‘[‘ || d==‘{‘ ){
          Push(d, &ParenStack);
       } else if ( d==‘)‘ || d==‘]‘ || d==‘}‘ ){
          if (Empty(&ParenStack)){
             printf(“More right parentheses than left parentheses\n”);
             return;
          } else {
             Pop(&ParenStack, &c);
             if (!Match(c,d)){
                printf(“Mismatched Parentheses: %c and %c\n”, c, d);
                return;
             }
          }
       }
       ++i;
    }
    if (Empty(&ParenStack)){
       printf(“Parentheses are balanced properly\n”);
    } else {
       printf(“More left parentheses than right parentheses\n”);
    }
}
```

The Program (cont’d)

```
int main(void)
{
      InputExpression=(char *)malloc(100);
      printf(“Give Input Expression without
blanks:”);
      scanf(“%s”, InputExpression);
      ParenMatch();

      return 0;
}
```

Using the Stack ADT to Evaluate Postfix Expressions

- Expressions are usually written in **infix** notation e.g., *(a+b)*2-c*. Parentheses are used to denote the order of operation.
- **Postfix** expressions are used to specify algebraic operations using a parentheses free notation. For example, *ab+2*c-.*
- The postfix notation *L R op* corresponds to the infix notation *L op R*.

Examples

Infix	Postfix
(a + b)	a b +
(x – y – z)	x y – z –
(x – y – z)/(u + v)	x y – z – u v + /
$(a^2 + b^2)$ * (m – n)	a 2 ^ b 2 ^ + m n – *

Prefix Notation

- There is also **prefix (or Polish) notation** in which the operator precedes the operands.
- **Example**: `+ 3 * 2 5` is the prefix form of `(2 * 5) + 3`

- Prefix and postfix notations **do not need parentheses** for denoting the order of operations.

The Algorithm

- To evaluate a postfix expression *P*, you **scan from left to right**.
- When you encounter an operand *X*, you **push** it onto an evaluation stack *S*.
- When you encounter an operator *op*, you **pop** the topmost operand stacked on *S* into a variable *R* (which denotes the right operand), then you **pop** another topmost operand stacked on *S* onto a variable *L* (which denotes the left operand).
- Finally, you **perform the operation** *op* on *L* and *R*, getting the value of the expression *L op R*, and you **push** the value back onto the stack *S*.
- When you finish scanning *P*, the value of *P* is the only item remaining on the stack *S*.

The Program

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <ctype.h>
#include <string.h>
#include “StackInterface.h”

Stack EvalStack;
char PostfixString[20];

void InterpretPostfix(void)
{
  float LeftOperand, RightOperand, Result;
  int i;
  char c;
  char s[]=‘x‘;

  InitializeStack(&EvalStack);
```

The Program (cont’d)

```
  for (i=0; i<strlen(PostfixString); ++i){
     s[0]=c=PostfixString[i];
     if (isdigit(c)){
       Push((float)(atof(s)), &EvalStack);
     } else if (c=='+' || c=='-' || c=='*' || c=='/' || c=='^'){
          Pop(&EvalStack, &RightOperand);
          Pop(&EvalStack, &LeftOperand);

          switch (c) {
             case '+': Push(LeftOperand+RightOperand, &EvalStack);
                       break;
             case '-': Push(LeftOperand-RightOperand, &EvalStack);
                       break;
             case '*': Push(LeftOperand*RightOperand, &EvalStack);
                       break;
             case '/': Push(LeftOperand/RightOperand, &EvalStack);
                       break;
             case '^': Push(pow(LeftOperand, RightOperand), &EvalStack);
                       break;
             default: break;
          }
      }
  }

  Pop(&EvalStack, &Result);
  printf("Value of postfix expression = %f\n", Result);
}
```

The Program (cont'd)

```
int main(void) {

     printf("Give input postfix string without
blanks:");
     scanf("%s", PostfixString);
     InterpretPostfix();

     return 0;
}
```

Implementing the Stack ADT

- We will present two implementations of the stack ADT based on:
  - arrays (sequential representation)
  - linked lists (linked representation)

- Both implementations can be used to realize the two applications we presented earlier.

The Implementation Based on Arrays

```
/* This is the file StackTypes.h   */

#define MAXSTACKSIZE 100

typedef char ItemType;
/* char is the type for our first application */
/* float is the type for our second application */

typedef struct{
           int Count;
           ItemType Items[MAXSTACKSIZE];
        } Stack;
```

The Implementation Based on Arrays (cont’d)

```
/* This is the file StackImplementation.c */

#include <stdio.h>
#include <stdlib.h>
#include "StackInterface.h"

void InitializeStack(Stack *S)
{
   S->Count=0;
}

int Empty(Stack *S)
{
   return (S->Count == 0);
}
```

The Implementation Based on Arrays (cont’d)

```
int Full(Stack *S){
   return(S->Count == MAXSTACKSIZE);
}

void Pop(Stack *S, ItemType *X)
{  if (S->Count ==0){
      printf(“attempt to pop the empty stack”);
   } else {
      --(S->Count);
      *X=S->Items[S->Count];
   }
}
```

The Implementation Based on Arrays (cont’d)

```
void Push(ItemType X, Stack *S)
{
   if (S->Count == MAXSTACKSIZE){
      printf(“attempt to push new item on a full
stack”);
   } else {
      S->Items[S->Count]=X;
      ++(S->Count);
   }
}
```

The Implementation Based on Linked Lists

```
/* This is the file StackTypes.h   */

typedef char ItemType;
/* char is the type for our first application */
/* float is the type for our second application */

typedef struct StackNodeTag {
           ItemType Item;
           struct StackNodeTag *Link;
        } StackNode;

typedef struct {
           StackNode *ItemList;
        } Stack;
```

The Implementation Based on Linked Lists (cont’d)

```
/* This is the file StackImplementation.c */

#include <stdio.h>
#include <stdlib.h>
#include “StackInterface.h”

void InitializeStack(Stack *S)
{
   S->ItemList=NULL;
}

int Empty(Stack *S)
{
   return (S->ItemList==NULL);
}

int Full(Stack *S)
{
  return 0;
}
/* We assume an already constructed stack is not full since it can potentially */
/* grow as a linked structure  */
```

The Implementation Based on Linked Lists (cont'd)

```
void Push(ItemType X, Stack *S)
{
   StackNode *Temp;

   Temp=(StackNode *) malloc(sizeof(StackNode));

   if (Temp==NULL){
      printf("system storage is exhausted");
   } else {
      Temp->Link=S->ItemList;
      Temp->Item=X;
      S->ItemList=Temp;
   }
}
```

The Implementation Based on Linked Lists (cont'd)

```
void Pop(Stack *S, ItemType *X)
{
   StackNode *Temp;

   if (S->ItemList==NULL){
     printf("attempt to pop the empty stack");
   } else {
      Temp=S->ItemList;
      *X=Temp->Item;
      S->ItemList=Temp->Link;
      free(Temp);
   }
}
```

Information Hiding Revisited

- The two previous specifications of the ADT stack **do not hide the details of the representation** of the stack since a client program can access the array or the list data structure because it includes `StackInterface.h` and therefore `StackTypes.h`.
- We will now present another specification which does a better job in hiding the representation of the stack.

The Interface File `STACK.h`

```
void STACKinit(int);
int STACKempty();
void STACKpush(Item);
Item STACKpop();
```

The type `Item` will be defined in a header file `Item.h` which will be included in the implementation of the interface and the client programs.

The Implementation of the Interface

- As previously, we will consider an array implementation and a linked list implementation of the ADT stack.

The Array Implementation

```
#include <stdlib.h>
#include "Item.h"
#include "STACK.h"
static Item *s;
static int N;

void STACKinit(int maxN)
{ s = malloc(maxN*sizeof(Item)); N = 0; }

int STACKempty()  { return N == 0; }

void STACKpush(Item item)
{ s[N++] = item; }

Item STACKpop()  { return s[--N]; }
```

Notes

- The variable `s` is a pointer to an item (equivalently, the name of an array of items defined by `Item s[]`).
- When there are `N` items in the stack, the implementation keeps them in array elements `s[0], …, s[N-1].`
- The variable `N` shows the top of the stack (where the next item to be pushed will go).
- `N` is defined as a **static** variable i.e., it **retains its value throughout calls** of the various functions that access it.
- The client program passes the maximum number of items expected on the stack as an argument to `STACKinit`.
- The previous code does not check for errors such as pushing onto a full stack or popping an empty one.

The Linked List Implementation

```
#include <stdlib.h>
#include "Item.h"

typedef struct STACKnode* link;
struct STACKnode { Item item; link next; };
static link head;

link NEW(Item item, link next)
{ link x = malloc(sizeof *x);
  x->item = item;
  x->next = next;
  return x;
}

void STACKinit(int maxN)  { head = NULL; }

int STACKempty()
{ return head == NULL; }

STACKpush(Item item)
{ head = NEW(item, head); }

Item STACKpop()
{ Item item = head->item;
  link t = head->next;
  free(head);
  head = t;
  return item;
}
```

Notes

- This implementation uses an auxiliary function `NEW` to allocate memory for a node, set its fields from the function arguments, and return a link to the node.
- In this implementation, we keep the stack in the reverse order of the array implementation; from most recently inserted elements to least recently inserting elements.
- **Information hiding:** For both implementations (with arrays or linked lists), the data structure for the representation of the stack (array or linked list) is defined **only** in the implementation file thus it is not accessible to client programs.

Translating Infix Expressions to Postfix

- Let us now use the latest implementation of the stack ADT to implement a translator of **fully parenthesized** infix arithmetic expressions to postfix.
- The **algorithm** for doing this is as follows. To convert (A+B) to the postfix form AB+, we ignore the left parenthesis, convert A to postfix, save the + on the stack, convert B to postfix, then, on encountering the right parenthesis, pop the stack and output the +.

Example

- We want to translate the infix expression `((5*(9+8))+7)` into postfix.
- The result will be `5 9 8 + * 7 +`.

Executing the Algorithm

Input	Output	Stack
(		
(		
5	5	
*		*
(		*
9	9	*
+		* +
8	8	* +
)	+	*
)	*	
+		+
7	7	+
)	+	

The Client Program

```
#include <stdio.h>
#include <string.h>
#include "Item.h"
#include "STACK.h"

main(int argc, char *argv[])
{
    char *a = argv[1];
    int i, N = strlen(a);

    STACKinit(N);
    for (i = 0; i < N; i++)
    {
       if (a[i] == ')')  printf("%c ", STACKpop());
       if ((a[i] == '+')  ||  (a[i] == '*'))   STACKpush(a[i]);
       if ((a[i] >= '0')  && (a[i] <= '9'))  printf("%c ", a[i]);
    }
    printf("\n");
}
```

The File `Item.h`

- The file `Item.h` can only contain a `typedef` which defines the type of items in the stack.
- For the previous program, this can be:

```
typedef char Item;
```

A Weakness of the 2nd Solution

- The 2nd solution for defining and implementing a stack ADT is weaker than the 1st one since it allows the construction and operation of a **single stack** by a client program.
- Conversely, the 1st solution allows us to define **many stacks** in the client program.

Exercise

- Modify the 1st solution so that it does better information hiding without losing the capability to be able to define many stacks in the client program.

Question

- Which implementation of a stack ADT should we prefer?

Answer

- It depends on the application.

- In the linked list implementation, push and pop take more time to allocate and de-allocate memory.
- If we need to do these operations a huge number of times then we might prefer the array implementation.
- On the other hand, the array implementation uses the amount of space necessary to hold the maximum number of items expected. This can be wasteful if the stack is not kept close to full.
- The list implementation uses space proportional to the number of items but always uses extra space for a link per item.

- Note also that the **running time** of push and pop in each implementation is **constant**.

How C Implements Recursive Function Calls Using Stacks

- When calling an instance of a function $F(a_1,a_2,\ldots,a_n)$ with actual parameters $a_1, a_2,\ldots,a_n$, C uses a **run-time stack**.
- A collection of information called a **stack frame** or **call frame** or **activation record** is prepared to correspond to the call and it is placed on top of other previously generated stack frames on the run-time stack.

Stack Frames

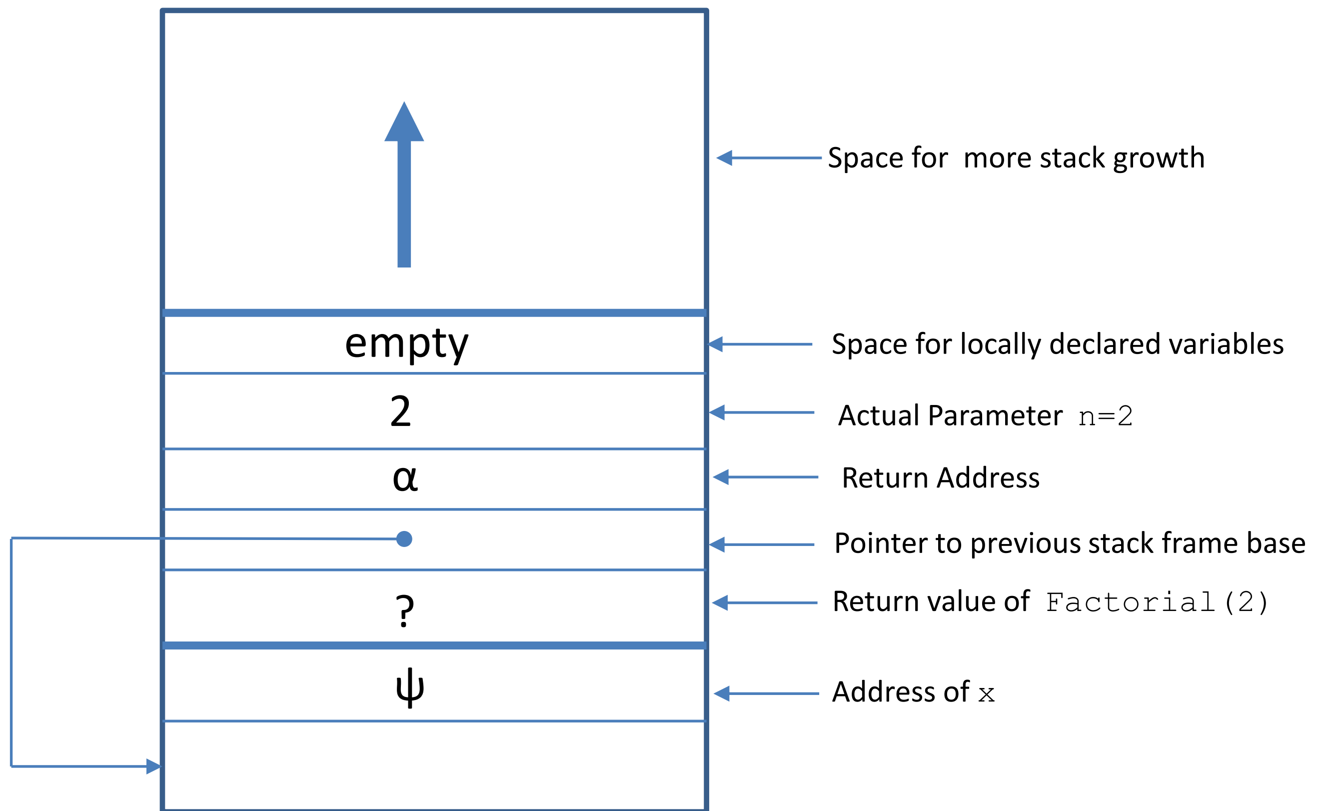
- The information in a stack frame consists of:
  - Space to hold the **value returned by the function**.
  - A **pointer to the base of the previous stack frame** in the stack.
  - A **return address**, which is the address of an instruction to execute in order to resume the execution of the caller of the function when the call has terminated.
  - Parameter storage sufficient to hold **the actual parameter values** used in the call.
  - A set of storage locations sufficient to hold the values of the **variables declared locally** in the function.

Example - Factorial

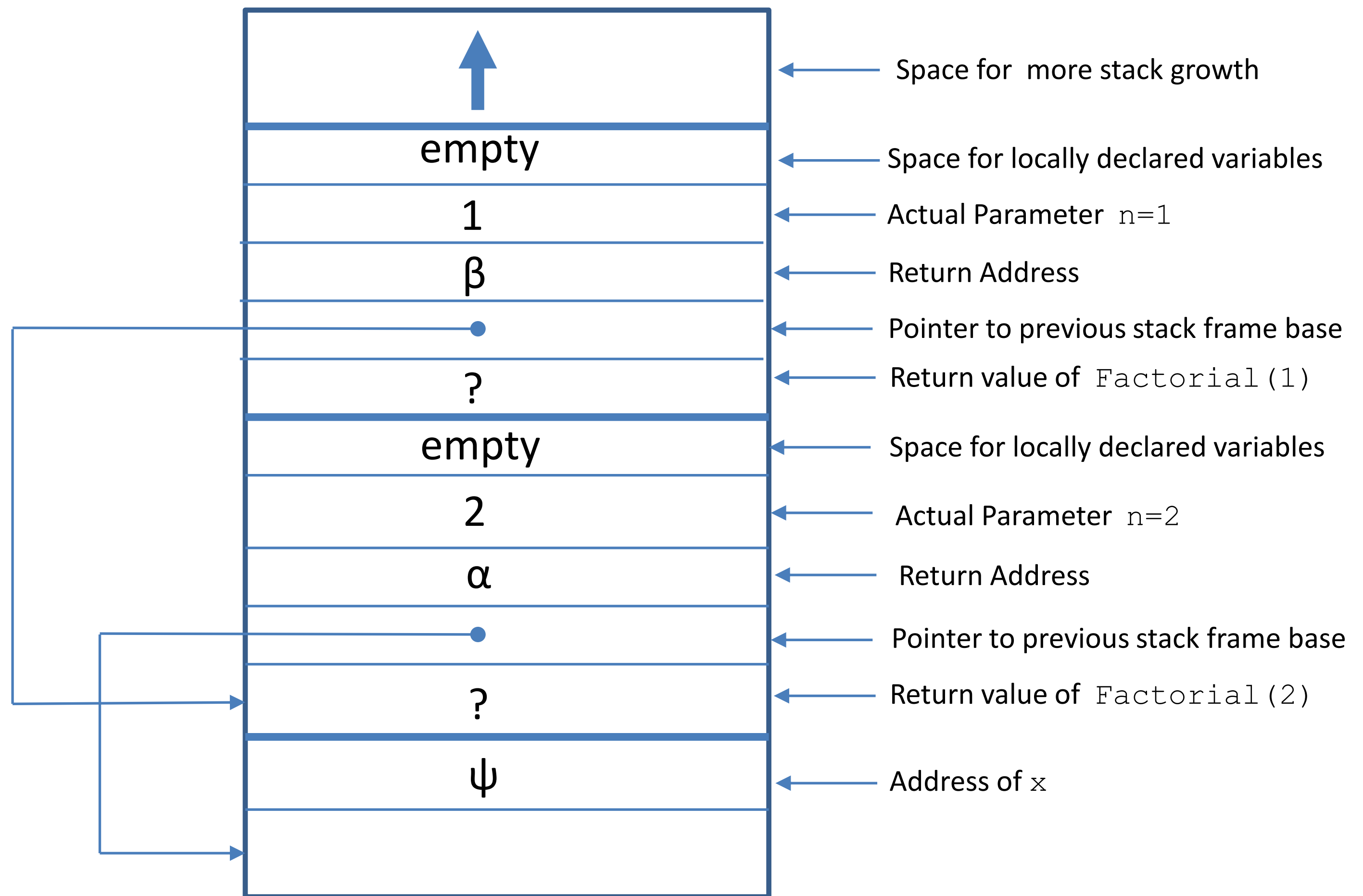
```
int Factorial(int n);
{
   if (n==1) {
      return 1;
   } else {
      return n*Factorial(n-1);
   }
}
```

Let us consider the call `x=Factorial(2).`

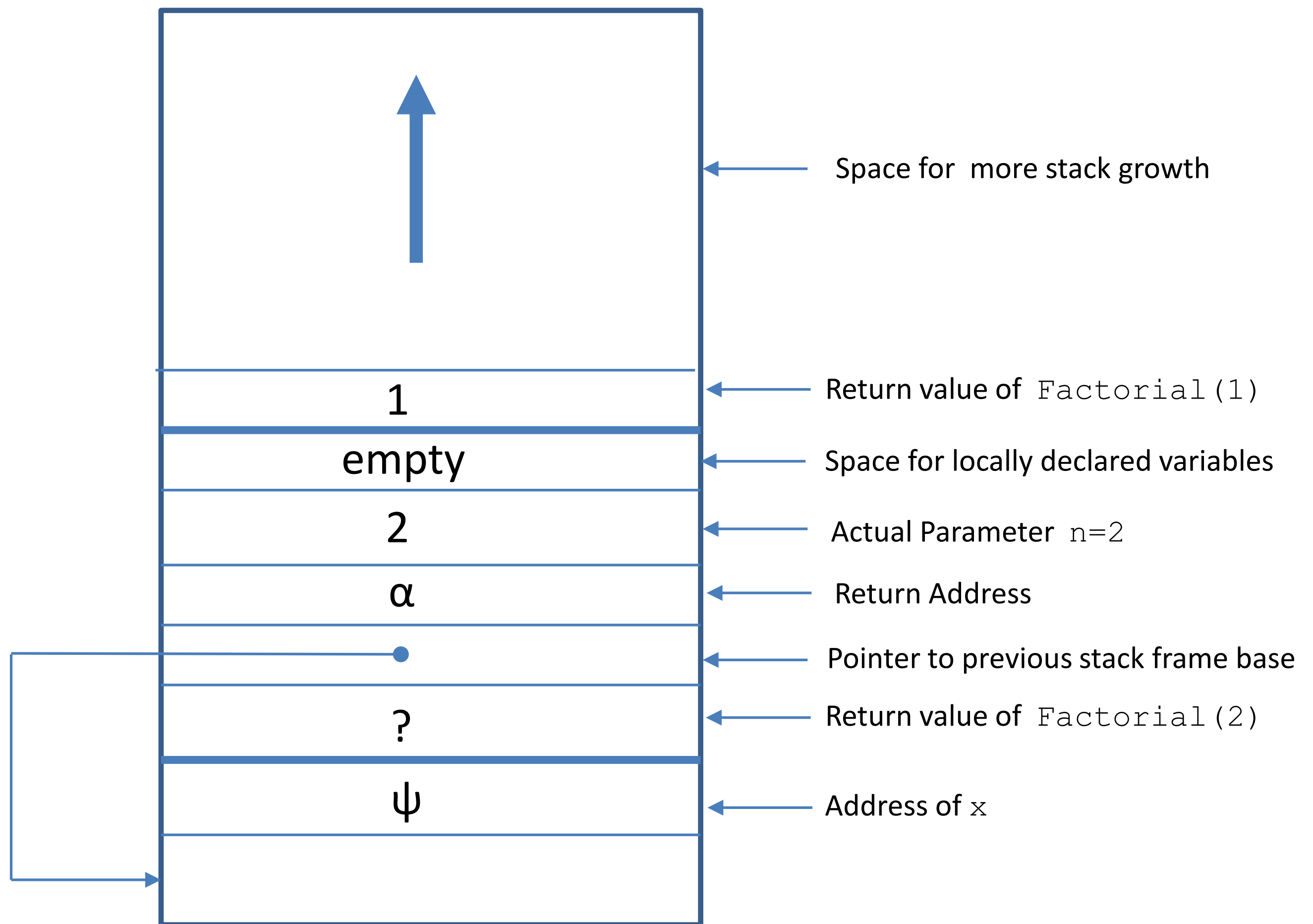
Stack Frame for `Factorial(2)`



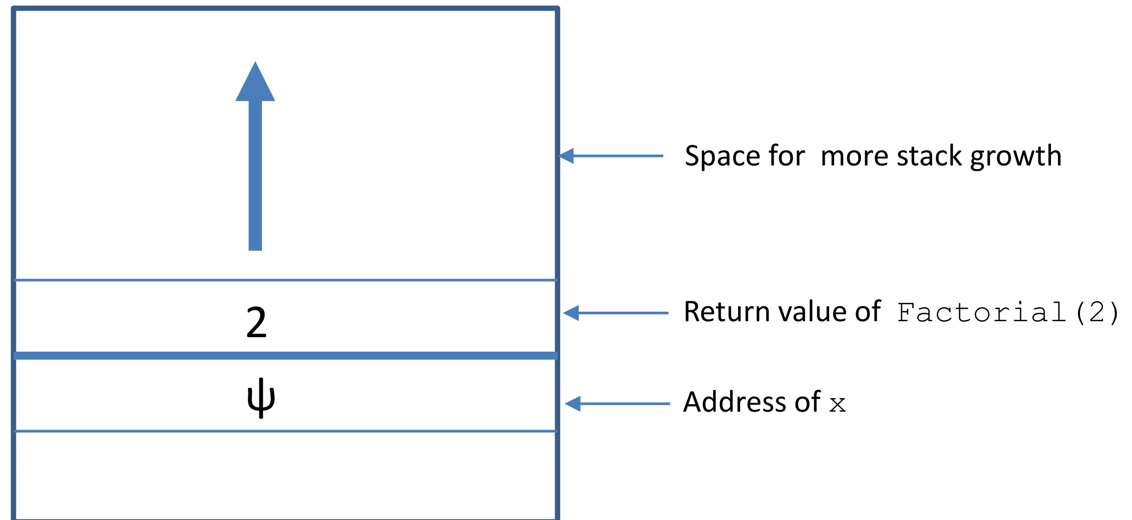
Stack Frame for `Factorial(2)` and `Factorial(1)`



Stack After Return from `Factorial(1)`



Stack After Return from `Factorial(2)`



More Details

- **Stack + Iteration** can implement **Recursion**.

- Run-time stacks are discussed in more details in a Compilers course.

Using Stacks

- Generally speaking, stacks can be used to implement any kind of **nested structure**.
- When processing nested structures, we can start processing the outermost level of the structure, and if we encounter a nested substructure, we can interrupt the processing of the outer layer to begin processing an inner layer by putting a record of the interrupted status of the outer layer's processing on top of a stack.
- In this way the stack contains **postponed obligations** that we should resume and complete.

Readings

- T. A. Standish. *Data Structures, Algorithms and Software Principles in C*.

  Chapter 7.
- R. Sedgewick. Αλγόριθμοι σε C.

  Κεφ. 4